IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 1 Issue 10, December 2014.
www.ijiset.com
ISSN 2348 – 7968

Some Results on Divisor Cordial Labeling of Graphs

P. Lawrence Rozario Raj[1] and R. Lawrence Joseph Manoharan[2]

[1] Assistant Professor, P.G. and Research Department of Mathematics, St. Joseph's College, Tiruchirappalli – 620 002, Tamil Nadu, India.

[2] Dean of Computational Studies and Head of the Department of Mathematics, Hindustan College of Arts and Science, Chennai – 603 103, Tamil Nadu, India.

Abstract

In this paper, the divisor cordial labeling of $S'(K_{2,m})$, $S'(K_{1,n,n})$, double fan P_n+2K_1, cone C_n+2K_1, Jewel graph J_n, $P_m(+)\overline{K}_n$, $(\overline{K}_n \cup P_m)+2K_1$ and $(P_n \cup P_m)+2K_1$ are presented.

Keywords: *Cordial labeling, Cordial Graphs, Divisor Cordial labeling, Divisor Cordial Graphs.*

1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [4]. For standard terminology and notations related to number theory we refer to Burton [2] and graph labeling, we refer to Gallian [3]. In [1], Cahit introduce the concept of cordial labeling of graph. In [14], Varatharajan et al. introduce the concept of divisor cordial labeling of graph. The divisor cordial labeling of various types of graph are presented in [5-13,15]. The brief summaries of definition which are necessary for the present investigation are provided below.

2. Definitions

Definition :2.1

A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph.

Definition :2.2

A mapping $f : V(G) \rightarrow \{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f. If for an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = | f(u) - f(v) |$. Then $v_f(i)$ = number of vertices of having label i under f and $e_f(i)$ = number of edges of having label i under f*.

Definition :2.3

A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$. A graph G is cordial if it admits cordial labeling.

Definition :2.4

Let a and b be two integers. If a divides b means that there is a positive integer k such that b = ka. It is denoted by a | b. If a does not divide b, then we denote $a \nmid b$.

Definition :2.5

Let G = (V(G), E(G)) be a simple graph and $f : V(G) \rightarrow \{1,2,...,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if f(u) | f(v) or f(v) | f(u) and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph with a divisor cordial labeling is called a divisor cordial graph.

Definition :2.6

For a graph G, the splitting graph S′(G) of a graph G is obtained by adding a new vertex v′ corresponding to each vertex v of G such that N(v) = N(v′).

Definition :2.7

The graph $P_m(+)\overline{K}_n$ is a graph with the vertex set V(G) = $\{u_i, v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and the edge set E(G) = $\{u_iu_{i+1}, u_1v_j, u_mv_j : 1 \leq i \leq m-1, 1 \leq j \leq n\}$.

Definition :2.8

The join $G_1 + G_2$ of G_1 and G_2 consists of $G_1 \cup G_2$ and all lines joining V_1 with V_2. The graph $P_n + 2K_1$ is called the double fan. The graph $C_n + 2K_1$ is called the double cone.

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3. Main Results

Theorem 3.1

The graph $S'(K_{2,m})$ is divisor cordial graph.
Proof :

Let $x_1, x_2, v_1, v_2, \ldots, v_m$ be the vertices of $K_{2,m}$.

Then $x_1,x_2,v_1,v_2, \ldots, v_m, x'_1, x'_2, v'_1, v'_2, \ldots, v'_m$ are the vertices of $S'(K_{2,m})$ and $|V(G)| = 2m+4$ and $|E(G)| = 6m$.

p_1 and p_2 are the largest and next largest prime numbers.

Define $f : V(G) \rightarrow \{1, 2, 3, \ldots, 2m+4\}$ by

$f(x_1) = 1$,
$f(x_2) = p_1$,
$f(x'_1) = 2$,
$f(x'_2) = p_2$,
$f(v_i) = 2+2i$, $\quad 1 \le i \le m$.
$f(v'_m) = 2m+4$,

Label the vertices $v'_1, v'_2, \ldots, v'_{m-1}$ with odd numbers from 3, 5, …, 2m+3 other than p_1 and p_2.

Then $e_f(0) = e_f(1) = 3m$ for any m.

Therefore, $|e_f(0) - e_f(1)| \le 1$.
Hence G is divisor cordial graph.

