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New weak forms of faint continuity

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Abstract.

The concept of M-open sets [15] can be applied in modifications of rough set approximations [36, 35] which is widely applied in many application fields. The aim of this paper is to introduce and study new forms of faint continuity which are called faint M-continuity. Moreover, basic properties and preservation theorems of faintly M-continuous functions are investigated. Also, the relationships between these functions and other forms are discussed.

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1. Introduction

In 1982, Long and Herrington [28] defined a weak form of continuity called faintly continuous by making use of θ-open sets. They obtained a large number of properties concerning such functions and among them, showed that every weakly continuous function is faintly continuous. Noiri and Popa [32] introduced and investigated three weaken forms of faint continuity which are called faint semicontinuity and faint precontinuity and faint β-continuity. Also, a nother weaker form of this class of functions called faint α-continuity and faint-b-continuity are introduced and investigated in [23, 30]. Caldas [6] exhibited and studied among others of new weaker form of this class of functions called faint e-continuity. Recently, A good number of researchers have also initiated different types of faintly continuous like functions in the papers [33, 31, 14]. In this paper, we introduce and investigate another form of faint continuity namely, faint M-continuity. Also, some of fundamental properties of them are studied.

2. Preliminaries.

Throughout this paper (X, τ) and (Y, σ) (Simply, X and Y) represent topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The closure of subset A of X, the interior of A and the complement of A is denoted by cl(A), int(A) and A^c or X\A respectively. A subset A of a space (X, τ) is called regular open [39] if A= int(cl(A)). A point x∈X is said to be a θ-interior point of

A [28] if there exists an open set U containing x such that U $\subseteq$ cl(U) $\subseteq$ A. The set of all θ-interior points of A is said to be the θ-interior set and denoted by $int_\theta(A)$. A subset A of X is called θ-open if A = $int_\theta(A)$.The family of all θ-open sets in a topological space (X,τ) forms a topology τ_θ on X. This topology is coarser than τ. A subset A of a space (X,τ) is called preopen [29] or locally dense [8] (resp. δ-preopen [37], α-open [31], β-open [1], semi-open [26], δ-semi-open [34], θ-semi-open [7] , e-open [11], e*-open [12], b-open [4] or γ-open [13] if A $\subseteq$ int(cl(A)) (resp. A $\subseteq$ $int(cl_\delta(A))$, A $\subseteq$ int(cl(int(A))), A $\subseteq$ cl(int(cl(A))), A $\subseteq$ cl(int(A)), A $\subseteq$ $cl(int_\delta(A))$, A $\subseteq$ $cl(int_\theta(A))$, A $\subseteq$ $cl(int_\delta(A)) \cup int(cl_\delta(A))$, A $\subseteq$ $cl(int(cl_\delta(A)))$, A $\subseteq$ cl(int(A)) $\cup$ int(cl(A)).

A subset A of a space (X,τ) is called M-open [15] if A $\subseteq cl(int_\theta(A)) \cup int(cl_\delta(A))$.

The complement of preopen (resp. δ-preopen, α-open, β-open, semi-open, γ-open, e-open, e*-open, δ-semi-open, θ-semi-open, θ-open, M-open) set is called preclosed (resp. δ-preclosed, α-closed, β-closed, semiclosed, γ-closed, e-closed, e*-closed, δ-semiclosed, θ-semiclosed , θ-closed, M-closed). The family of all preopen (resp. δ-preopen, α-open, β-open, semi-open, γ-open, θ-semi-open, e-open, e*-open, δ-semi-open , θ-open, M-open) is denoted by PO(X) (resp. δ-PO(X), αO(X), βO(X), SO(X), γO(X), θ-SO(X), e-O(X), e*O(X), δ-SO(X), θ-O(X), MO(X)). The union of all M-open (resp. θ-open, θ-semi-open, δ-preopen, e-open) sets contained in A is called the M-interior [15] (resp. θ-interior [28], θ-semi-interior [7], δ-pre-interior [37], e-interior [11]) of A and it is denoted by M-int(A) (resp. $int_\theta(A)$, $sint_\theta(A)$, $pint_\delta(A)$, e-int(A)). The intersection of all M-closed (resp.θ-semi-closed, δ-preclosed, e-closed) sets containing A is called the M-closure [15] (resp. θ-semi-closure [7], δ-preclosure [37], e-closure [11]) of A and it is denoted by M-cl(A) (resp. $scl_\theta(A)$, $pcl_\delta(A)$). A point $x \in$ X is called a θ-cluster [28] (resp. δ-cluster [40]) point of A if cl(U) $\cap$ A $\neq \phi$ (resp. int(cl(U)) $\cap$ A $\neq \phi$) for every open set U of X containing x. The set of all θ-cluster (resp. δ-cluster) points of A is called the θ-closure (resp. δ-closure) of A and is denoted by $cl_\theta(A)$ (resp. $cl_\delta(A)$). We observe that for any topological space (X, τ) the relation $\tau_\theta \subseteq \tau_\delta \subseteq \tau$ always holds. We also have A $\subseteq$ cl(A) $\subseteq cl_\delta(A) \subseteq cl_\theta(A)$, for any subset A of X.

