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Entire Sequence Space of Modals

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Abstract

In the past decades, modal analysis has become a major technology in the quest for determining, improving and optimizing dynamic characteristics of engineering structures. Not only has it been recognized in mechanical and aeronautical engineering, but modal analysis has also been discovered in profound applications for civil and building structures, space structures, transportation and nuclear problems. Shortly, modal analysis relies on mathematics to establish theoretical models for a dynamic system and to analyze data in various forms. Since modals are used in different branches of engineering in order to contribute to modal analysis, we have constructed some sequence spaces $\Gamma(gI)$ and $\Lambda(gI)$ of modal intervals. Also , we have given some new definitions and theorems about the sequence space of modals.

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1. Introduction

Interval arithmetic was first suggested by Dwyer [2] in 1951. Development of interval arithmetic as a formal system and evidence of its value as a computational device was provided by Moore [9] in 1959 and Moore and Yang [10] 1962. Furthermore, Moore and others [11] have developed applications to differential equations.

Chiao in [5] introduced sequence of interval numbers and defined usual convergence of sequences of interval number. Sengönül and Eryilmax [12] in 2010 studied bounded and convergent sequence space of interval numbers and showed that these spaces are complete metric space. Recently, Zararsiz and Sengönül[13] introduced null, bounded and convergent sequence space of modals.

Let us denote the set of all real valued closed interval by I ,the set of positive integers by N and the set of all real numbers by $\Re$. Any element of I is called interval number and it is denoted by $\hat{x}$. That is $\hat{x} = \{x \in \Re : \underline{x} \le x \le \overline{x}\}$. An interval number $\hat{x}$ is a closed subset of real numbers. Let $\underline{x}$ *and* $\overline{x}$ be respectively first and last points of the interval number $\hat{x}$.Therefore, when $\underline{x} > \overline{x}$, $\hat{x}$ is not an interval number. But in modal analysis $[\overline{x}, \underline{x}]$ is a valid interval. A modal $\tilde{x} = \{[\underline{x}, \overline{x}] : \underline{x}, \overline{x} \in \Re\}$ is defined by a pair of real numbers $\overline{x}, \underline{x}$. Let us denote the set of all modals by gI .Let us suppose that $\tilde{x}, \tilde{y} \in gI$. Then the algebraic operations between $\tilde{x}$ and $\tilde{y}$ are defined in the Kaucher arithmetic, [7]. For a modal $\tilde{x} = [\underline{x}, \overline{x}]$ *dual* operator is defined as $dual\tilde{x} = [\overline{x}, \underline{x}]$. Thus, if $\tilde{x} \in gI$,then $\tilde{x} - dual\tilde{x} = [0,0] = \tilde{0}$, $dual\tilde{x} \in gI$. Let us suppose that $\tilde{x} \in gI$, then $\tilde{x}$ is called symmetric modal if $\underline{x} = -\overline{x}$ or vice-versa.

The set of all modals gI is metric space defined as

$$d(\tilde{x}_1, \tilde{x}_2) = \max\{|\underline{x_1} - \underline{x_2}|, |\overline{x}_1, \overline{x}_2|\} \qquad (1)$$

If $\tilde{x}, \tilde{y} \in gI$ and $\underline{x} \le \overline{x}, \underline{y} \le \overline{y}$ then the set gI is reduced ordinary set of interval numbers which is complete metric space with the metric d defined in Eq.(1)[7].If we take $\tilde{x}_1 = [a, a]$ and $\tilde{x}_2 = [b, b]$,we obtain the usual metric of $\Re$ with $d(\tilde{x}_1, \tilde{x}_2) = |a - b|$, where $a, b \in \Re$

Let f be a function from N to gI which is defined by $k \to f(k) = \tilde{x}, \tilde{x} = (\tilde{x}_k)$ .Then $(\tilde{x}_k)$ is called sequence of modals. We will denote the set of all sequences of modals by $w(gI)$.

