

Vibration Analysis of Cracked Cantilever Beam With Suitable Boundary Conditions

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Abstract

Vibration analysis of a beam is an important and peculiar subject of study in mechanical engineering. Many developments have been carried out in order to try to quantify the effects produced by dynamic loading. Examples of structures where it is particularly important to consider dynamic loading effects are the construction of tall buildings, long bridges under wind-loading conditions and buildings in earthquake zones, etc. Dynamic structures subjected to periodic loads compose a very important part of industrial machineries. One of the major problems in these machineries is the fatigue and the cracks initiated by the fatigue. These cracks are the most important cause of accidents and failures in industrial machinery. In addition, existing of the cracks may cause vibration in the system. Thus an accurate and comprehensive investigation about vibration of cracked dynamic structures seems to be necessary. On the base of these investigations the cracks can be identified well in advance and appropriate measures can be taken to prevent more damage to the system due to the high vibration level. Typical situations where it is necessary to consider more precisely the response produced by dynamic loading are vibrations due to equipment or machinery, impact load produced by traffic, snatch loading of cranes, impulsive load produced by blasts, earthquakes or explosions. So it is very important to study the dynamic nature of structures.

Keywords: *Vibration, crack, FEM, CEM*

1. Introduction

Beams are fundamental models for the structural elements of many engineering applications and have been studied extensively. There are many examples of structures that may be modeled with beam-like elements, for instance, long span bridges, tall buildings, and robot arms, beams as well as the presence of cracks in the structural components can have a significant influence on the dynamic responses of the whole structure; it can lead to the catastrophic failure of the structure. To predict the Failure, vibration monitoring can be used to detect changes in the dynamic responses and/or dynamic characteristics of the structure. Knowledge of the effects of cracks on the vibration of the structure is of importance. Efficient techniques for the forward analysis of cracked beams are required. In this

paper various techniques or approaches that can analyze the vibration of beams or structures with or without cracks.

A promising approach for developing a solution for structural vibration problems is provided by an advanced numerical discretisation scheme, such as; finite element method (FEM). The finite element method (FEM) is the dominant discretization technique in structural mechanics. The differential quadrature method (DQM) was first advanced by Bellman and his associates in the early 1970s aiming towards offering an efficient numerical method for solving non-linear partial differential equations. The method has since been

applied successfully to various problems. In third order shear deformation theory free vibration of beams with different boundary conditions is analyzed. The boundary conditions of beams are satisfied by using Lagrange multipliers.

Fourier series will be utilized for the solution of simply supported beams with different loadings in order to arrive at a free vibration. The equation of the free vibration is $\{(\delta^2 y / \delta t^2) / (\delta^4 y / \delta x^4)\}$. One of the methods of solving this type of equation is the separation of the variables which assumes that the solution is the product of two functions, one defines the deflection shape and the other defines the amplitude of vibration with time. Modes of deflection with and without time along the beam were drawn for certain cases. To this end, the composite element method is then extended for free and forced vibration analysis of cracked beams. The principal advantage of the proposed method is that it does not need to partition the stepped beam into uniform beam segments between any two successive discontinuity points and the whole beam can be treated as a uniform beam. Moreover, the presented work can easily be extended to cracked beams with an arbitrary number of non-uniform segments.

2. Introduction to Crack

A crack in a structural member introduces local flexibility that would affect vibration response of the structure. This

property may be used to detect existence of a crack together its location and depth in the structural member. The presence of a crack in a structural member alters the local compliance that would affect the vibration response under external loads.

1.4.1 Classification of Crack

Based on geometries, cracks can be broadly classified as follows:

- (1) **Transverse crack** : These are cracks perpendicular to beam axis. These are the most common and most serious as they reduce the cross section as by weaken the beam. They introduce a local flexibility in the stiffness of the beam due to strain energy concentration in the vicinity or crack tip.
- (2) **Longitudinal cracks**: These are cracks parallel to beam axis. They are not that common but they pose danger when the tensile load is applied at right angles to the crack direction i.e. perpendicular to beam axis.
- (3) **Open cracks** : These cracks always remain open. They are more correctly called “notches”. Open cracks are easy to do in laboratory environment and hence most experimental work is focused on this type of crack
- (4) **Breathing crack** : These are cracks those open when the affected part of material is subjected to tensile stress and close when the stress is reversed. The component is most influenced when under tension. The breathing of crack results in non-linearity in the vibration behavior of the beam. Most theoretical research efforts are concentrated on “transverse breathing” cracks due to their direct practical relevance.
- (5) **Slant cracks** : These are cracks at an angle to the beam axis, but are not very common. Their effect on lateral vibration is less than that of transverse cracks of comparable severity.
- (6) **Surface cracks** : These are the cracks that open on the surface. They can normally be detected by dye-penetrates or visual inspection.
- (7) **Subsurface cracks** : Cracks that do not show on the surface are called subsurface cracks. Special techniques such as ultrasonic, magnetic particle, radiography or shaft voltage drop are needed to detect them.

3. Introduction to Beam

A beam is generally considered to be any member subjected to principally to transverse gravity or vertical loading. The term transverse loading is taken to include end moments.

There are many types of beams that are classified according to their size, manner in which they are supported, and their location in any given structural system.

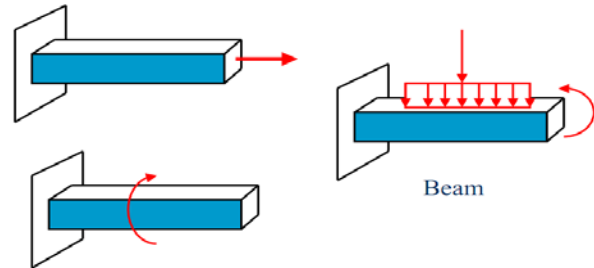


Fig. 1 Loading on beams.

Beams can be Straight as shown in Figure 1.

- For example the straight member bde. Curved as shown in c.

- For example the curved member abc.

Beams are generally classified according to their geometry and the manner in which they are supported.

Geometrical classification includes such features as the shape of the cross section, whether the beam is Straight or – Curved Or whether the beam is Tapered, or – Has a constant cross section. Beams can also be classified according to the manner in which they are supported. Some types that occur in ordinary practice are shown in Figure 2, the names of some of these being fairly obvious from direct observation. Note that the beams in (d), (e), and (f) are statically indeterminate. A beam is a horizontal structural member used to support loads. Beams are used to support the roof and floors in buildings,

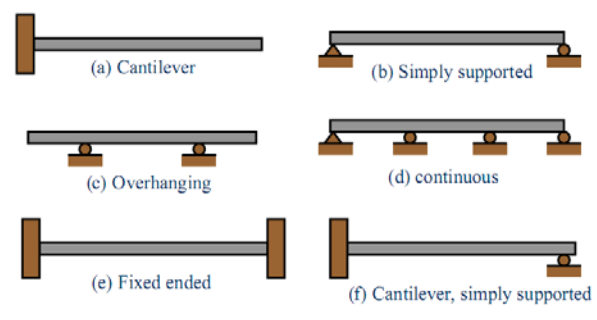


Fig. 2 Types of Beams based on the manner in which they are supported.

Common materials are steel and wood. The parallel portions on an I-beam or H-beam are referred to as the *flanges*. The portion that connects the flanges is referred to as the *web*. Beams are supported in structures via different configurations. Beams are designed to support various types of loads and forces.

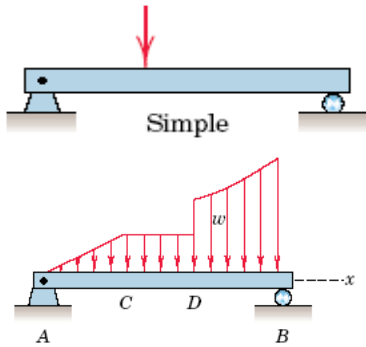


Fig. 3 Load on Beam, a) Concentrated Load, (b) Distributed Load

4. Experimental Analysis of Cracked Beam

The PULSE software analysis was used to measure the frequency ranges to which the foundations of various machines are subjected to when the machine is running with no load and full load. This will help us in designing the foundations of various machines in such a way that they are able to resist the vibration caused in them. Figure 4 shows photograph for pulse analyser and display unit. Table 1 gives the different specifications for the required beam. Table 2 gives the natural frequencies for the uncracked beam by using Finite Element method and by using Experimental method. The % variation between these two are less than 10%. so the validation is approved for the experimentation.

