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Optimization of Monitoring Points on Computer System from Poor Knowledge Risk Point of View

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Abstract

In the training processes, computer systems are often used today. During monitoring/diagnostics of computer systems (CS), it is very important to optimize the number of monitoring points in order to ensure a reliable operation, availability and to minimize the risk of poor knowledge. This paper presents an analytical expression for determining the optimal number of monitoring points of computer system used for training.

Keywords: *Computer system, Reliability analysis, Monitoring, Optimization.*

1. Introduction

In carrying out the monitoring of computer system (CS), by measuring the relevant parameter y, a technical check is performed for any j-th device in order to verify if the following condition is met [1,5]:

$$\Delta_1 \le y_j \le \Delta_2 \qquad (1)$$

where:

Δ_1 -lower limit of the parameter y

Δ_2 - upper limit of the parameter y

In risk technical systems [2] the parameter y is considered as random variable (taking into account population of many devices), the realizations of each of them are $y_1,\ldots,y_j,\ldots,y_J$ - values of the monitored parameter y of the j-th device from a given kind. In turn the monitored parameter y is a function of a continuous argument which varies in the range $[0, X]$, i.e. $y = y(x), \quad x \in [0, X]$. In this case, the monitoring of the j-th device is performed, by checking the fulfillment of the condition [3]:

$$\Delta_1(x) \le y_j(x) \le \Delta_2(x), \; x \in [0, X] \qquad (2)$$

where:

$\Delta_1(x), \Delta_2(x)$ - lower and upper limit of the monitored parameter;

$y_j(x)$ - realization of the random function $y(x)$.

2. Theoretical Background

In practice widely used is a discrete monitoring of random functions, which concludes in the verification of the fulfillment of the condition (1) in case of I (I-number of discrete values of the argument x), i.e. when [4]:

$$\Delta_1(x_i) \le y_j(x_i) \le \Delta_2(x_i), \; i = \overline{1, I} \qquad (3)$$

Normally $x_1 = 0, \; x_I = X$. If condition (3) is fulfilled- then the j-th device is recognized to be in up sate, and otherwise is rejected (down state). It is obvious that the larger the number of CS monitored points, so more confident will be the reasoning for up state (down state) of j-th product in the performance (or non-execution) of inequality (3). On the other hand, is a natural aspiration to reduce the workload of the monitoring, as I is reduced. As a result of this arises the task of justifying the choice of rational number monitored points. However, the following axiom is inserted [6]:

Axiom: Optimal number monitored items ranging in $[0, X]$ of the argument x of the studied function $y(x)$ representing the basic parameter of the CS is called this their minimum number I_P that provides a specified level of effectiveness of the monitoring and the reliability $P_{БР}\left[y(x), t\right]$, determined in accordance with the formula (2.4.2) for the parametric reliability [1,2]:

$$P_{БР}\left[y(x),t\right]=P\left[\left(Y_H-\Delta Y_{ДОП}\right)<y(x)<\left(Y_H+\Delta Y_{ДОП}\right)\right]= \\ =\frac{1}{\sigma_Y(x).\sqrt{2\pi}}\cdot\int\limits_{Y_H-\Delta Y_{ДОП}}^{Y_H+\Delta Y_{ДОП}}\exp\left\{-\frac{\left[y(x)-m_Y(x)\right]^2}{2.\sigma_Y(x)}\right\}\cdot dx \tag{4}$$

where:

Y_H - is the nominal value of the studied function $y(x)$ of CS in general technical specification (TS) of the manufacturer;

$\Delta Y_{ДОП}$ -the allowable tolerances of $y(x)$ in accordance with TS;

$m_Y(x)$ and $\sigma_Y(x)$- a mathematical expected value and standard deviation, defined statistically by the measured values $y_j(x_i)$ according to the equation (3).

The task of selecting I_P occurs for instance in the following cases of solving practical technical problems [7,8]:

- select the section in which you need to monitor the diameter of the mechanical shaft;
- when you select the time interval (step) in which you need to measure the parameters of technological process in its discrete monitoring;
- when checking the measuring tools, then you need to choose the number of validated scale graduations.