For two sequences of modals $(\tilde{x}_k)$ and $(\tilde{y}_k)$, the addition, scalar product and multiplication are defined as follows

$(\tilde{x}_k + \tilde{y}_k = [\underline{x}_k + \underline{y}_k, \overline{x}_k + \overline{y}_k]$, $(\alpha\tilde{x}_k = [\alpha\underline{x}_k, \alpha\overline{x}_k], \alpha \in \Re$, $(\tilde{x}_k\tilde{y}_k = [\underline{x}_k\underline{y}_k, \overline{x}_k\overline{y}_k]$ respectively.

The set $w(gI)$ is a vector space since the vector space rules are clearly provided. The zero element of $w(gI)$ is the sequence $\tilde{\theta} = (\tilde{\theta}_k) = ([0,0])$ all terms of which are zero interval. If $(\tilde{x}_k) \in w(gI)$ then inverse of $(\tilde{x}_k)$, according to addition, is $dual(\tilde{x}_k)$.

Let $\lambda(gI) \subset w(gI)$. If a sequence space contains a sequence $(\tilde{e}_n)$ of modals with the property that for every $\tilde{u} \in \lambda(gI)$ there is a unique sequence of scalars $(\tilde{t}_n)$ such that $\lim_n d(\tilde{u}, \tilde{t}_1\tilde{e}_1 + ... + \tilde{t}_n\tilde{e}_n) \to \tilde{0}$ then $(\tilde{e}_n)$ is called a

Schauder modal basis for $\lambda(gI)$. The series $\sum_{k=1}^{\infty}\tilde{t}_k\tilde{e}_k$ which has the sum $\tilde{u}$ is then called the expansion of $\tilde{u}$ with respect to $(\tilde{e}_n)$, and we write $\tilde{u}=\sum_{k=1}^{\infty}\tilde{t}_k\tilde{e}_k$.

Let $\lambda(gI)$ and $\mu(gI)$ be linear space of modals. Then a function $\tilde{A}:\lambda(gI)\to\mu(gI)$ is called a linear transformation if and only if, for all $\tilde{u}_1,\tilde{u}_2\in\lambda(gI)$ and all $\tilde{t}_1,\tilde{t}_2\in gI$, $\tilde{A}(\tilde{t}_1\tilde{u}_1+\tilde{t}_2\tilde{u}_2)=\tilde{t}_1\tilde{A}\tilde{u}_1+\tilde{t}_2\tilde{A}\tilde{u}_2$.

1.1 Proposition

If $(\tilde{x}_k)$, $(\tilde{y}_k)$, $(\tilde{r}_k)$ are sequences of symmetric modal, then the following equality holds:

$$(\tilde{x}_k)\{(\tilde{y}_k)-(\tilde{r}_k)\}=(\tilde{x}_k)(\tilde{y}_k)-(\tilde{x}_k)(\tilde{r}_k) \qquad (2)$$

1.2 Definition

A sequence $\tilde{x}=(\tilde{x}_k)\in w(gI)$ of modals is said to be convergent to the modal $\tilde{x}_0$ if for each $\varepsilon>0$ there exists a positive integer n_0 such that $d(\tilde{x}_k,\tilde{x}_0)<\varepsilon$ for all $k\geq n_0$ and we denote it by writing $\lim_k\tilde{x}_k=\tilde{x}_0$. Thus, $\lim_{k\to\infty}\tilde{x}_k=\tilde{x}_0\Leftrightarrow\lim_{k\to\infty}\underline{x}_k=\underline{x}_0$ *and* $\lim_{k\to\infty}\bar{x}_k=\bar{x}_0$.

1.3 Definition

A sequence of modals, $\tilde{x}=(\tilde{x}_k)\in w(gI)$, is said to be modal fundamental sequence if for every $\varepsilon>0$ there exists $k_0\in\mathrm{N}$ such that $d(\tilde{x}_k,\tilde{x}_n)<\varepsilon$ whenever $n,k>k_0$.