Example 3.1

The divisor cordial labeling of $S'(K_{2,4})$ is given in figure 3.1.

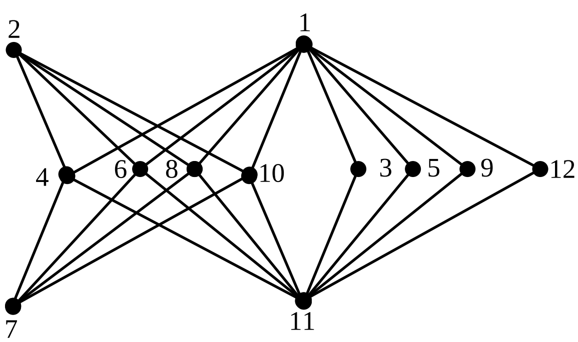

Figure 3.1

Theorem 3.2

The graph $S'(K_{1,n,n})$ is divisor cordial graph.
Proof .

Let $x, v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}, \ldots, v_{2n}$ be the vertices of $K_{1,n,,n}$.

Then $x, v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}, \ldots, v_{2n}, x', v'_1, v'_2, \ldots, v'_n, v'_{n+1}, v'_{n+2}, \ldots, v'_{2n}$ are the vertices of $S'(K_{1,n,n})$.

Then $|V(G)| = 4n+2$ and $|E(G)| = 6n$.

Define $f : V(G) \rightarrow \{1, 2, 3, \ldots, 4n+2\}$ by

$f(x) = 1$,
$f(x') = 2$,
$f(v_i) = 4+(i-1)$, $\quad 1 \le i \le n$.
$f(v_{n+i}) = 5+(i-1)$, $\quad 1 \le i \le n$.
$f(v'_i) = 6+(i-1)$, $\quad 1 \le i \le n$.
$f(v'_{n+i}) = 3+(i-1)$, $\quad 1 \le i \le n$.

Then $e_f(0) = e_f(1) = 3n$ for any n.

Therefore, $|e_f(0) - e_f(1)| \le 1$.
Hence G is divisor cordial graph.

Example 3.2

The divisor cordial labeling of $S'(K_{1,4,4})$ is given in figure 3.2.

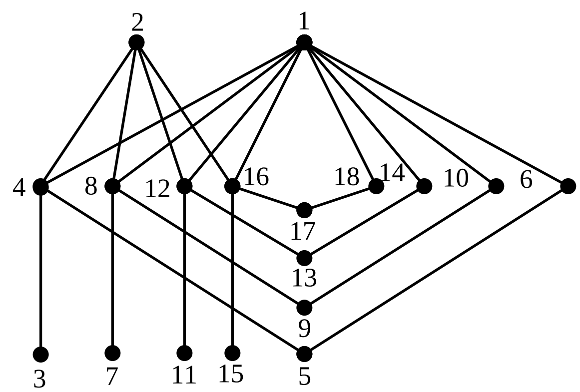

Figure 3.2

Theorem 3.3

The double fan $P_n + 2K_1$ is divisor cordial graph.
Proof.

Let $u_1,u_2,\ldots,u_n$ be the vertices of path P_n.

Let G be a graph $P_n + 2K_1$.

Let $V(G) = \{u_i,v,w : 1 \le i \le n\}$ and

$E(G) = \{ u_iu_{i+1}, vu_j, wu_j : 1 \le i \le n-1, 1 \le j \le n \}$.

Then $|V(G)| = n+2$ and $|E(G)| = 3n-1$.

Define $f : V(G) \rightarrow \{1, 2, 3, \ldots, n+2\}$ by

$f(v) = 1$,
$f(w) = 2$,
$f(u_i) = 3+ (i-1)$, $\quad 1 \le i \le n$.

Thus, n is odd, then $e_f(0) = e_f(1) = \frac{3n-1}{2}$ and n is even, then $e_f(0) = \frac{3n-2}{2}$ and $e_f(1) = \frac{3n}{2}$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.
Hence G is divisor cordial graph.

Example 3.3

The divisor cordial labeling of P_5+2K_1 is given in figure 3.3.