Table 3 gives the natural frequencies for the cracked beam for first four modes. The crack depths are taken 2mm, 6mm and 8mm and different locations on the beam. Figure 4 and 5 shows the frequency response curves for the different conditions. Figure 7 shows the graphical presentation for the natural frequencies for the Finite Element Method and Experimental method for different modes for uncracked beam. Figure 8 and 9 shows the graphical presentation for the natural frequencies for the Finite Element Method and Experimental method for different modes for cracked beam, crack at different locations.



Fig.4. Pulse Analyser and Display Unit

Table 1: Beam specification

Software used	FFT analyzer and accessories, pulse lab shop version 9.0
Parameter	Frequency
Length of cantilever	20 cm
Section dimintions	0.0095 X 0.0095 m ²
Boundary conditions	One end fixed and another free
Material	Aluminum
Mass density	2659 kg m ³
Elastic modulus	68.0 E09 N m ²
Poison's ratio	0.205

Table 2: Natural Frequencies for Beam without crack:-

Mode	Frequency (Hz)(by FEM)	Frequency (Hz)(by Experiment method)	Percentage of error
First	197.42	187.00	5.27%
Second	1227.66	1180.00	3.88%
Third	3393.65	3308.00	2.52%
forth	6547.61	6425.00	1.87%

Table 3: : Natural Frequencies (Hz) for Beam with crack:-

		Crack depth	1st mode	2nd mode	3rd mode	4th mode
1.	Cracks at center	2mm	144	928	2088	2792
2.		6mm	136	887	2064	2744
3.		8mm	48	560	1448	2744
4.	Crack at 0.25L	2mm	128	960	1936	2832
5.		6mm	112	876	1720	2456
6.		8mm	88	448	856	1736
7.	No crack	nil	118	756	3308	6425

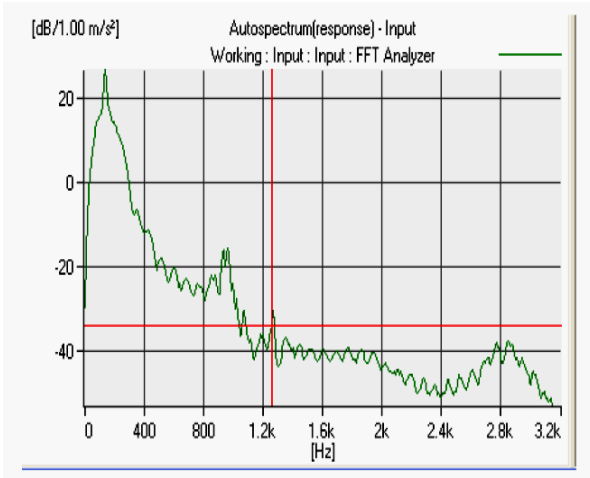


Fig.5. Frequency response for 2mm crack at 0.25 times Length of beam

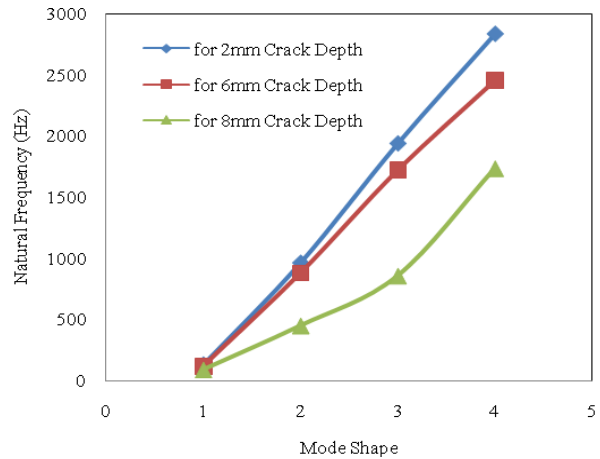


Fig.8. Mode shape vs natural frequency for cracked beam (Crack at 0.25 times length from Fixed end)