We define convergent series, bounded series and p - absolute convergent series of sequences spaces of the symmetric modals which are denoted $cs(gI),bs(gI),l_p(gI)$ respectively, that is

$$cs(gI)=\left\{\tilde{x}=(\tilde{x}_k)\in w(gI):\lim_n\left(d(\sum_{k=1}^{n}\tilde{x}_k,\tilde{x}_n)\right)=\tilde{0}\right\},$$

$$bs(gI)=\left\{\tilde{x}=(\tilde{x}_k)\in w(gI):\sup_n\left(d(\sum_{k=1}^{n}\tilde{x}_k,\tilde{0})\right)<\infty\right\},$$

$$l_p(gI)=\left\{\tilde{x}=(\tilde{x}_k)\in w(gI):\left(\sum_{k=1}^{n}[d(\tilde{x}_k,\tilde{0})]^p\right)^{1/p}<\infty,p\geq1\right\}$$

Clearly we see that the spaces $cs(gI),bs(gI)$ and $l_p(gI)$ are sub vector spaces in accordance with scalar product and addition on $w(gI)$ which are metric spaces.

2. Entire Sequence space of modals.

We define the entire sequence spaces of symmetric modals which are denoted by $\Gamma(gI)$ and $\Lambda(gI)$ respectively. $\Gamma(gI)=\left\{\tilde{x}=(\tilde{x}_k)\in w(gI):\lim_k\left(D(\tilde{x}_k,\tilde{0})\right)=\tilde{0}\right\}$,

$$\Lambda(gI)=\left\{\tilde{x}=(\tilde{x}_k)\in w(gI):\sup_k\left(D(\tilde{x}_k,\tilde{0})\right)<\infty\right\}$$

Where $D(\tilde{x}_k,\tilde{y}_k)=\max\left\{\left|\underline{x}_k-\underline{y}_k\right|^{1/k}\left|\bar{x}_k-\bar{y}_k\right|^{1/k}\right\}$

The function $\tilde{d}$ defined by

$$\tilde{d}(\tilde{x}_k,\tilde{y}_k)=\sup_k\max\left\{\left|\underline{x}_k-\underline{y}_k\right|^{1/k}\left|\bar{x}_k-\bar{y}_k\right|^{1/k}\right\}=\sup_k D(\tilde{x}_k,\tilde{y}_k) \qquad (3)$$

which satisfies the metric space axioms .

2.1.Theorem

The sequence space $\Gamma(gI)$ is a complete metric space with respect to the metric defined by (3)

Proof . Let $(\tilde{x}^{(n)})$ be a fundamental sequence of generalized interval numbers in $\Gamma(gI)$.Then for a given $\varepsilon>0$ there exists a positive integer n_0 such that

$\tilde{d}(\tilde{x}_k^{(n)},\tilde{x}_k^{(m)})=\sup_k D(\tilde{x}_k^{(n)},\tilde{x}_k^{(m)})<\varepsilon$ for all $n,m\geq n_0$

This is true for all k, we have $D(\tilde{x}_k^{(n)},\tilde{x}_k^{(m)})<\varepsilon$ for all $n,m\geq n_0$

$\max\left\{\left|\underline{x}_k^{(n)}-\underline{x}_k^{(m)}\right|^{1/k},\left|\bar{x}_k^{(n)}-\bar{x}_k^{(m)}\right|^{1/k}\right\}<\varepsilon$ for all $n,m\geq n_0$

$\left|\underline{x}_k^{(n)}-\underline{x}_k^{(m)}\right|^{1/k}<\varepsilon^k$ *and* $\left|\bar{x}_k^{(n)}-\bar{x}_k^{(m)}\right|^{1/k}<\varepsilon^k$ for all $n,m\geq n_0$

This leads to the fact $\tilde{x}_k^{(n)}$ is a fundamental sequence in gI .Since gI is a complete metric space, $\tilde{x}_k^{(n)}$ is convergent $\lim_n\tilde{x}_k^{(n)}=\tilde{x}_k$ for each $k\in\mathrm{N}$

This is true for all k, $\sup_k D(\tilde{x}_k^{(n)},\tilde{x}_k)<\varepsilon$

so $\tilde{x}_k^{(n)}\to\tilde{x}_k$ *as* $n\to\infty$ *in* $\Gamma(gI)$

we have to show that $\tilde{x}=(\tilde{x}_k)\in\Gamma(gI)$

since $\tilde{x}_k^{(n)}\in\Gamma(gI)$, we have $\tilde{d}(\tilde{x}_k^{(n)},\tilde{0})<\varepsilon$

Consider

$$\tilde{d}(\tilde{x}_k,\tilde{0})=\sup_k D(\tilde{x}_k,\tilde{0})\leq\sup_k D(\tilde{x}_k^{(n)},\tilde{x}_k)+\sup_k D(\tilde{x}_k^{(n)},\tilde{0})$$
$$<\varepsilon+\varepsilon=2\varepsilon$$

Hence $(\tilde{x}_k)\in\Gamma(gI)$

This completes the proof.