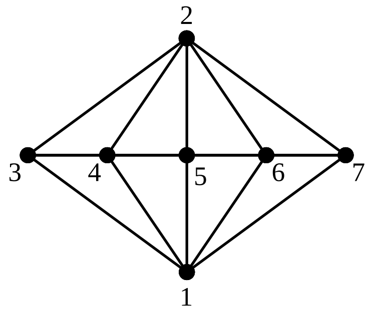

Figure 3.3

Theorem 3.4

The double cone $C_n + 2K_1$ is divisor cordial graph.

Proof.

Let $u_1,u_2,\ldots,u_n$ be the vertices of path C_n.

Let G be a graph $C_n + 2K_1$.

Let $V(G) = \{u_i,v,w: 1 \le i \le n\}$ and

$E(G) = \{ u_iu_{i+1}, vu_j, wu_j , u_nu_1 : 1 \le i \le n-1, 1 \le j \le n \}$.

Then $|V(G)| = n+2$ and $|E(G)| = 3n$.

Define $f : V(G) \rightarrow \{1, 2, 3, \ldots, n+2\}$ by

$f(v) = 1$,

$f(w) = 2$,

and $f(u_n) = p$, were p is the largest prime and $p \le n+2$.

Label the vertices $u_1,u_2,\ldots,u_{n-1}$ continuously with numbers from 3 to n+2 other than p.

Thus, n is odd, then $e_f(0) = \frac{3n+1}{2}$ and $e_f(1) = \frac{3n-1}{2}$

and n is even, then $e_f(0) = e_f(1) = \frac{3n}{2}$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.

Hence G is divisor cordial graph.

Example 3.4

The divisor cordial labeling of C_5+2K_1 is given in figure 3.4.

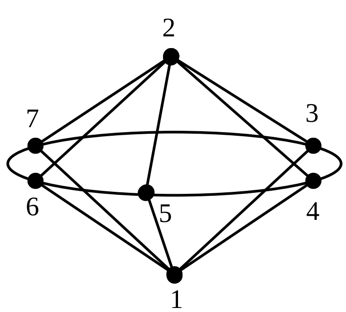

Figure 3.4

Theorem 3.5

The Jewel graph J_n is divisor cordial graph.

Proof.

Let G be Jewel graph J_n.

Let $V(J_n) = \{u, x, v, y, u_i : 1 \le i \le n\}$ and

$E(J_n) = \{ux, vx, uy, vy, xy, uu_i, vu_i : 1 \le i \le n\}$.

Then $|V(G)| = n+4$ and $|E(G)| = 2n+5$.

Define $f : V(J_n) \rightarrow \{1,2,\ldots, n+4\}$ as follows

$f(x) = 2$,

$f(y) = 3$,

$f(u) = 1$,

and $f(v) = p$, where p is the largest prime number and $p \le n+4$ and label the vertices $u_1, u_2, \ldots, u_n$ with numbers from 4 to n+4 other than p.

Thus, $e_f(0) = n+3$ and $e_f(1) = n+2$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.

Hence G is divisor cordial graph.

Example 3.5

The divisor cordial labeling of J_5 is shown in figure 3.5.

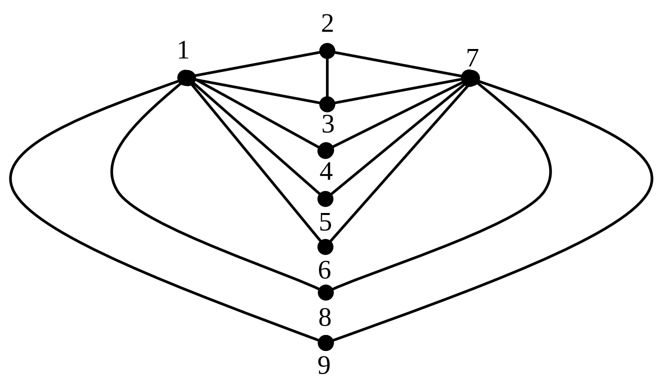

Figure 3.4

Theorem 3.6

The graph $P_m (+) \overline{K}_n$ is divisor cordial graph.

Proof.

Let $u_1, u_2, \ldots, u_m$ be the vertices of the path P_m.

Let $v_1, v_2, \ldots, v_n$ be the vertices of the path $\overline{K}_n$.

