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Approximation of the Quartic Double Centralizers and Quartic multipliers on Banach algebras

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ABSTRACT

In this paper, we establish stability of quartic double centralizers and quartic multipliers on Banach algebras. We also prove the superstability of quartic double centralizers on Banach algebras which are quartic commutative and quartic without order, and of quartic multipliers on Banach algebras which are quartic without order.

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1.INTRODUCTION

The stability problem of functional equations originated from a question of Ulam [14] in 1940, concerning the stability of group homomorphisms. Let $(G_1, *)$ be a group and let $(G_2, *)$ be a metric group with the metric d(.,.). Given $\varepsilon > 0$, does there exist a $\delta > 0$, such that if a mapping $h:G_1 \to G_2$ satisfies the inequality $d(h(x, y), \ h(x) * h(y) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \to G_2$ with $d(h(x), \ H(x)) < \varepsilon$ for all $x \in G_1$? In the other words, Under what condition does there exist a homomorphism near an approximate homomorphism? The concept of stability for functional equation arises when we replace the functional equation by an inequality which acts as a perturbation of the equation. In 1941, D.H. Hyers [6] gave a first affirmative answer to the question of Ulam for Banach space.
In 1950, T. Aoki [1] was the second author to treat this problem for additive mapping. Finally in 1978, Th. M. Rassias [13] proved the following Theorem:
Theorem (Th. M. Rassias). Let $f : E \to E'$ be a mapping from a norm vector space E into a Banach space E' subject to the inequality

$$\left\| f(x+y) - f(x) - f(y) \right\| \le \varepsilon(\left\|x\right\|^{p} + \left\|y\right\|^{p})$$

for all $x, y \in E$, where ε and p are constants with $\varepsilon > 0$ and $p < 1$. Then there exists a unique additive mapping $T : E \to E'$ such that

$$\left\| f(x) - T(x) \right\| \le \frac{2\varepsilon}{2 - 2^{P}} \left\|x\right\|^{P}$$

for all $x \in E$. *Also, if* the function $t \to f(tx)$ from R into E' *is* continuous for each fixed x *in* E, then T is linear.
This stability phenomenon *of* this kind is called the Hyers-Ulam-Rassias stability. In *1991, Z.* Gajda *[3]* answered the question for the case $p < 1$, which was raised by Rassias. In *1994,* a generalization *of* the Rassias' theorem was obtained by Gavruta as follows [4].

The functional equation is called stable *if* any function satisfying that functional equation "approximately" is near to a true solution of functional equation. We say that a functional equation is superstable if every approximately solution is an exact solution of it.

The functional equation

$f(2x+y)+f(2x-y)=4f(x+y)+4f(x-y)+24f(x)-6f(y)$ *(1.1)*

is called the quartic functional equation, since the function $f(x)=x^4$ is a solution of this functional equation. Note that f is called quartic because of the identity

$$(2x+y)^4+(2x-y)^4=4(x+y)^4+4(x-y)^4+24x^4-6y^4 \qquad (1.2)$$

Every solution of the quartic functional equation is said to be a quartic mapping. It is In proved in *[8]* that a function $f: X \to Y$ between real normed spaces is quartic if and only if there exists a symmetric biquadratic function $f: X\times X \to Y$ such that $f(x)=F(x,x)$ for all $x\in X$. The first result on the stability of the quartic functional equation was obtained by J. M. Rassias *[12]*. *Also L.* Cadariu *[2]*.H. -M. Kim [7], S. H. Lee, S. M. Im and I. S. Hwang *[8]*, Najati *[10]* and C. Park *[11]* investigated the stability of quartic functional equation.

Moslehian, Rahbarnia and Sahoo *[9]* established the stability of double centralizers to Cauchy functional equations in the framework of Banach algebras. They also proved the superstability of double centralizers on Banach algebras which are strongly without order.

Gordji, Ebadian, Ramezani and Park [5] proved the stability quadratic double centralizers on Banach algebras.

In this paper, we introduce the quartic double centralizers and quartic multipliers on Banach algebras, and we establish the stability of both of them. We also prove the superstability of quartic double centralizers on Banach algebras which are quartic without order and quartic commutative, and of quartic multipliers on quartic without order Banach algebras.