The study of rough sets on an approximation space was initiated by [36, 35]. Rough set theory is one of the new methods that connect information systems and data processing to mathematics in general and especially to the theory of topological structures and spaces. A large number of authors [2, 25, 3, 21, 24, 27, 41, 5] had turned their attention to the generalization of approximation spaces which is widely applied in many applications fields.

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We recall the following definitions and results, which are useful in the sequel.

Definition 2.1. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called faintly continuous [28] (resp. faintly α-continuous [30], faintly precontinuous [32], faintly semicontinuous [32], faintly β-continuous [32], faintly γ-continuous [30], faintly e-continuous [6], faintly e*-continuous, faintly δ-precontinuous [10], faintly δ-semicontinuous [9], faintly θ-semicontinuous) if, $f^{-1}(V)$ is open (resp. α-open, preopen, semi-open, β-open, γ-open, e-open, e*-open, δ-preopen, δ-semi-open, θ-semi-open) in X for every θ-open set V of Y.

Definition 2.2. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called:

(i) θ-continuous [20] if, $f^{-1}(V) \in \theta\text{-}O(X)$ for every $V \in \sigma$,

(ii) quasi θ-continuous [22] if, $f^{-1}(V) \in \theta\text{-}O(X)$ for every $V \in \theta\text{-}O(Y)$.

Definition 2.3. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called

(i) M-continuous [17] if, $f^{-1}(U) \in MO(X)$, for each $U \in \sigma$,

(ii) M-irresolute [17] if, $f^{-1}(U) \in MO(X)$, for each $U \in MO(Y)$,

(iii) pre-M-open [19] if, $f(U) \in MO(Y)$, for each $U \in MO(X)$,

(iv) pre-M-closed [19] if, $f(U) \in MC(Y)$, for each $U \in MC(X)$.

Definition 2.4. [18] A mapping $f : (X, \tau) \to (Y, \sigma)$ is called :

(i) weakly-M-continuous if, for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in MO(X)$ such that $x \in U$ and $f(U) \subseteq cl(V)$,

(ii) contra-M-continuous, if $f^{-1}(U) \in MC(X)$, for every open set U of Y.

Lemma 1.1. For a topological space (X, τ) and $A \subseteq X$, then the following statements are hold:

(i) If $A \subseteq F_i$, F_i is an M-closed set of X, then $A \subseteq M\text{-}cl(A) \subseteq F_i$,

(ii) If $G_i \subseteq A$, G_i is an M-open set of X, then $G_i \subseteq M\text{-}int(A) \subseteq A$.

Proposition 1.1. Let (X, τ) be a topological space and $A \subseteq X$. Then, the following statements are hold:

(i) $b_\theta(A) = cl_\theta(A) \setminus int_\theta(A)$ [22],

(ii) $M\text{-}b(A) = M\text{-}cl(A) \setminus M\text{-}int(A)$ [15],

(iii) $M\text{-}Bd(A) = A \setminus M\text{-}int(A)$ [17].

The set of θ-boundary (resp. M-boundary, M-border) of A is denoted by $b_\theta(A)$ (resp. M-b(A), M-Bd(A)).

