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Necessary Optimality Conditions for Problem With Integral Boundary and Distributed Conditions

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Abstract

Many of the physics, techniques and mechanics systems can better be described by integral boundary conditions. At this paper problems of optimal control with integral boundary and distributed conditions are considered. Starting from the necessary conditions for optimality represented by a Hamiltonian system, we solve the Hamilton-Jacobi equation for a generating function for a specific canonical transformation. Impulsive differential equations have become important in recent years as mathematical models.

Mathematics subject classification: 34 B10,49J22

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1. Introduction

The problems of optimal control are met in many of the physical systems can better be described by integral boundary conditions. The problem of optimal control for non-linear system with integral conditions in considered in the given paper i.e. it is considered minimization of the functional:

Minimize

$$J(u) = \varphi\big(x(t_\circ), x(T)\big) + \int_{t_\circ}^{T} F(x,u,t)dt \qquad (1)$$

On solutions of the system

$$\frac{dx}{dt} = f(x,u,t) \qquad ; t \in [t_\circ, T] \qquad (2)$$

Under Non-liner Conditions

$$\int_{t_\circ}^{T} \mathcal{K}(t)x(t)dt + Ax(T) = B \qquad (3)$$

Where

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$$u = u(0) \in U = \left\{u(t) \in L_2^r[t_\circ, T]; u(t) \in V \subset R^r\right\}$$

a.e $\quad t \in [t_\circ, T] \quad (4)$

and $\mathcal{K}(t)$ is an $n \times n$ matrix function, u is a control parameter ; $U \in R^r$ is an open set.

2. Statement of the problem

We will use the following assumptions:

I. $\varphi(x(t_\circ), x(T))$ and $F(x, u, t)$ are continuous by there own arguments and have continuous and bounded partial derivatives with respect to x, y and U up to second order, inclusively.

II. $\|M\| < 1$ for the matrix M defined by the formula $M = \int_{t_0}^{T} \mathcal{K}(t)dt$

III. The functions $f: [t_0, T] \times R^n \times R^r \to R^n$
And $I_i: R^n \to R^n; i = 1,2, \dots, P$ are continuous functions where
$x(t_i^+) - x(t_i) = I_i\big(x(t_i)\big); i = 1,2, \dots p$
$t_0 < t_1 < t_2 < \cdots < t_p < T$
And $L_i \geq 0 \quad ; \quad i = 1,2, \dots p$,

Such that $|f(x, t, u) - f(y, t, u| \leq p|x - y|$, $t \in [t_0, T]$, $x, y \in R^n, u \in R^r$

And $|I_i(y) - I_i(x)| \leq L_i|x - y|$; $x, y \in R^n$

IV. $L = (1 - \|\kappa\|^{-1}\left[PTN + \sum_{i=1}^{p} l_i\right] < 1$
Where $N = Max\|N(t, s)\|$
$0 \leq t_0$, $\quad s \leq T$

And $N\ (t, s) = \begin{cases} E + \int_{t_0}^{s} k(t)d\xi \ ; \ t_0 \leq t \leq s \\ -\int_{s}^{T} m(\xi) \ ; \ s \leq t \leq T \end{cases}$

V. Suppose that if the condition $\left\|\det(\int_{t_0}^{T} \kappa(t)dt + A)\right\| \neq 0$

VI. Note, that if the condition $\left\|\det \int_{t_0}^{T} \kappa(t)dt\right\| < 1$ holds, then matrix $(\int_{t_0}^{T} \kappa(t)dt)$ is reversible.

3. Problem formulation

Let $\{\mathrm{u}, x = x(\mathrm{t}, \mathrm{u})\}$and $\{\tilde{\mathrm{u}} = u + \Delta u, \tilde{x} = x + \Delta x = x(t, \tilde{u})\}$

Be two admissible processes. Applying (2) , (3) we obtain the boundary-Value problem

$$\Delta\dot{x} = \Delta f(x, t, u) \qquad ; t \in [t_0, T] \qquad (5)$$

$$\int_{t_0}^{T} \kappa(t)\Delta x(t)dt + A\Delta x(T) = 0 \qquad (6)$$

Where

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$$\Delta f(x,t,u) = \Delta f(\tilde{x},t,\tilde{u}) - f(x,t,u) \qquad (7)$$

Denotes he total increment of the function $f(x,t,u)$ be continuous by t,x,t for $x \epsilon R^n, u \epsilon V$, $t_0 \le t \le T$ and derivatives $f(t,x,u)$ with respect to x exist.

Let $(x(t),u(t))$ and $u(t)+\tilde{u}(t), x(t)+\tilde{x}(t)$ be two solutions of system (2), (3).

At this solutions increment of functional (1) is of the form:

$$\Delta J(u) = J(\tilde{u}) - J(u) = \Delta\varphi\big(x(t_0),x(T)\big) + \int_{t_0}^{T}\Delta F(x,u,t)dt + \int_{t_0}^{T} < \Psi(t), \Delta x - \quad \Delta f(x,u,t) > dt + < \lambda, A\Delta x(T) + \int_{t_0}^{T}\kappa(t)\Delta x(t)dt > 0 \qquad (8)$$

Where $\Psi(t) \in R^n$ Vector function and vector $\lambda \in R^n$.

Let us define the Hamiltonian -Jacobi as:

$$\mathrm{H}(\Psi,x,t,u) =< \Psi(\mathrm{t}), \mathrm{f}(x,\mathrm{t},\mathrm{u}) > -F(x,\mathrm{t},\mathrm{u}) \qquad (9)$$

Where, they are obtained differentiating the Hamiltonian with respect to the control (if the Hamiltonian is differentiable with respect to the control variable), state and constate variables.