2.Stability of quartic double centralizer

In this section, let A be a complex Banach algebra. We establish the stability of quartic double centralizers.

Definition 2.1. A mapping $L: A\to A$ is a quartic left centralizer if L satisfies the following properties:

1) L is a quartic mapping,

2) L is a quartic homogeneous, that is, $L(\lambda a)=\lambda^4 L(a)$ for all $a\in A$ and $\lambda\in C$,

3) $L(ab)=L(a)b^4$ for all $a,b\in A$.

Definition 2.2. A mapping $R: A\to A$ is a quartic right centralizer if R satisfies the following properties:

1) R is a quartic mapping,

2) R is quartic homogeneous, that is, $R(\lambda a)=\lambda^4 R(a)$ for all $a\in A$ and $\lambda\in C$,

3) $R(ab)=a^4R(b)$ for all $a,b\in A$.

Definition 2.3. A quartic double centralizer of an algebra A is a pair (L,R), where L is a quartic left centralizer, R is a quartic right centralizer and $a^4L(b)=R(a)b^4$ for all $a,b\in A$.

The following example introduces a quartic double centralizer.

Example 2.4. Let $(A,\|.\|)$ be a Banach algebra. Let $B=A\times A\times A\times A\times A$.We define $\||a\|| = \|a_1\|+\|a_2\|+\|a_3\|+\|a_4\|+\|a_5\|$ for all $a=(a_1,a_2,a_3,a_4,a_5)$ in B. It is not hard to see that $(B,\||.\||)$ is a banach space for arbitrarily elements $a=(a_1,a_2,a_3,a_4,a_5)$ and $b=(b_1,b_2,b_3,b_4,b_5)$ in B, we define $ab=(0,a_1b_5,a_2b_4,a_3b_3,0)$. since A is a Banach algebra, we conclude that B is a Banach algebra.

It is easy to see that $B^5=\{abcde: a,b,c,d,e\in B\}=\{0\}$ But $B^4=\{abcd: a,b,c,d\in B\}$ is not zero. Now we consider the mapping $T: B\to B$ defined by

$$T(a)=a^4 (a\in B).$$

Then T *is a quartic mapping and quartic* homogeneous. *Since* $B^5=\{0\}$, *we get*

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$$T(ab) = (ab)^4 = 0 = a^4 b^4 = T(a)b^4 = a^4 T(b)$$

and

$$a^4 T(b) = a^4 b^4 = 0 = T(a)b^4$$

For all $a, b \in B$. Hence (T, T) is a quartic double centralizer of B.
In the above example, B *is* a quartic commutative algebra, but it is not commutative.

Theorem 2.5. Suppose that $s \in \{-1,1\}$ and that $f : A \to A$ is a mapping with $f(0) = 0$ for which there exist a mapping $g : A \to A$ with $g(0) = 0$ and functions $\phi_j, \psi_i : A \times A \to [0,\infty)$ $(1 \le j \le 2, 1 \le i \le 3)$ such that

$$\tilde{\phi}_j(a,b) := \sum_{k=0}^{\infty} \frac{\phi_j(2^{sk}a, 2^{sk}b)}{16^{sk}} < \infty \qquad (1 \le j \le 2), \tag{2.1}$$

$$\lim_{n\to\infty} \frac{\psi_i(2^{sn}a, b)}{16^{sn}} = 0 = \lim_{n\to\infty} \frac{\psi_i(a, 2^{sn}b)}{16^{sn}} \qquad (1 \le j \le 3),$$

$$\left\| f(2\lambda a + \lambda b) + f(2\lambda a - \lambda b) - 4\lambda^4 f(a+b) - 4\lambda^4 f(a-b) - 24\lambda^4 f(a) + 6\lambda^4 f(b) \right\| \le \phi_1(a,b) \tag{2.2}$$

$$\left\| g(2\lambda a + \lambda b) + g(2\lambda a - \lambda b) - 4\lambda^4 g(a+b) - 4\lambda^4 g(a-b) - 24\lambda^4 g(a) + 6\lambda^4 g(b) \right\| \le \phi_2(a,b)$$