When using probabilistic efficiency criteria of monitoring, the task of selecting a rational number monitoring points of the random function $y(x)$can be determined analytically if the following conditions are met [9,11]:

- $y(x)$ appears to be a normal random function of the argument x;
- the function $y(x)$ is stationary function throughout the range of variation of the argument x (or range $[0,\ X]$ can be divided into sections for each of which the function could be considered stationary);
- the probability of lying the function $y(x)$ outside of permitted limits is little (i.e., small number of rejected products in control);
- the relative error of the method of monitoring and measurement tools used, which verifies the fulfillment of condition (3) is negligible compared to the width of the tolerance of the monitored parameter;
- a priori is known or can be determined with a sufficient level of confidence following statistical parameters, mathematical expectation $m_Y(x)$ and the autocorrelation function $r_\tau(x)$ of $y(x)$.

For a given type of CS in the absence of any a prior information, the hypothesis that the distribution of $y(x)$, where $x\in[0,\ X]$ is a normal (Gaussian) can be verified, for example through the criteria χ^2 [10]. Verification of the hypothesis for stationarity of $y(x)$ practically is performed by checking the hypothesis of equality of variance of the random function $y_0(x_i)$ in I (where: I - the number of monitoring points) points by using the criterion of Bartlett [12,13].

3. Application

As a statistical material in these calculations may serve the values $y_{ij}\ \left(i=\overline{1,I};\ j=\overline{1,J}\right)$ of the measurement results of the monitored parameter in I control points of a random sample of units of a given type with volume J. This data can be used in determining the estimates of $\hat{m}_Y(x)$ and $\hat{r}_\tau(x)$. In the following calculations assumes that the conditions related to $y(x)$ are met. It follows that it is possible that the task to find I_P to be solved through the use of theory of outliers from the random processes [14] and the theory of parametric reliability [5].

In determining the criteria for the effectiveness of the monitoring of technical systems, it is assumed that the function $y(x)$ of the CS is centered with respect to the nominal value Y_H and is a stationary random function with the tolerance limits $\Delta Y_{ДОП}=\pm\Delta$.

In the literature [2] instead of estimating $P_{ИЗЛ}\{\eta=1\}$ - the probability of lying of the function $y(x)$ outside the tolerance $\pm\Delta$, one calculates the mathematical expectation $M\{\eta\}$ of the number of outliers (spikes) η.

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This replacement is possible in fulfillment of the condition for dispersion parameter $W_{ПРК}$, set by:

$$W_{ПРК} = \frac{M\{\eta\} - P_{ИЗЛ}\{\eta = 1\}}{M\{\eta\}} << 1 \qquad (5)$$

In equation (5) upper limit of $W_{ПРК}$ marked by $W_{ПРК,Г}$ shall be evaluated by the expression:

$$W_{ПРК,Г} = \frac{M\{\eta^2\} - M\{\eta\}}{M\{\eta\}} \qquad (6)$$

In reference [5] is shown that:

$$W_{ПРК,Г} = \int_0^x A(x).\exp\left[-\frac{\Delta^2}{\sigma_Y^2(x) + r_\tau(x)}\right].dx \qquad (7)$$

where:

$\sigma_Y^2(x) = D(x)$ - the variance of the studied function $y(x)$

$r_\tau(x)$ - autocorrelation function of $y(x)$;

$A(x)$ - function defined when $x \to \infty$ by

$$A(x) \to \frac{1}{2\pi}\sqrt{\frac{-D''(x)}{D(x)}}$$

From equations (5) and (7) follows that:

$$W_{ПРК} \le \frac{M\{\eta^2\} - M\{\eta\}}{M\{\eta\}} \le \frac{x}{2\pi}\sqrt{\frac{-D''(x)}{D(x)}} \cdot \exp\left[-\frac{\Delta^2}{2.D(x)}\right] \qquad (8)$$

If $W_{ПРК} << 1$, then the probability of the occurrence of spikes is determined by the formula:

$$P_{ИЗЛ}\{\eta = 1\} = \frac{x}{\pi}\sqrt{\frac{-D''(x)}{D(x)}} \cdot \exp\left\{-\frac{\Delta^2}{2.D(x)}\right\} \qquad (9)$$

From equations (8) and (9) follows that:

$$W_{ПРК}(x) \le 0{,}5.P_{ИЗЛ}\{\eta = 1\} \qquad (10)$$

If the probability $P_{ИЗЛ}\{\eta = 1\}$ of lying of the function $y(x)$ outside of the tolerance $\pm\Delta$ is $P_{ИЗЛ}\{\eta = 1\} \le 0{,}2$ then the dispersion parameter $W_{ПРК}$ when performing the monitoring will respond to the condition $W_{ПРК} < 0{,}1$. This value of $W_{ПРК}$ can be considered small enough to replace $P_{ИЗЛ}\{\eta = 1\}$ it with $M\{\eta\}$.