2.2. Theorem

A necessary and sufficient condition that $D(\sum\tilde{x}_k\tilde{y}_k,\tilde{0})$ should be convergent for every $(\tilde{x}_k)$ for which $\lim_k\left(D(\tilde{x}_k,\tilde{0})\right)=\tilde{0}$ is that $D(\tilde{y}_k,\tilde{0})$ should be bounded.

Proof. Suppose $D(\tilde{y}_k,\tilde{0})$ is bounded. Then we can find M so that $D(\tilde{y}_k,\tilde{0})\leq M$ *for* $k\geq1$

Since $D(\tilde{x}_k,\tilde{0})\to\tilde{0}$ as $k\to\infty$,

We can find k_0 so that $D(\tilde{x}_k,\tilde{0})\leq\frac{1}{2M},k\geq k_0$

$[D(\tilde{x}_k\tilde{y}_k,\tilde{0})]^k \le [D(\tilde{x}_k,\tilde{0})]^k[D(\tilde{y}_k,\tilde{0})]^k$

$$< \left(\frac{1}{2M}\right)^k M^k = \frac{1}{2^k}$$

So $\sum[D(\tilde{x}_k\tilde{y}_k,\tilde{0})]^k$ converges

Converse:

Suppose $D(\tilde{y}_k,\tilde{0})$ is not bounded .Then we can find an increasing sequence $\{k_p\}$ of integers such that

$D(\tilde{y}_{k_p},\tilde{0}) \ge p \;\; p = 1,2,..$

That is, $[D(\tilde{y}_{k_p},\tilde{0})]^{k_p} \ge p^{k_p}$, $p = 1,2,..$

Take $\tilde{x}_k = \begin{cases} [1/p^k,0]\; if\;\; k = k_p \\ [0,0]\; if\;\; k \ne k_p \end{cases}$

Then $\lim_k \left(D(\tilde{x}_k,\tilde{0})\right) = \tilde{0}$

But

$[D(\tilde{x}_k\tilde{y}_k,\tilde{0})]^{k_p} \ge [D(\tilde{x}_k,\tilde{0})]^{k_p}[D(\tilde{y}_k,\tilde{0})]^{k_p}$

$$= \left[\left(\frac{1}{p^{k_p}}\right)^{1/k_p}\right]^{k_p}\left[\left(p^{k_p}\right)^{1/k_p}\right]^{k_p} = \frac{1}{p}.p = 1$$

$for\; k = k_p$

so that $\sum[D(\tilde{x}_k\tilde{y}_k,\tilde{0})]$ does not converges.

Hence $D(\tilde{y}_k,\tilde{0})$ is bounded.

Hence the theorem

2.3. Theorem.

The sets $\Gamma(gI)$ and $\Lambda(gI)$ sequence spaces of modal is solid.

Proof: We consider $\Gamma(gI)$ Now let $\tilde{d}(\tilde{y}_k,\tilde{0}) \le \tilde{d}(\tilde{x}_k,\tilde{0})$ for all $k \in N$

and for some $\tilde{x} \in \Gamma(gI)$.

Then we have

$\sup_k \max\left\{\left|\underline{y}_k\right|^{1/k}\left|\bar{y}_k\right|^{1/k}\right\} \le \sup_k \max\left\{\left|\underline{x}_k\right|^{1/k}\left|\bar{x}_k\right|^{1/k}\right\}$

$\underline{y}_k \le \underline{x}_k \; and \; \bar{y}_k \le \bar{x}_k$

That is $\tilde{y} \le \tilde{x}$

It is clear that $\tilde{y} \in \Gamma(gI)$

Therefore $\Gamma(gI)$ is solid.