$$\left\| f(ab) - f(a)b^4 \right\| \le \psi_1(a,b) \tag{2.3}$$

$$\left\| g(ab) - a^4 g(b) \right\| \le \psi_2(a,b)$$

$$\left\| a^4 f(b) - g(a)b^4 \right\| \le \psi_3(a,b) \tag{2.4}$$

for all $a, b \in A$ and all $\lambda \in T = \{\lambda \in T : |\lambda| = 1\}$. Also, if for each fixed $a \in A$ the mappings $t \to f(ta)$ and $t \to g(ta)$ from R to A are continuous, then there exists a unique quartic double centralizer (L, R) on A satisfying

$$\left\| f(a) - L(a) \right\| \le \frac{1}{32} \tilde{\phi}_1(a,a), \tag{2.5}$$

$$\left\| g(a) - R(a) \right\| \le \frac{1}{32} \tilde{\phi}_2(a,a), \tag{2.6}$$

for all $a \in A$.

Proof: Let $s = 1$. Putting $b = 0$ and $\lambda = 1$ in *(2.2)*, we have

$$\left\| f(2a) - 16 f(a) \right\| \le \frac{1}{2} \phi_1(a,a)$$

for all $a \in A$. One can use induction to show that

$$\left\| \frac{f(2^n a)}{16^n} - \frac{f(2^m a)}{16^m} \right\| \le \frac{1}{32} \sum_{k=m}^{n-1} \frac{\phi_1(2^k a, 2^k a)}{16^k} \tag{2.7}$$

for all $n > m \ge 0$ and all $a \in A$. It follows from *(2.7)* and *(2.1)* that sequence $\left\{ \frac{f(2^n a)}{16^n} \right\}$ is Cauchy. Since A *is* a banach algebra, this sequence is convergent. Define

$$L(a) := \lim_{n\to\infty} \frac{f(2^n a)}{16^n}. \tag{2.8}$$

Replacing a and b by $2^n a$ and $2^n b$, respectively, in *(2.2)*, we get

$$\|\frac{f(2^n(2\lambda a+\lambda b))}{16^n}+\frac{f(2^n(2\lambda a-\lambda b))}{16^n}-4\lambda^4\frac{f(2^n(a+b))}{16^n}-4\lambda^4\frac{f(2^n(a-b))}{16^n}$$

$$-24\lambda^4\frac{f(2^n a)}{16^n}+6\lambda^4\frac{f(2^n a)}{16^n}\|\le\frac{\phi_1(2^n a,2^n b)}{16^n}$$

Taking the limit as $n\to\infty$, we obtain

$$L(2\lambda a+\lambda b)+L(2\lambda a-\lambda b)=4\lambda^4 L(a+b)+4\lambda^4 L(a-b)+24\lambda^4 L(a)-6\lambda^4 L(b) \tag{2.9}$$

for all $a,b\in A$ and all $\lambda\in T$. Putting $\lambda=1$ in *(2.9)*, *we* obtain that L *is* a quartic mapping. Setting $b:=0$ in *(2.9)*, we get

$$L(2\lambda a)=16\lambda^4 L(a)$$

for all $a\in A$, $\lambda\in T$. But L *is* a quartic mapping. So

$$L(\lambda a)=\lambda^4 L(a)$$

for all $a\in A$ *and all* $\lambda\in T$. Under the assumption that $f(ta)$ *is* continuous in $t\in R$ for each fixed $a\in A$, by the same reasoning as in the proof of *[16]*, $L(\lambda a)=\lambda^4 L(a)$ for all $a\in A$ and all $\lambda\in R$. Hence

$$L(\lambda a)=L(\frac{\lambda}{|\lambda|}|\lambda|a)=\frac{\lambda^4}{|\lambda|^4}L(|\lambda|a)=\frac{\lambda^4}{|\lambda|^4}|\lambda|^4 L(a)$$

for all $a\in A$ and $\lambda\in C(\lambda\ne 0)$. This means that L *is* quartic homogeneous. It follows from *(2.3)* and *(2.8)* that

$$\left\|L(ab)-L(a)b^4\right\|=\lim_{n\to\infty}\frac{1}{16^n}\left\|f(2^n ab)-f(2^n a)b^4\right\|\le\lim_{n\to\infty}\frac{\psi_1(2^n a,b)}{16^n}=0$$

