SIMILARITY SOLUTION OF THE RAYLEIGH PROBLEM FOR NON-NEWTONIAN MHD FLUID PAST SEMI-INFINITE PLATE

Pankaj Sonawane¹, M. G. Timol² and J.N. Salunke³

1. Department of Mathematics, Dhanaji Nana Mahavidyalaya, Faizpur, Dist Jalgaon, India
2. Department of mathematics, Veer Narmad South Gujarat University, Surat.
3. Department of Mathematics, Swami Ramanand Tirth Marathawada University, Nanded.

Abstract

The group theoretic transformation technique is applied to study the MHD of Rayleigh problem past semi-infinite plate impulsively set into a motion, moves with continuous velocity in a non-Newtonian power law fluid. The similarity equation is derived using one parameter group transformation to the Rayleigh problem for a power law non-Newtonian conducting fluid. This similarity equation which is highly non-linear ordinary differential equation is solved numerically using non-linear finite difference method. The results obtained are found in good agreement with those available in literature.

Keywords: Rayleigh problem, Group-theoretic method, MHD.

Introduction

Stokes’ first problem for the flat plate originated in 1851, and is also known as the Rayleigh–Stokes problem. This problem for non-Newtonian fluid has received much attention due to its practical applications in industry, geophysics, chemical and petroleum engineering [1]. Some investigations are notably important in industries related to paper, food stuff, personal care products, textile coating and suspension solutions. Salah Faisal et al. [2] investigated new exact solution for Rayleigh-Stokes problem of Maxwell fluid in porous medium and rotating frame. Parmar et al. [3] investigated group theoretic analysis of Rayleigh problem for a non-Newtonian MHD Sisko fluid past semi infinite plate. Rossow [4] was probably first to study magnetic Rayleigh problem, where a semi finite plate is given an impulsive motion and there after moves a constant velocity in a Newtonian fluid of infinite extend. He has studied both the case: when transfers magnetic field is fixed to the plate and fixed to the fluids. In 1970, Sapunkov [5] studied non Newtonian flow of electrically conducting fluids. He obtained approximate solution to the problem solved in his paper but only in the special case of very strong or very weak magnetic fields. The solution was obtained only for a power law fluid for \( n = 2 \).

In fact, the symmetries of a differential equation are those continuous group of transformations under which the differential equation remains invariant, that is, a symmetry group maps any solution to another solution. The interesting point is that, having obtained the symmetries of a specific problem, one can proceed further to find out the group invariant solutions, which
are nothing but the well-known similarity solutions. The similarity solutions are quite popular because the result in the reduction of the independent variables of the problem. Sonawane and Timol [6] investigated similarity solution for electrically conducting non-Newtonian fluids over a vertical porous-elastic surface.

Now a day for similarity analysis many techniques are available, among them the similarity methods which invoke the invariance under the group of transformations are known as group theoretic methods. These methods are more recent and are mathematically elegant and hence they are widely used in different fields. It was first reported by Birkhoff [7] and later a number of authors like Hansen [8], Bluman and Cole [9], Seshadri and Na [10] have contributed much to the development of the theory. The method has been applied intensively by Hansen and Na [11], Timol and Kalthia [12], Pakdemirli [13], Patil and Timol [14], Nita Jain et al [15].

Abd-El-Malek et al. have studied [16], the solution of the Rayleigh problem for power law non-Newtonian conducting fluid. Motivated by that study, in the present paper we present solution of the Rayleigh–Stokes problem for a power law non-Newtonian fluid via one parameter linear group theoretic technique. Under the transformation, the partial differential equation with boundary conditions is reduced to an ordinary differential equation with appropriate corresponding conditions. The transformed ordinary equation is then solved numerically for various parameters and effect of the parameters on the velocity of fluid studied and result are plotted.

**Mathematical Formulation**

Consider the equation of the motion of the semi-infinite flat plate in the infinite power law non-Newtonian fluid of the form:

\[
\frac{\partial^2 u}{\partial t^2} - \nu \left( \frac{\partial u}{\partial y} \right)^{n-1} \left( \frac{\partial^2 u}{\partial y^2} \right) + MH^2 u = 0
\]  \hspace{1cm} (1.1)

The boundary conditions and initial conditions are

\[
u(0,t) = V, \quad t > 0
\]

\[
u(\infty,t) = 0, \quad t > 0
\]

\[
u(y,0) = 0, \quad y > 0
\]

Equation (1.1) can be written as,

\[
\frac{\partial u}{\partial t} - \nu \left( \frac{\partial u}{\partial y} \right)^{n-1} \left( \frac{\partial^2 u}{\partial y^2} \right) + MH^2 u = 0
\]  \hspace{1cm} (1.2)

Assume

\[
u(y,t) = \nu F(y,t)
\]  \hspace{1cm} (1.3)

Where \( F(y,t) \) unknown function and its proper form will be determined.