2.4.Theorem

The sequence $(\tilde{e}_1,\tilde{e}_2....\tilde{e}_k,..)$ is schauder modal base for $\Gamma(gI)$, where $\tilde{e}_k = \{\tilde{0},\tilde{0},...[1,1],\tilde{0},...\}$

Proof: Let $\tilde{x} = (\tilde{x}_k) \in \Gamma(gI)$. Therefore for every $\varepsilon > 0$ there exists a positive integer $n \in N$ such that $k \ge n$,

$\tilde{d}(\tilde{x}_k,\tilde{0}) = \sup_k D(\tilde{x}_k,\tilde{0}) < \varepsilon$

Now we should show the following statement.

$\lim_{k\to\infty} \tilde{d}((\tilde{x}_k - \sum \tilde{e}_k\tilde{x}_k),\tilde{0}) = \tilde{0}$

From here we can write next steps

$\tilde{d}((\tilde{x}_k - \sum_{k=1}^{n}\tilde{e}_k\tilde{x}_k),\tilde{0}) = \tilde{d}(([\underline{x}_1,\bar{x}_1],[\underline{x}_2,\bar{x}_2],......[\underline{x}_k,\bar{x}_k],...)$

$- ([\underline{x}_1,\bar{x}_1],[\underline{x}_2,\bar{x}_2],......[\underline{x}_n,\bar{x}_n]),\tilde{0})$

$= \tilde{d}(([\tilde{0},\tilde{0},......[\underline{x}_{n+1},\bar{x}_{n+1}],[\underline{x}_{n+2},\bar{x}_{n+2}]),\tilde{0})$

$= \sup_{k\ge n+1} \max\left\{\left|\underline{x}_k\right|^{1/k}\left|\bar{x}_k\right|^{1/k}\right\} \to \tilde{0} \; as \; n \to \infty$

We have $\tilde{x}_k = \sum_{k=1}^{\infty}\tilde{e}_k\tilde{x}_k$ (4)

Let us show uniqueness of the representation given by Eq.(4) for $\tilde{x} = (\tilde{x}_k) \in \Gamma(gI)$

Suppose that there exists a representation $\tilde{x}_k = \sum_{k=1}^{\infty}\tilde{e}_k\tilde{y}_k$,

then for $n \to \infty$,we have

$\tilde{d}(\sum_{k=1}^{n}(\tilde{x}_k - \tilde{y}_k)\tilde{e}_k,\tilde{0}) = \sum_{k=1}^{n}\tilde{d}((\tilde{x}_k - \tilde{y}_k)\tilde{e}_k,\tilde{0})$

$= \sup_{k\ge n+1}\max\left\{\left|\underline{y}_k - \underline{x}_k\right|^{1/k}\left|\bar{y}_k - \bar{x}_k\right|^{1/k}\right\} \to \tilde{0} \; as\; n \to \infty$

$\left|\underline{y}_k - \underline{x}_k\right|^{1/k} \to \tilde{0} \; and \; \left|\bar{y}_k - \bar{x}_k\right|^{1/k} \to \tilde{0} \; as \; n \to \infty$

Therefore $\underline{y}_k = \underline{x}_k \; and \; \bar{y}_k = \bar{x}_k$

That is $\tilde{y} = \tilde{x}$

3. α,β,γ duals of sequence space of modals.

For the sequence spaces $\lambda(gI)$ and $\mu(gI)$, we define the set $S(\lambda(gI),\mu(gI))$ by

$S(\lambda(gI),\mu(gI)) = \{(\tilde{y}_k) \in w(gI) : (\tilde{x}_k\tilde{y}_k) \in \mu(gI)\}$

$for\; all\; \tilde{x}_k \in \lambda(gI)$ (5)

with the notation of (5), the α,β,γ duals of a sequence space $\lambda(gI)$ which are denoted by