for all $a,b\in A$. Hence L *is* a quartic left centralizer on A. Applying *(2.7)* with $m=0$, we get $\left\|L(a)-f(a)\right\|\le\frac{1}{32}\tilde{\phi}_1(a,a)$ for all $a\in A$. It is well known that the quartic mapping L satisfying *(2.5) is* unique. A similar argument gives us a unique quartic right centralizer R defined by

$$R(a):=\lim_{n\to\infty}\frac{g(2^n a)}{16^n}$$

which satisfies *(2.6). Now* we let $a,b\in A$ arbitrarily. Since L *is* a quartic homogeneous, it follows from *(2.4)* and *(2.5)* that

$$\left\|a^4L(b)-R(a)b^4\right\|=\frac{1}{16^n}\left\|a^4L(2^n b)-16R(a)b^4\right\|$$

$$\le\frac{1}{16^n}[\left\|a^4L(2^n b)-a^4f(2^n b)\right\|+\left\|a^4f(2^n b)-g(a)(16^n b^4)\right\|$$

$$+\left\|16^n g(a)b^4-16^n R(a)b^4\right\|]$$

$$\le\frac{1}{16^{n+1}}\tilde{\phi}_1(2^n b,2^n a)\ \left\|a\right\|^4+\frac{\psi_3(a,2^n b)}{16^n}+\left\|g(a)-R(a)\right\|\ \left\|b\right\|^4.$$

The right hand side of the last inequality tends to $\left\|g(a)-R(a)\right\|\ \left\|b\right\|^4$ as $n\to\infty$.

By *(2.6)*, *we* obtain

$$\left\|a^4L(b)-R(a)b^4\right\|=\frac{1}{32}\tilde{\phi}_2(a,a)\left\|b^4\right\|.$$

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Since R *is* a quartic mapping, we thus obtain

$$\left\| a^4 L(b) - R(a)b^4 \right\| = \frac{1}{16^n} \left\| 16^n a^4 L(b) - R(2^n a)b^4 \right\|$$

$$\leq \frac{1}{32} \tilde{\phi}_2(2^n a, 2^n a) \; \|a\|^4$$

$$= \frac{1}{32} \sum_{k=n}^{\infty} \frac{\phi_2(2^k a, 2^k a)}{16^k} \left\| b^4 \right\|.$$

Passing to the limit as $n \to \infty$, we conclude $a^4 L(b) = R(a)b^4$. *Thus* (L, R) is a quartic double centralizer.
The proof for $s = -1$ is similar to $s = 1$.

Corollary 2.6. Suppose that $f : A \to A$ is a mapping for which there exist a mapping $g : A \to A$ and constants $\varepsilon > 0$ and $0 < p < A$ such that

$$\left\| f(2\lambda a + \lambda b) + f(2\lambda a - \lambda b) - 4\lambda^4 f(a+b) - 4\lambda^4 f(a-b) - 24\lambda^4 f(a) + 6\lambda^4 f(b) \right\| \leq \varepsilon(\|a\|^p + \|b\|^p),$$

$$\left\| g(2\lambda a + \lambda b) + g(2\lambda a - \lambda b) - 4\lambda^4 g(a+b) - 4\lambda^4 g(a-b) - 24\lambda^4 g(a) + 6\lambda^4 g(b) \right\| \leq \varepsilon(\|a\|^p + \|b\|^p),$$

$$\left\| f(ab) - f(a)b^4 \right\| \leq \varepsilon \|a\|^p \|b\|^p,$$

$$\left\| g(ab) - a^4 g(b) \right\| \leq \varepsilon \|a\|^p \|b\|^p,$$

$$\left\| a^4 f(b) - g(a)b^4 \right\| \leq \varepsilon \|a\|^p \|b\|^p$$

for all $a, b \in A$ and all $\lambda \in T$ .Also, if for each fixed $a \in A$ the mappings $t \to f(ta)$ and $t \to g(ta)$ from R to A are continuous, then there exists a unique quartic double centralizer (L, R) on A satisfying

$$\left\| f(a) - L(a) \right\| \leq \frac{\varepsilon}{|16 - 2^p|} \|a\|^p,$$

$$\left\| g(a) - R(a) \right\| \leq \frac{\varepsilon}{|16 - 2^p|} \|a\|^p$$

for all $a \in A$.