Let $G = P_m (+) \overline{K}_n$ be the graph with the vertex set $V(G) = \{u_i,v_j : 1 \le i \le m, 1 \le j \le n\}$ and the edge set $E(G) = \{u_iu_{i+1}, u_1v_j, u_mv_j : 1 \le i \le m-1, 1 \le j \le n\}$.

Then $|V(G)| = m+n$ and $|E(G)| = m+2n-1$.

Define $f : V(G) \rightarrow \{1, 2, \ldots, m+n\}$ as follows

p is the largest prime number and $p \le m+n$.

Case (i) : $p \le m - 1$.

$f(u_m) = p$ and label the vertices $u_1, u_2, \ldots, u_{m-1}$ with following order other than p.

$$\begin{array}{lllll} 1, & 2, & 2^2, & \ldots, & 2^{k_1,} \\ 3, & 3\times 2 & 3\times 2^2 & \ldots, & 3\times 2^{k_2}, \\ 5, & 5\times 2 & 5\times 2^2 & \ldots, & 5\times 2^{k_3}, \\ \ldots & \ldots & \ldots & \ldots & \ldots, \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{array}$$

where $(2t-1)2^{k_t} \le m$ and $t \ge 1$, $k_t \ge 0$.

Observe that $(2t-1)2^a$ divides $(2t-1)2^b$ $(a < b)$ and $(2t-1)2^{k_i}$ does not divide $(2t+1)$ and $2(2t+1)$.

In the above labeling, see that the consecutive adjacent vertices having the labels even numbers and consecutive adjacent vertices having labels odd and even numbers contribute 1 to each edge.

Similarly, the consecutive adjacent vertices having the labels odd numbers and consecutive adjacent vertices having labels even and odd numbers contribute 0 to each edge.

$f(v_j) = m + j, \qquad 1 \le j \le n$

Case (ii) : $m \le p \le m+n$.

$f(u_m) = p$ and label the vertices $u_1, u_2, \ldots, u_{m-1}$ with following order.

$$\begin{array}{lllll} 1, & 2, & 2^2, & \ldots, & 2^{k_1,} \\ 3, & 3\times 2 & 3\times 2^2 & \ldots, & 3\times 2^{k_2}, \\ 5, & 5\times 2 & 5\times 2^2 & \ldots, & 5\times 2^{k_3}, \\ \ldots & \ldots & \ldots & \ldots & \ldots, \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{array}$$

where $(2t-1)2^{k_t} \le m-1$ and $t \ge 1$, $k_t \ge 0$.

Observe that $(2t-1)2^a$ divides $(2t-1)2^b$ $(a < b)$ and $(2t-1)2^{k_i}$ does not divide $(2t+1)$.

In the above labeling, see that the consecutive adjacent vertices having the labels even numbers and consecutive adjacent vertices having labels odd and even numbers contribute 1 to each edge.

Similarly, the consecutive adjacent vertices having the labels odd numbers and consecutive adjacent vertices having labels even and odd numbers contribute 0 to each edge.

Label the vertices $v_1, v_2, \ldots, v_n$ with numbers from m+1 to m+n other than p.

In both cases, m is odd, then $e_f(0) = e_f(1) = \frac{2n+m-1}{2}$ and m is even, then $e_f(0) = \frac{2n+m-2}{2}$ and $e_f(1) = \frac{2n+m}{2}$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.

Hence G is divisor cordial graph.

Example 3.6

The divisor cordial labeling of P_7 (+) $\overline{K}_5$ is shown in figure 3.6.

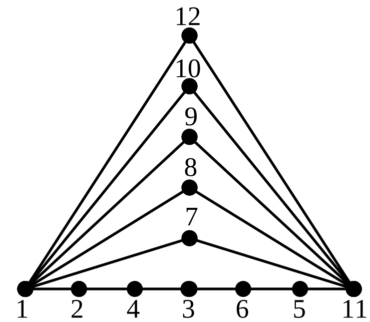

Figure 3.6

Theorem 3.7

The graph $(\overline{K}_n \cup P_m) + 2K_1$ is divisor cordial graph.

Proof :

Let x, y, $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_m$ be the vertices of $\overline{K}_n$ and P_m respectively.