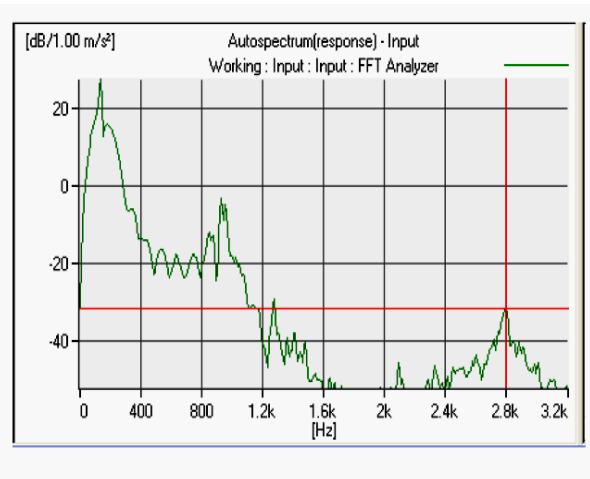


Fig.6. Frequency response for 2mm crack at 0.5 times Length of beam

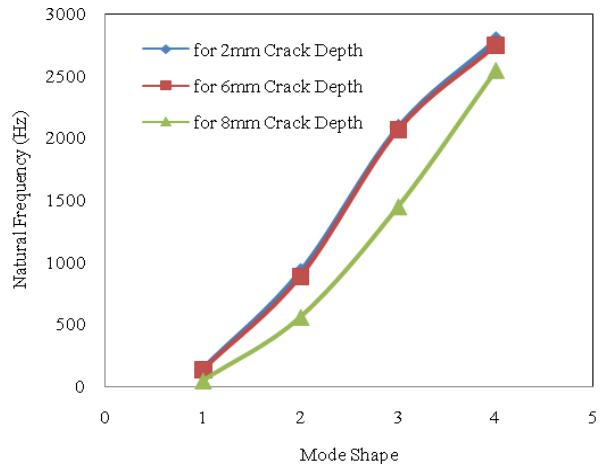


Fig.9. Mode shape vs natural frequency for cracked beam (Crack at Center of beam)

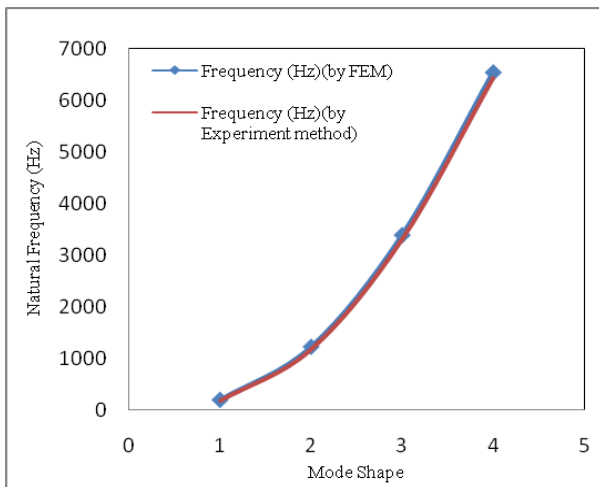


Fig.7. Mode shape vs natural frequency for uncracked beam

5. Conclusions

The vibration analysis of a structure holds a lot of significance in its designing and performance over a period of time. The verification of the analytical approach with a considerable amount of experimental data and with the results of calculations showed that the analytical approach enables one to obtain well-founded relationships between different dynamic characteristics and crack parameters and to solve the inverse problem of damage diagnostics with sufficient accuracy for practical purposes.

In case of cracks the frequencies of vibration of cracked beams decrease with increase of crack depth for crack at any particular location due to reduction of stiffness. The effect of crack is more pronounced near the fixed end than

at far free end. The first natural frequency of free vibration decreases with increase in number of cracks. The natural frequency decreases with increase in relative crack depth. The results obtained are accurate and are expected to be useful to other researchers for comparison. The study in this work is necessary for a correct and thorough understanding of the Vibration analysis techniques.

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