Proof: For $j = 1,2,$ *putting* $\phi_j(a,b) = \varepsilon(\|a\|^p + \|b\|^p)$ and for $i = 1,2,$ *putting* $\psi_i(a,b) = \varepsilon \|a\|^p \|b\|^p$ *in Theorem 2.5, we* get the desired results.

3. Stability of quartic multipliers

Throughout this section, assume that A is a complex Banach algebra.

Definition 3.1. We say that a mapping $T : A \to A$ is a quartic multiplier if T satisfies the following properties:

1) T is a quartic mapping,

2) T is quartic homogeneous, that is, $T(\lambda a) = \lambda^4 T(a)$ for all $a \in A$ and $\lambda \in C$,

3) $a^4 T(b) = T(a)b^4$ for all $a, b \in A$.

Example 2.4 introduces a quartic multiplier. We investigate the stability of quartic. multipliers.

Theorem 3.2. Suppose that $s \in \{-1,1\}$ and that $f : A \to A$ is a mapping with $f(0) = 0$ for which there exist functions, $\psi : A \times A \to [0, \infty)$ such that

$$\tilde{\phi}(a,b) := \sum_{k=0}^{\infty} \frac{\phi(2^{sk}a, 2^{sk}b)}{16^{sk}} < \infty, \tag{3.1}$$

$$\lim_{n\to\infty} \frac{\psi(2^{sn}a,b)}{16^{sn}} = 0 = \lim_{n\to\infty} \frac{\psi(a,2^{sn}b)}{16^{sn}},$$

$$\left\| f(2\lambda a + \lambda b) + f(2\lambda a - \lambda b) - 4\lambda^4 f(a+b) - 4\lambda^4 f(a-b) - 24\lambda^4 f(a) + 6\lambda^4 f(b) \right\| \le \phi(a,b),$$

$$\left\| a^4 f(b) - f(a) b^4 \right\| \le \psi(a,b) \tag{3.2}$$

for all $a,b \in A$ and all $\lambda \in T$. Also, if for each fixed $a \in A$ the mappings $t \to f(ta)$ from R to A are continuous, then there exists a unique quartic multiplier T on A satisfying

$$\left\| f(a) - T(a) \right\| \le \frac{1}{32} \tilde{\phi}(a,a), \tag{3.3}$$

for all $a \in A$.

Proof. Let $s = 1$. By the same reasoning as in the proof of Theorem *2.5*, there exists a unique quartic mapping $T : A \to A$ defined by

$$T(a) := \lim_{n\to\infty} \frac{f(2^n a)}{16^n}$$

with satisfying $T(\lambda a) = \lambda^4 T(a)$ for all $a \in A$ and all $\lambda \in C$. *Also,* $\left\| f(a) - T(a) \right\| \le \frac{1}{32} \tilde{\phi}(a,a)$ for all $a \in A$ .Let $a,b \in A$ be arbitrarily. Then T is quartic homogeneous.

By using *(3.2)* and *(3.3)*, we have

$$\left\| a^4 T(b) - T(a) b^4 \right\| = \frac{1}{16^n} \left\| a^4 T(2^n b) - 16^n T(a) b^4 \right\|$$

$$\le \frac{1}{16^n} [\left\| a^4 T(2^n b) - a^4 f(2^n b) \right\| + \left\| a^4 f(2^n b) - f(a)(16^n b^4) \right\|$$

$$+ \left\| 16^n f(a) b^4 - 16^n T(a) b^4 \right\|]$$

$$\le \frac{1}{16^{n+1}} \tilde{\phi}(2^n b, 2^n a) \ \|a\|^4 + \frac{\psi(a, 2^n b)}{16^n} + \frac{1}{32} \tilde{\phi}(a,a) \ \|b\|^4 .$$

It follows from *(3.1)* that

$$\left\| a^4 T(b) - T(a) b^4 \right\| = \frac{1}{32} \tilde{\phi}(a,a) \ \|b\|^4 .$$

Finally, we obtain

$$\left\| a^4 T(b) - T(a) b^4 \right\| = \frac{1}{16^n} \left\| 16^n a^4 T(b) - T(2^n a) b^4 \right\|$$

$$\le \frac{1}{32} \tilde{\phi}_2(2^n a, 2^n a) \ \|b\|^4$$

$$= \frac{1}{32} \sum_{k=n}^{\infty} \frac{\phi(2^k a, 2^k a)}{16^k} \|b\|^4$$

$$\to 0 \qquad as \quad n \to \infty.$$

So $a^4 T(b) = T(a) b^4$. *Hence* T is a quartic multiplier.