Let G be the graph $(\overline{K}_n \cup P_m)+2K_1$ and let vertex set $V(G) = \{x,y,u_i,v_j : 1\le i\le n, 1\le j\le m\}$ and the edge set $E(G) = \{xu_i, yu_i, v_kv_{k+1}, xv_j, yv_j : 1 \le i \le n, 1 \le j \le m, 1 \le k \le m-1\}$.

Then $|V(G)| = n+m+2$ and $|E(G)| = 2n+3m-1$.

Define vertex labeling $f : V(G) \to \{1, 2, \ldots, n+m+2\}$ as follows

$f(x) = 1$, $f(y) = p$, where p is the largest prime number.

Case (i) : $p \le m+1$.

Label the vertices $v_1, v_2, \ldots, v_m$ with following order other than 1 and p.

$$\begin{array}{lllll} 1, & 2, & 2^2, & \ldots, & 2^{k_1,} \\ 3, & 3\times 2 & 3\times 2^2 & \ldots, & 3\times 2^{k_2}, \\ 5, & 5\times 2 & 5\times 2^2 & \ldots, & 5\times 2^{k_3}, \\ \ldots & \ldots & \ldots & \ldots & \ldots, \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{array}$$

where $(2t-1)2^{k_t} \le m+2$ and $t \ge 1$, $k_t \ge 0$.

Observe that $(2t-1)2^a$ divides $(2t-1)2^b$ $(a < b)$ and $(2t-1)2^{k_i}$ does not divide $(2t+1)$ and $(2t+3)$.

In the above labeling, see that the consecutive adjacent vertices having the labels even numbers and consecutive adjacent vertices having labels odd and even numbers contribute 1 to each edge.

Similarly, the consecutive adjacent vertices having the labels odd numbers and consecutive adjacent vertices

ISSN 2348 – 7968

having labels even and odd numbers contribute 0 to each edge.

Label the vertices $u_1, u_2, ..., u_n$ from m+3 to n+m+2.

Thus, n is odd, then $e_f(0) = e_f(1) = \frac{2n+3m-1}{2}$ and n is even, then $e_f(0) = \frac{2n+3m}{2}$ and $e_f(1) = \frac{2n+3m-2}{2}$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.

Case (ii) : p > m+1.

Label the vertices $v_1, v_2, ..., v_m$ with following order other than 1.

$$\begin{array}{lllll} 1, & 2, & 2^2, & ..., & 2^{k_1}, \\ 3, & 3\times 2 & 3\times 2^2 & ..., & 3\times 2^{k_2}, \\ 5, & 5\times 2 & 5\times 2^2 & ..., & 5\times 2^{k_3}, \\ ... & ... & ... & ... & ..., \\ ... & ... & ... & ... & ... \end{array}$$

where $(2t-1)2^{k_t} \le m+1$ and $t \ge 1$, $k_t \ge 0$.

Observe that $(2t-1)2^a$ divides $(2t-1)2^b$ (a < b) and $(2t-1)2^{k_i}$ does not divide (2t+1) and (2t+3).

In the above labeling, see that the consecutive adjacent vertices having the labels even numbers and consecutive adjacent vertices having labels odd and even numbers contribute 1 to each edge.

Similarly, the consecutive adjacent vertices having the labels odd numbers and consecutive adjacent vertices having labels even and odd numbers contribute 0 to each edge.

Label the vertices $u_1, u_2, ..., u_n$ from m+2 to n+m+2 other than p.

Thus, n is odd, then $e_f(0) = e_f(1) = \frac{2n+3m-1}{2}$ and n is even, then $e_f(0) = \frac{2n+3m}{2}$ and $e_f(1) = \frac{2n+3m-2}{2}$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.

Hence G is divisor cordial graph.

Example 3.7

The divisor cordial labeling of $(\overline{K}_4 \cup P_4) + 2K_1$ is given in figure 3.7.

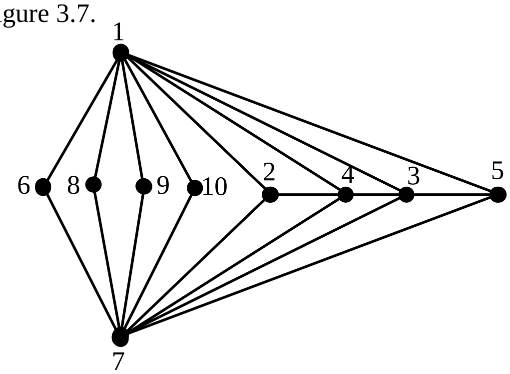

Figure 3.7

Theorem 3.8

The graph $(P_n \cup P_m) + 2K_1$ is divisor cordial graph, where $n,m \ge 2$.