$\lambda^{\alpha}(gI), \lambda^{\beta}(gI) \; and \; \lambda^{\gamma}(gI)$ are defined by

$$\lambda^{\alpha}(gI) = S(\lambda(gI), l_1(gI)),$$
$$\lambda^{\beta}(gI) = S(\lambda(gI), cs(gI))$$
$$\lambda^{\gamma}(gI) = S(\lambda(gI), bs(gI))$$

3.1.Theorem

The β dual of sequence space $\Gamma(gI)$ is $\Lambda(gI)$

Proof: Let us suppose that and $\tilde{y} = (\tilde{y}_k) \in \Lambda(gI)$ for every $\tilde{x} = (\tilde{x}_k) \in \Gamma(gI)$, then $\sup D(\tilde{y}_k,\tilde{0}) < \infty$

We can write

$$\lim_n D(\sum_{k=1}^{n} \tilde{x}_k \tilde{y}_k, \tilde{0}) = \lim_n D(\sum_{k=1}^{n} [\underline{y}_k, \overline{y}_k], [\underline{x}_k, \overline{x}_k], \tilde{0})$$

$$= \lim_n D(\sum_{k=1}^{n} [\underline{y}_k \underline{x}_k, \overline{y}_k \overline{x}_k], \tilde{0})$$

$$= \lim_n \max\left\{ \left| \sum_{k=1}^{n} \underline{y}_k \underline{x}_k \right|^{1/k}, \left| \sum_{k=1}^{n} \overline{y}_k \overline{x}_k \right|^{1/k} \right\}$$

$$\le \lim_n \max\left\{ \sum_{k=1}^{n} \left| \underline{y}_k \underline{x}_k \right|^{1/k}, \sum_{k=1}^{n} \left| \overline{y}_k \overline{x}_k \right|^{1/k} \right\}$$

$$= \lim_n M \max\left\{ \sum_{k=1}^{n} \left| \underline{x}_k \right|^{1/k}, \sum_{k=1}^{n} \left| \overline{x}_k \right|^{1/k} \right\}$$

Where $M = \max\{M_1, M_2\}$

$$M_1 = \sup_k \left| \underline{y}_k \right|^{1/k}, M_2 = \sup_k \left| \overline{y}_k \right|^{1/k}$$

$$\lim_n D(\sum_{k=1}^{n} \tilde{x}_k \tilde{y}_k, \tilde{0}) \le \lim_n MD(\sum_{k=1}^{n} \tilde{x}_k, \tilde{0})$$

$$= MD(\sum_{k=1}^{\infty} \tilde{x}_k, \tilde{0}) = M \sum_{k=1}^{\infty} D(\tilde{x}_k, \tilde{0}) < \infty$$

Therefore, we get $\tilde{x}_k \tilde{y}_k \in cs(gI)$. Hence $(\tilde{y}_k) \in \Gamma^{\beta}(gI)$

$$\Lambda(gI) \subset \Gamma^{\beta}(gI) \qquad (6)$$

Let $\tilde{y} = (\tilde{y}_k) \in \Gamma^{\beta}(gI)$,then $\sum D(\tilde{x}_k \tilde{y}_k, \tilde{0})$ converges for every $\tilde{x} = (\tilde{x}_k) \in \Gamma(gI)$

By theorem 2.2, $D(\tilde{y}_k, \tilde{0})$ is bounded.

$\sup_k D(\tilde{y}_k, \tilde{0})$ is bounded, then $\tilde{y} = (\tilde{y}_k) \in \Lambda(gI)$

$$\Gamma^{\beta}(gI) \subset \Lambda(gI) \qquad (7)$$

From (6),(7) $\Gamma^{\beta}(gI) = \Lambda(gI)$

3.2. Theorem. $\Gamma^{\alpha}(gI) = \Gamma^{\beta}(gI) = \Gamma^{\gamma}(gI) = \Lambda(gI)$

Proof: From theorem 3.1, $\Gamma^{\beta}(gI) = \Lambda(gI)$

From theorem 2.3 and theorem 2.4

$\Gamma^{\alpha}(gI) = \Gamma^{\beta}(gI) = \Gamma^{\gamma}(gI)$

Hence $\Gamma^{\alpha}(gI) = \Gamma^{\beta}(gI) = \Gamma^{\gamma}(gI) = \Lambda(gI)$

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