Theorem 2.1. [18] If a mapping $f : (X, \tau) \to (Y, \sigma)$ is contra-M-continuous and Y is regular, then f is M-continuous.

3. Faintly M-continuous mappings.

Definition 3.1. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called faintly M-continuous, if

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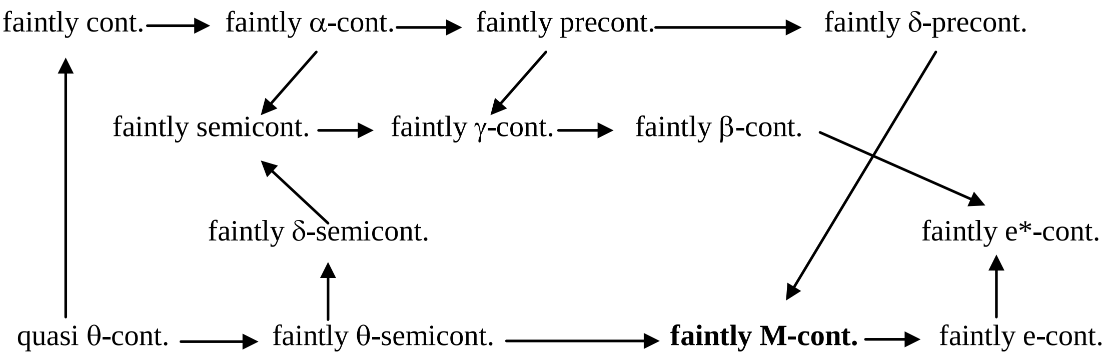
$f^{-1}(U) \in MO(X)$, for every θ-open set U of Y.

Remark 3.1. The implication between some types of mappings of Definitions 2.1, 3.1, are given by the following diagram.



None of these implications is reversible by [39, 29, 37, 31, 1, 26, 34, 11, 7, 12, 13, 15, 22] and the following examples.

Example 3.1. Let X = Y = {a, b, c, d} with topologies τ = {X, ϕ, {a},{b},{a, b}} and σ = {Y, ϕ, {a}, {a, d}, {b, c}, {a, b, c}}. Then the identity mapping $f : (X, \tau)\rightarrow(Y, \sigma)$ is faintly e-continuous but not faintly M-continuous. Since, $f^{-1}(\{a, d\}) = \{a, d\} \notin MO(X)$.

Example 3.2. Let X = Y = {a, b, c, d} with topologies τ = {X, ϕ, {a},{c},{a, b}, {a, c}, {a, b, c}, {a, c, d}} and σ = {Y, ϕ,{a, b}, {c, d}}. Then the identity mapping

$f : (X, \tau)\rightarrow (Y, \sigma)$ is faintly M-continuous but not faintly θ-semicontinuous. Since,

$f^{-1}(\{a, b\}) = \{a, b\} \notin \theta\text{-}SO(X)$.

Furthermore, $f : (X, \tau)\rightarrow (Y, \sigma)$ is faintly M-continuous but not faintly δ-precontinuous. Since, $f^{-1}(\{c, d\}) = \{c, d\} \notin \delta\text{-}PO(X)$.

The following example is an application of the concept of M-open sets in the rough set approximations.

Example 3.3. If we have the following information system. The objects{x_1, x_2, x_3, x_4} represent the ID of students, the attributes {EL(1), MA, AL(1)} are three salyets studied by the students, EL(1) is English language (1), MA is Mathematics and AL(1) is Arabic language(1). The values are the numbers scored by the students in an exam in the following table.