Proof :

Let x, y, $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_m$ be the vertices of P_n and P_m respectively.

Let G be the graph $(P_n \cup P_m) + 2K_1$ and let vertex set $V(G) = \{x, y, u_i, v_j : 1 \le i \le n, 1 \le j \le m\}$ and the edge set $E(G) = \{xu_i, yu_i, u_ru_{r+1}, v_kv_{k+1}, xv_j, yv_j : 1 \le i \le n, 1 \le j \le m, 1 \le r \le n-1, 1 \le k \le m-1\}$.

Then $|V(G)| = n+m+2$ and $|E(G)| = 3n+3m-2$.

Define vertex labeling $f : V(G) \to \{1, 2, ..., n+m+2\}$ as follows

$f(x) = 1$,

$f(y) = p$, where p is the largest prime number.

Label the vertices $u_2, ..., u_n, v_1, v_2, ..., v_m$ in the following order other than p and n+m+1.

$$\begin{array}{lllll} 2 & 2^2, & 2^3, & ..., & 2^{k_1}, \\ 3, & 3\times 2 & 3\times 2^2 & ..., & 3\times 2^{k_2}, \\ 5, & 5\times 2 & 5\times 2^2 & ..., & 5\times 2^{k_3}, \\ ... & ... & ... & ... & ..., \\ ... & ... & ... & ... & ... \end{array}$$

where $(2s-1)2^{k_s} \le n+m+2$ and $s \ge 1$, $k_s \ge 0$.

In the above labeling, see that the consecutive adjacent vertices having the labels even numbers and consecutive adjacent vertices having labels odd and even numbers contribute 1 to each edge.

Similarly, the consecutive adjacent vertices having the labels odd numbers and consecutive adjacent vertices having labels even and odd numbers contribute 0 to each edge.

Case (i) : n+m is odd and $f(v_1)$ is even.

Then, $e_f(0) = e_f(1) + 1 = \frac{3n+3m-1}{2}$.

Case (ii) : n+m is odd and $f(v_1)$ is odd.

Then, $e_f(1) = e_f(0)+1 = \frac{3n+3m-1}{2}$.

Case (iii) : n+m is even and $f(v_1)$ is even.

Then, $e_f(0) = e_f(1) = \frac{3n+3m-2}{2}$.

Case (iv) : n+m is even and $f(v_1)$ is odd.

Subcase (a) : n+m = 6 and $f(v_1)$ is odd.

Interchange the labels of u_1 and v_1.

Then, $e_f(0) = e_f(1) = 8$

Subcase (b) : $n+m \ne 6$ and $f(v_1)$ is odd.

Interchange the labels of v_1 and v_m.

Then, $e_f(0) = e_f(1) = \frac{3n+3m-2}{2}$.

Therefore, $|e_f(0) - e_f(1)| \leq 1$.

Hence G is divisor cordial graph.

Example 3.8

The divisor cordial labeling of $(P_4 \cup P_4) + 2K_1$ is given in figure 3.8.

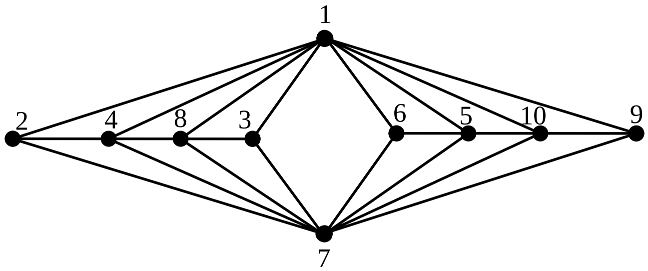

Figure 3.8

4. Conclusions

In this paper, we prove the divisor cordial labeling of $S'(K_{2,m})$, $S'(K_{1,n,n})$, double fan P_n+2K_1, cone C_n+2K_1, Jewel graph J_n, P_m (+) $\overline{K}_n$, $(\overline{K}_n \cup P_m) + 2K_1$ and $(P_n \cup P_m)+2K_1$.

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