In equation (9) instead $D(x)$ and $-D''(x)$ one can use their estimates S^2 and S_d^2 that are calculated on the results of the measurement of the parameter y_{ij} of the sample of products with a volume J:

$$S^2 = \frac{1}{I}\sum_{i=1}^{I} S_i^2 = \frac{1}{J-1}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[(y_{ij} - m_i) - \frac{1}{J}\sum_{j=1}^{J}(y_{ij} - m_i)\right]^2 \qquad (11)$$

where:

$m_i = \hat{m}(x_i) = \frac{1}{J}\sum_{j=1}^{J} y_{ij}$ - mathematical expected value of $y(x)$ in the i-th monitoring point $\left(i = \overline{1, I}\right)$.

$$S_d^2 = \frac{1}{I-1} \cdot \sum_{i=1}^{I-1} S_{d_i}^2 \qquad (12)$$

$$S_{d_i}^2 = \frac{J.(I-1)^2}{(J-1)^2.x^2}\sum_{j=1}^{J}\left\{\left[(y_{i+1,j} - m_{i+1}) - (y_{ij} - m_i)\right] - \frac{1}{J}\sum_{j=1}^{J}\left[(y_{i+1,j} - m_{i+1}) - (y_{ij} - m_i)\right]\right\}^2$$

It is not hard to show that S_d^2 is a consistent and unbiased estimate of $-D''(x)$. By substituting (9) in the formulas (11) and (12), then for the fulfillment of a condition $P_{ИЗЛ}\{\eta = 1\} \le 0{,}2$, it follows that:

$$\frac{S_d}{S} \cdot x.\exp\left[-\frac{\Delta^2}{2.S^2}\right] \le 0{,}2.\pi \qquad (13)$$

This inequality comes in a quality criterion for the use of the proposed method for specific types of devices. Implementation of condition (13) means that $W_{ПРК} < 0{,}1$, i.e. it is a fair assumption for the use of $M\{\eta\}$ instead of $P_{ИЗЛ}\{\eta = 1\}$.

For the criterion of monitoring effectiveness (CME), one can select the ratio ξ_{CME} of the two probabilities, defined by:

$$\xi_{CME} = P\{A/B\}/P\{\eta = 1\} \tag{14}$$

where:

$P\{A/B\}$ - a posterior probability of the occurrence of the unnoticed spike provided that CS was accepted for operation (a posterior probability (type 2 error) in monitoring);

A, B - events, consisting of the appearance of unnoticed spike and recognition of CS as to be in up-state after carrying out the monitoring.

In the literature [5], it is considered that monitoring is effective when $\xi_{CME} = 0{,}01$. On the basis of the assumption for a small number of scrapped (not subject to recovery) CS during their monitoring, instead of the equation (14) for ξ_{CME} can be written:

$$\xi_{CME} = \frac{P\{AB\}}{P\{\eta = 1\}} = 1 - \sqrt{2\pi}\,\frac{2.D(x).(I_P - 1)}{x.\Delta.\sqrt{-D''(x)}} \cdot \Phi\left(\frac{x.\Delta.\sqrt{-D''(x)}}{2.D(x).(I_P - 1)}\right) = 0{,}01 \tag{15}$$

where:

$P\{AB\}$ - a prior probability of undetected spike;

Φ - function of the normal distribution.

After solving the equation (15), the following is obtained for I_P:

$$I_P = 2x \cdot \frac{\Delta}{D(x)} \cdot \sqrt{-D''(x)} + 1 \tag{16}$$

Equation (16) is an analytical expression for determining the optimal number of monitoring points of the relevant cybernetic system (including CS) for training process.

4. Conclusions

From the performed analysis on the technical state of CS for providing knowledge, follow the conclusions:

1. the derived equation (16) ensure the necessary reliability of the studied CS during the training process.
2. by the performed monitoring/diagnostics of the existing hardware of CS, one can ensure reliability of the intellectual product received by our students.

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