Object(U)	a_1 EL(1)	a_2 MA	a_3 AL(1)
x_1	86	83	77
x_2	93	85	81
x_3	89	60	78
x_4	88	60	82

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And consider the relation R_i on the set of objects defined by :

$x\ R_i\ y$ iff $|a_i(x) - a_i(y)| \leq 2$, i = 1,2,3. Then we can get the following classifications corresponding to every subclass of attributes:

$S_{EL(1)} = \{\{x_1\}, \{x_1, x_4\}, \{x_2, x_3\}\}$, $S_{MA} = \{\{x_4\},\{x_1, x_3\},\{x_2, x_4\}\}$, $S_{AL(1)} = \{\{x_1\},\{x_2\}, \{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}\}$ and hence the topologies generated by the above classes are: $\tau_{El(1)} = \{U, \phi, \{x_1\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}\}$, $\tau_{MA} = \{U, \phi, \{x_4\}, \{x_1, x_3\},\{x_2, x_4\}, \{x_1, x_3, x_4\}\}$ and $\tau_{AL(1)} = \{U, \phi, \{x_1\}, \{x_2\},\{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}\}$. Hence, the identity mapping $f : (U, \tau_{MA}) \to (U, \tau_{El(1)})$ is faintly M-continuous. But the identity mapping $g : (U, \tau_{AL(1)}) \to (U, \tau_{El(1)})$ is not faintly M-continuous.

Theorem 3.1. For a mapping $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:

(i) f is faintly M-continuous,

(ii) For each $x \in X$ and each θ-open V of $f(x)$ in Y, there exists an M-open set U of x in X such that $f(U) \subseteq V$,

(iii) $f^{-1}(F)$ is M-closed in X, for every θ-closed set F of Y,

(iv) $M\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(cl_\theta(B))$, for each $B \subseteq Y$,

(v) $f(M\text{-}cl(A)) \subseteq cl_\theta(f(A))$, for each $A \subseteq X$,

(vi) $f^{-1}(int_\theta(B)) \subseteq M\text{-}int(f^{-1}(B))$, for each $B \subseteq Y$,

(vii) $M\text{-}Bd(f^{-1}(B)) \subseteq f^{-1}(Bd_\theta(B))$, for each $B \subseteq Y$,

(viii) $M\text{-}b(f^{-1}(B)) \subseteq f^{-1}(b_\theta(B))$, for each $B \subseteq Y$.

Proof. (i)→(ii). Let $x \in X$ and $V \subseteq Y$ be a θ-open set containing $f(x)$. Then $x \in f^{-1}(V)$. Hence by hypothesis, $f^{-1}(V)$ is M-open set of X containing x. We put $U = f^{-1}(V)$, then $x \in U$ and $f(U) \subseteq V$.

(ii)→(iii). Let $F \subseteq Y$ be θ-closed. Then Y\F is θ-open and $x \in f^{-1}(Y\setminus F)$. Then $f(x) \in Y\setminus F$. Hence by hypothesis, there exists an M-open set U containing x such that $f(U) \subseteq Y\setminus F$, this implies that, $x \in U \subseteq f^{-1}(Y\setminus F)$. Therefore, $f^{-1}(Y\setminus F) = \cup\{U : x \in f^{-1}(Y\setminus F)\}$ which is M-open in X. Therefore, $f^{-1}(F)$ is M-closed.

(iii)→(i). Let $V \subseteq Y$ be a θ-open set. Then Y\V is θ-closed in Y. By hypothesis, $f^{-1}(Y\setminus V) = X\setminus f^{-1}(V)$ is M-closed and hence $f^{-1}(V)$ is M-open. Therefore, f is faintly M-continuous.

(i)→ (iv). Since $B \subseteq cl_\theta(B) \subseteq Y$ which is a θ-closed set. Then by hypothesis, $f^{-1}(cl_\theta(B))$ is M-closed in X . Hence, by Lemma 1.1, $M\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(cl_\theta(B))$ for each $B \subseteq Y$.

(iv) →(v). Let $A \subseteq X$. Then $f(A) \subseteq Y$, hence by hypothesis, $M\text{-}cl(A) \subseteq M\text{-}cl(f^{-1}(f(A))) \subseteq f^{-1}(cl_\theta(f(A)))$. Therefore, $f(M\text{-}cl(A)) \subseteq f f^{-1}(cl_\theta(f(A))) \subseteq cl_\theta(f(A))$,