Substitution from (1.3) into (1.2), we get

\[
\frac{\partial F}{\partial t} - \nu \nu^{n-1} \left( \frac{\partial F}{\partial y} \right)^{n-1} \left( \frac{\partial^2 F}{\partial y^2} \right) + MH^2 F = 0
\]  \hspace{1cm} (1.4)

The boundary conditions and initial conditions are

\[
u(0,t) = 1, \quad t > 0
\]

\[
u(y,0) = 0, \quad y > 0
\]

\[
u(\infty,t) = 0, \quad t > 0
\]  \hspace{1cm} (1.5)
Method of Solution

First introduced a one parameter group transformation of the form

\[ \tilde{t} = p^a t^b, \quad \tilde{y} = p^a y^b \]
\[ \tilde{F} = p^a F^b, \quad \tilde{H} = p^a H \quad (1.6) \]

Where \( a_1, a_2, a_3, a_4 \) and \( P \) are Constants

Applying above linear group transformation given by equation (1.6) in to the equation (1.4-1.5) result in,

\[ \frac{\partial F}{\partial t} - \alpha_1 \frac{\partial F}{\partial t} - \alpha_2 \gamma^{n-1} (\varphi' - \alpha_3 \gamma^{n-1} \int \frac{\partial F}{\partial \gamma})^{n-1} \]
\[ p^{2 \alpha_2 - \alpha_3} \frac{\partial^2 F}{\partial \gamma^2} + M p^{2 \alpha_2} H^2 p^{\alpha_3} = 0 \quad (1.7) \]

The differential equation are completely invariant to the proposed linear transformation, the following coupled algebraic equations are obtained

\[ \frac{\alpha_2}{\alpha_1} = - \frac{1}{2}, \quad \frac{\alpha_2}{\alpha_2} = \frac{1}{n+1}, \quad \alpha_3 = 0 \quad (1.8) \]

Introducing equation (1.8) in to equation (1.6) result in

\[ \eta = \frac{\tilde{y}}{t^{\frac{1}{n+1}}} , \quad F = \varphi(\eta), \quad H(\eta) = g(\eta) t^{-\frac{1}{2}} \quad (1.9) \]

Using the relation (1.9), equation (1.4) reduce to

\[ -\eta \beta \varphi' (\eta) - \gamma^{n-1} \int \varphi' (\eta) (\varphi'(\eta))^{n-1} \varphi'' (\eta) \]
\[ + M H^2 t \varphi(\eta) = 0 \quad (1.10) \]

From equation (1.10), we get

\[ n \gamma^{n-1} \varphi'' (\eta) (\varphi' (\eta))^{n-1} \gamma^{-\frac{1}{2}} \beta \varphi (\eta) \]
\[ - \gamma \beta \varphi' (\eta) - M E^2 \varphi (\eta) = 0 \quad (1.11) \]

For (1.11) to be reduced to an ordinary differential equation in one variable \( \eta \), it is necessary that the coefficients should be constants or functions of \( \eta \) only.

\[ H^2 t = E^2 \quad (1.12) \]
\[ \beta = \frac{1}{n+1} \quad (1.13) \]

Hence equation (1.11) as

\[ n \gamma^{n-1} \varphi'' (\eta) (\varphi' (\eta))^{n-1} + \gamma \beta \varphi' (\eta) - N \varphi (\eta) = 0 \quad (1.14) \]

where \( w = \gamma^{n-1} \) and \( N = ME^2 \quad (1.15) \)

Under the similarity variable boundary conditions \( \eta \), the boundary conditions (1.5) are

\[ \varphi (\eta) = 1, \quad \eta = 0 \]
\[ \varphi (\eta) = 0, \quad \eta = \infty \quad (1.16) \]

Result and Discussions

1. Effect of \( N \) on the normalized velocity

Consider \( n = 1, w = 0.1 \) and \( t = 1 \).

The result for different values of \( N \) is plotted in figure (1.1).
Figure (1.1) shows that, the velocity of the fluid flow increases as the constant \( N \) decreases.

2. **Effect of \( w \) on the normalized velocity**

Consider \( n = 1, w = 0.1 \) and \( t = 1 \).

The result for different values of \( w \) is plotted in figure (1.2)

![Fig. (1.2) Effect of \( w \) on the normalized velocity](image)

Figure (1.2) shows that, the velocity of the fluid flow increases as \( w \) increases.

3. **Effect of \( n \) on the normalized velocity**

Consider \( n = 1, N = 3 \) and \( w = 0.1 \).

The result for different values of \( n \) is plotted in figure (1.3)

![Fig. (1.3) Effect of \( n \) on the normalized velocity](image)

Figure (1.3) shows that, the velocity of the fluid flow increases with increase of time \( t \).

4. **Effect of \( t \) on the normalized velocity**

Consider \( t = 1, N = 3 \) and \( w = 0.1 \).

The result for different values of \( t \) is plotted in figure (1.4)

![Fig. (1.4) Effect of \( n \) on the normalized velocity](image)

Figure (1.4) shows for \( n = 1 \), the behavior of the fluid is Newtonian and for \( n = 2 \), the behavior of the fluid is Dilatant.

**Conclusion**

In the present paper, similarity solution is produced for a Rayleigh problem for non-Newtonian MHD fluid past semi-infinite plate. The similarity equation is derived using one parameter group transformation to the Rayleigh problem for a power law non-Newtonian conducting fluid. This similarity equation which is highly non-linear ordinary differential equation is solved numerically. An interesting effect of \( N, w, \) and \( n \) on the normalized velocity is observed.
References