We can write formula them as:

$$\Delta\varphi(x(\mathrm{t}_0),x(\mathrm{T}) =< \frac{\partial\varphi}{\partial x(\mathrm{t}_0)}, \Delta x(\mathrm{t}_0) > + < \frac{\partial\varphi}{\partial x(\mathrm{t}_0)}, \Delta x(\mathrm{T}) > + \mathrm{o}_\varphi(\|\Delta x(\mathrm{t}_0)\|, \|\Delta x(\mathrm{T})\|) \qquad (10)$$

We obtain:

$$\int_{\mathrm{t}_0}^{\mathrm{T}} < \Psi(\mathrm{t}), \Delta\dot{x}(\mathrm{t}) > dt =< \Psi(T), \Delta x(\mathrm{T}) > - < \Psi(\mathrm{t}_0), \Delta x(\mathrm{t}_0) >$$

$$-\int_{\mathrm{t}_0}^{\mathrm{T}} < \dot{\Psi}(t), \Delta x(t) > dt \qquad (11)$$

Taking into account (9)-(11) in (8) for in cerement of functional (6), (7), We get the following expression:

$$\Delta J(u) = -\int_{t_0}^{T} < \dot{\Psi}(t), \Delta x(t) > dt - \int_{t_0}^{T}\Delta_{\tilde{\mathrm{x}}\tilde{\mathrm{u}}} H(\Psi,x,t,u)dt + \int_{t_0}^{T} < \acute{\kappa}(t)\lambda, \Delta x(t) > dt$$

$$+ < \left[\frac{\partial\varphi}{\partial \mathrm{x}(\mathrm{T})} - \Psi(T) + \acute{A}\lambda\right], \Delta x(T) > + O_\varphi(\|\Delta x(T)\| \qquad (12)$$

Where

$$\Delta_{\tilde{\mathrm{x}}\tilde{\mathrm{u}}} H(\Psi,\tilde{x},t,\tilde{u}) = \Delta_{\tilde{x}}(\Psi,\tilde{x},t,\tilde{u}) - H(\Psi,x,t,u) \qquad (13)$$

And

$$\Delta_{\tilde{x}\tilde{u}} H(\Psi, x, t, u) = \Delta_{\tilde{x}}(\Psi, x, t, \tilde{u}) + \Delta_{\tilde{u}}(\Psi, x, t, u) \qquad (14)$$

Where

$$\Delta_{\tilde{x}} H(\Psi, x, t, \tilde{u}) =< \frac{\partial H(\Psi, x, t, \tilde{u})}{\partial x} \ , \ \ \Delta_x(t) > +O_H(\|\Delta x(t)\|) \qquad (15)$$

And

$$\frac{\partial H(\Psi, x, t, \tilde{u})}{\partial x} = \Delta_{\tilde{u}} \frac{\partial H(\Psi, x, t, u)}{\partial x} + \frac{\partial H(\Psi, x, t, u)}{\partial x} \qquad (16)$$

Furthermore, Putting the two equations (8),(16) together,

$$\Delta J(u) = -\int_{t_0}^{T} \Delta_{\tilde{u}} H(\Psi, x, t, u)dt - \int_{t_0}^{T} < \Delta_{\tilde{u}} \frac{\partial H(\Psi,x,t,u)}{\partial x} + \frac{\partial H(\Psi,x,t,u)}{\partial x} + \acute{\kappa}(t)\lambda + \dot{\Psi}(t), \qquad \Delta x(t) > dt - < \left[\frac{\partial \varphi}{\partial x(T)} - \Psi(T)\acute{A}\lambda\right], \Delta x(T) > +O_{\varphi}(\|\Delta x(T)\|)$$

$$-\int_{t_0}^{T} O_H\,(\|\Delta x(t)\|)dt \qquad (17)$$

Here, Consider a more general problem where there are n constraints, i.e, $\Psi \in R^n$ Let $\lambda \in R^n$ an constant vector is solution the following ad joint problem (called Lagrange multipliers) and define the function by state,

$$\dot{\Psi}(t) = \frac{\partial H(\Psi,x,t,u)}{\partial x} + \acute{\kappa}(t)\lambda \quad ; \qquad t \in [t_0, T] \qquad (18)$$

$$\frac{\partial \varphi}{\partial x(t_0)} - \Psi(t_0) + \lambda = 0 \qquad (19)$$

$$\frac{\partial \varphi}{\partial x(T)} + \Psi(T) = 0 \qquad (20)$$

$$\Psi(t_i^+) - \Psi(t_i) = -\acute{I}_{ix}\left(x(t_i)\right)\left(\acute{I}_{ix}\left(x(t_i)\right) + E\right)^{-1}\Psi(t_i) \qquad (21)$$

$$i = 1,2, \dots p$$

This means, that the functional (1) at restrictions (2), (4) is differentiable.

Theorem 1. Let condition (1)-(4) be fulfilled and $(u(t), x(t))$ for all $S \in R^m$ and $r \in [t_0, T]$. Then for optimality of the control $(u(t), x(t)) \in U$ it is necessary the fulfillment of the inequality

$$\acute{S}\, \frac{\partial^2 H(r, \Psi(r), x(r), u(r))}{\partial u^2} s \leq 0 \qquad (22)$$

The proof of this theorem is carried with the help of the method from [1],(see pp. 40)

Acknowledgments.

In this paper, We studied the problem for Implosive systems with integral boundary and distributed conditions, when the state of the system is described by the impulsive differentical equations.

Applying the banach contraction principle the existence and uniqueness of the solution is proved for the corresponding boundary problem by the fixed admissible control.

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