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The proof for $s=-1$ is similar.

Corollary *3.3.* Suppose that $f: A \to A$ is a mapping for which there exist nonnegative real numbers ε and p with $p \neq 4$ such that

$$\left\| f(2\lambda a+\lambda b)+f(2\lambda a-\lambda b)-4\lambda^4 f(a+b)-4\lambda^4 f(a-b)-24\lambda^4 f(a)+6\lambda^4 f(b)\right\| \le \varepsilon(\|a\|^p+\|b\|^p),$$

$$\left\| a^4 f(b)-f(a)b^4 \right\| \le \varepsilon \|a\|^p \|b\|^p$$

for all $a,b \in A$ and all $\lambda \in T$. Also, if for each fixed $a \in A$ the mappings $t \to f(ta)$ from R to A are continuous, then there exists a unique quartic multiplier T on A satisfying

$$\|f(a)-T(a)\| \le \frac{\varepsilon}{|16-2^p|}\|a\|^p$$

for all $a \in A$.

Proof: Putting $\phi(a,b)=\varepsilon(\|a\|^p+\|b\|^p)$ and $\psi(a,b)=\varepsilon\|a\|^p\|b\|^p$ in Theorem 3.2, we get the desired results.

4. Superstability of quartic double centralizers

In this section, we prove the super stability of quartic double centralizers on Banach algebras which are quartic without order and quartic commutative.

Theorem *4.1.* Suppose that A is a Banach algebra quartic without order and quartic commutative and $s \in \{-1,1\}$. Let $L,R: A \to A$ are mappings for which there exists a function $\psi: A \times A \to [0,\infty)$ such that

$$\lim_{n\to\infty} n^{-4s}\psi(n^s x,y)=0=\lim_{n\to\infty} n^{-4s}\psi(x,n^s y)$$

$$\left\| x^4 L(y)-R(y)^4 \right\| \le \psi(x,y)$$

for all $x,y \in A$. Then (L,R) is a quartic double centralizer.

Proof: We first show that L is a quartic homogeneous. To do this, pick $\lambda \in C$ and $x,y \in A$. We have

$$\left\| n^{4s} z^4 (L(\lambda x)-\lambda^4 L(x))\right\| = \left\| n^{4s} z^4 L(\lambda x)-\lambda^4 n^{4s} z^4 L(x)\right\|$$

$$\le \left\| n^{4s} z^4 L(\lambda x)-R(n^s z)(\lambda x)^4\right\| + \left\| \lambda^4 R(n^s z)x^4-\lambda^4 n^{4s} z^4 L(x)\right\|$$

$$\le \psi(n^s z,\lambda x)+|\lambda|^4 \psi(n^s z,x).$$

So

$$\left\| z^4 (L(\lambda x)-\lambda^4 L(x))\right\| \le n^{-4s}\psi(n^s z,\lambda x)+\lambda^4 n^{-4s}\psi(n^s z,x).$$

Since A *is* quartic without order, we conclude that $L(\lambda x)=\lambda^4 L(x)$ The quarticity of L follows from