(v) →(i). Let $V \subseteq Y$ be a θ-closed set. Then, $f^{-1}(V) \subseteq X$. Hence, by hypothesis, $f(M\text{-}cl(f^{-1}(V))) \subseteq cl_\theta(f(f^{-1}(V))) \subseteq cl_\theta(V) = V$. Thus $M\text{-}cl(f^{-1}(V)) \subseteq f^{-1}(V)$ and hence $f^{-1}(V) \in MC(X)$. Hence, f is faintly M-continuous,

(i) → (vi). Since $int_\theta(B) \subseteq B \subseteq Y$ is θ-open. Then by hypothesis, $f^{-1}(int_\theta(B))$ is an M-open set in X. Hence, by Lemma 1.1, $f^{-1}(int_\theta(B)) \subseteq M\text{-}int(f^{-1}(B))$, for each $B \subseteq Y$.

(vi) → (i). Let $V \subseteq Y$ be a θ-open set. Then by assumption, $f^{-1}(V) = f^{-1}(int_\theta(V)) \subseteq M\text{-}int(f^{-1}(V))$. Hence, $f^{-1}(V)$ is M-open in X. Therefore, f is faintly M-continuous.

(vi) → (vii). Let $V \subseteq Y$. Then by hypothesis, $f^{-1}(int_\theta(V)) \subseteq M\text{-}int(f^{-1}(V))$ and so $f^{-1}(V) \setminus M\text{-}int(f^{-1}(V)) \subseteq f^{-1}(V) \setminus f^{-1}(int_\theta(V)) = f^{-1}(V \setminus int_\theta(V))$. By Proposition 1.1, $M\text{-}Bd(f^{-1}(V)) \subseteq f^{-1}(Bd_\theta(V))$.

(vii) → (vi). Let $V \subseteq Y$. Then by hypothesis, $f^{-1}(V) \setminus M\text{-}int(f^{-1}(V)) \subseteq f^{-1}(V) \setminus f^{-1}(int_\theta(V))$. Therefore, $f^{-1}(int_\theta(V)) \subseteq M\text{-}int(f^{-1}(V))$.

(vi) → (viii). Let $B \subseteq Y$. Then by (vi), $f^{-1}(int_\theta(B)) \subseteq M\text{-}int(f^{-1}(B))$. Hence by (iv), $M\text{-}cl(f^{-1}(B)) \setminus M\text{-}int(f^{-1}(B)) \subseteq f^{-1}(cl_\theta(B)) \setminus f^{-1}(int_\theta(B))$. So, by Proposition 1.1, $M\text{-}b(f^{-1}(B)) \subseteq f^{-1}(b_\theta(B))$, for each $B \subseteq Y$.

(viii)→(vi). Let $B \subseteq Y$. Then by Proposition 1.1, $M\text{-}b(f^{-1}(B)) = M\text{-}cl(f^{-1}(B)) \setminus M\text{-}int(f^{-1}(B)) \subseteq f^{-1}(cl_\theta(B)) \setminus f^{-1}(int_\theta(B))$ this implies that $f^{-1}(int_\theta(B)) \subseteq M\text{-}int(f^{-1}(B))$, for each $B \subseteq Y$.

Proposition 3.1. If, $f : (X, \tau) \to (Y, \sigma)$ is an M-continuous mapping then f is faintly M-continuous.

Proof. Let $x \in X$ and $V \subseteq Y$ be θ-open containing $f(x)$. By fact that every θ-open set is open, then V is open in Y. Since f is M-continuous, then $f^{-1}(V) \in MO(X)$ and containing x. If we put $U = f^{-1}(V)$, then $f(U) \subseteq V$. Hence f is faintly M-continuous.

Remark 3.1. The converse of the above Proposition is not true as shown by the following example.

Example 3.4. Let X = Y = {a, b, c, d} with topologies τ = {X, ϕ, {a},{b},{a, b}} and σ = {Y, ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, {b, c, d}}. Then the identity mapping $f : (X, \tau) \to (Y, \sigma)$ is faintly M-continuous but not M-continuous. Since, $f^{-1}(\{b, c\}) = \{b, c\} \notin MO(X)$.

Theorem 3.2. If a mapping $f : (X, \tau) \to (Y, \sigma)$ is weakly M-continuous, then it is faintly M-continuous.