$$\left\|z^4(L(2x+y)+L(2x-y)-4L(x+y)-4L(x-y)-24L(x)+6L(y))\right\|$$
$$=n^{-4s}\left\|n^{4s}z^4L(2x+y)+n^{4s}z^4L(2x-y)-4n^{4s}z^4L(x+y)-4n^{4s}z^4L(x-y)\right.$$
$$\left.-24n^{4s}z^4L(x)+6n^{4s}z^4L(y)\right\|$$
$$\leq n^{-4s}[\left\|n^{4s}z^4L(2x+y)-R(n^sz)(2x+y)^4\right\|+\left\|n^{4s}z^4L(2x-y)-R(n^sz)(2x-y)^4\right\|$$
$$+4\left\|R(n^sz)(x+y)^4-n^{4s}z^4L(x+y)\right\|+4\left\|R(n^sz)(x-y)^4-n^{4s}z^4L(x-y)\right\|$$
$$+24\left\|R(n^sz)x^4-n^{4s}z^4L(x)\right\|+6\left\|R(n^sz)y^4-n^{4s}z^4L(y)\right\|]$$
$$\leq n^{-4s}[\psi(n^sz,2x+y)+\psi(n^sz,2x-y)+4\psi(n^sz,x+y)+4\psi(n^sz,x-y)$$
$$+24\psi(n^sz,x)+6\psi(n^sz,y)]$$

for all $x,y\in A$.

Finally, since A is a quartic commutative Banach algebra, we have

$$\left\|z^4(L(xy)-L(x)y^4)\right\|=n^{-4s}\left\|n^{4s}z^4L(xy)-n^{4s}z^4L(x)y^4\right\|$$
$$\leq n^{-4s}[\left\|n^{4s}z^4L(xy)-R(n^sz)(xy)^4\right\|$$
$$+\left\|R(n^sz)x^4y^4-n^{4s}z^4L(x)y^4\right\|]$$
$$\leq n^{-4s}[\psi(n^sz,xy)+\psi(n^sz,x)\left\|y\right\|^4]$$

for all $x,y\in A$. So $L(xy)=L(x)y^4$. Thus L *is* a quartic left centralizer. One can similarly prove that R *is* a quartic right centralizer. Since L *is* quartic homogeneous, $L(x)=n^{-4s}L(n^sx)$ for all $n\in N$ and $x\in A$. Thus

$$\left\|x^4(L(y)-R(x)y^4)\right\|=n^{-4s}\left\|x^4L(n^sy)-R(x)(n^{4s}y^4)\right\|$$
$$\leq n^{-4s}\psi(x,n^sy)$$

and hence by (4. 1) we infer that $x^4L(y)=R(x)y^4$ for all $x,y\in A$. Thus (L,R) *is* a quartic centralizer.

Corollary 4.2. Suppose A is a Banach algebra quartic without order and quartic commutative and $L,R:A\to A$ are mappings for which there exist a nonnegative real number ε and a real number p either greater than 4 or less than 4, such that

$$\left\|x^4T(y)-R(x)y^4\right\|\leq\varepsilon\left\|x\right\|^p\left\|y\right\|^p$$

for all $x,y\in A$. Then (L,R) is a quartic double centralizer.

Proof: Using Theorem 4.1 with $\psi(x,y)\leq\varepsilon\left\|x\right\|^p\left\|y\right\|^p$ we get the desired result.

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5. Superstability of quartic multipliers

In this section, we prove the superstability of quartic multipliers on Banach algebras which are quartic without order.

Theorem 5.1. Suppose that A is a Banach algebra with quartic without order and $s \in \{-1,1\}$. Let $T : A \to A$ are mappings for which there exists a function $\psi : A \times A \to [0,\infty)$ such that

$$\lim_{n\to\infty} n^{-4s}\psi(n^s x, y) = 0 = \lim_{n\to\infty} n^{-4s}\psi(x, n^s y)$$

$$\left\| x^4 L(y) - R(y)^4 \right\| \le \psi(x,y)$$

for all $x, y \in A$. Then (L,R) is a quartic multiplier.

Proof: By the same reasoning as in the proof of Theorem 4.1, putting $L = R = T$, we can show that the mapping T is a quartic multiplier.

Corollary 5.2. Suppose that A is a quartic without order Banach algebra and that $T : A \to A$ is a mapping for which there exist a nonnegative real number ε and a real number p either greater than 4 or less than 4**,** such that

$$\left\| x^4 T(y) - T(x) y^4 \right\| \le \varepsilon \|x\|^p \|y\|^p$$

for all $x, y \in A$. Then T is a quartic multiplier.

Proof: Using Theorem 5.1 with $\psi(x,y) = \varepsilon \|x\|^p \|y\|^p$,we get the result.

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