Proof. Let $x \in X$ and $V \subseteq Y$ be a θ-open set containing $f(x)$. Then, there exists an open set W such that $f(x) \in W \subseteq cl(W) \subseteq V$. Since f is weakly M-continuous, then there exists an M-open set U containing x such that $f(U) \subseteq cl(W) \subseteq V$. Therefore, f is faintly M-continuous.

Remark 3.2. The converse of the above Theorem is not true as shown by the following example.

Example 3.5. In Example 3.4, $c \in X$, $\{c\} \in \sigma$ and $f(c) = c \in \{c\}$ but we not find $U \in MO(X)$ such that $c \in U$ and $f(U) \subseteq cl(\{c\}) = \{c, d\}$. Then f is faintly M-continuous but not weakly M-continuous.

If (Y, σ) is a regular space, we have $\sigma = \sigma_\theta$ and the next theorem follows immediately from the definitions

Theorem 3.3. Let Y be a regular space. Then for a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ the following properties are equivalent :

(i) f is M-continuous,

(ii) f is faintly M-continuous.

Theorem 3.4. If a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra M-continuous and Y is regular, then it is faintly M-continuous.

Proof. Obvious from Theorem 2.1 and Proposition 3.1.

Remark 3.3. The composition of two faintly M-continuous mappings need not be faintly M-continuous as shown by the following example.

Example 3.6. Let $X = Y = Z = \{a, b, c\}$, with topologies $\tau_x = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$,
$\tau_y = \{Y, \phi, \{a\}\}$ and $\tau_z = \{Z, \phi, \{a\}, \{b, c\}\}$. Then the identity mappings
$f : (X, \tau_x) \rightarrow (Y, \tau_y)$ and $g : (Y, \tau_y) \rightarrow (Z, \tau_z)$ are faintly M-continuous , but $g \circ f$ is not faintly M-continuous. Since, $f^{-1}(\{a\}) = \{a\} \notin MO(X)$.

In the following, we give some properties of the composition of two faintly M-continuous mappings.

Theorem 3.5. Let $f : (X, \tau_x) \rightarrow (Y, \tau_y)$ and $g : (Y, \tau_y) \rightarrow (Z, \tau_z)$ be two mappings. Then the following statements are hold:

(i) If, f is faintly M-continuous and g is quasi θ-continuous, then $g \circ f$ is faintly M-continuous,

(ii) If, f is faintly M-continuous and g is θ-continuous, then $g \circ f$ is M-continuous,

(iii) If, f is M-continuous and g is faintly continuous, then $g \circ f$ is faintly M- continuous,

(iv) If, f is M-irresolute and g is faintly M-continuous, then $g \circ f$ is faintly M-continuous.

Proof. (i) Let $V \subseteq Z$ be θ-open set and g be quasi θ-continuous, then $g^{-1}(V) \in \theta\text{-}O(Y)$. But f is faintly M-continuous, then $(g \circ f)^{-1}(V) \in MO(X)$. Hence, $g \circ f$ is faintly M-continuous.

(ii) Let $V \in \tau_z$ and g be θ-continuous. Then $g^{-1}(V) \in \theta\text{-}O(Y)$. But f is faintly M-continuous, then $(g \circ f)^{-1}(V) \in MO(X)$. Hence, $g \circ f$ is M-continuous.

(iii) Let $V \subseteq Z$ be θ-open set and g be faintly continuous. Then $g^{-1}(V) \in \sigma$. But f is M-continuous, then $(g \circ f)^{-1}(V) \in MO(X)$. Hence, $g \circ f$ is faintly M-continuous.

(iv) Let $V \subseteq Z$ be θ-open set and g be faintly M-continuous. Then $g^{-1}(V) \in MO(Y)$. But f is M-irresolute, then $(g \circ f)^{-1}(V) \in MO(X)$. Hence, $g \circ f$ is faintly M-continuous.

Theorem 3.6. For two mappings $f : (X, \tau_x) \rightarrow (Y, \tau_y)$ and $g : (Y, \tau_y) \rightarrow (Z, \tau_z)$, the following properties are hold:

(i) If, f is a surjective pre-M-open and $g \circ f : X \rightarrow Z$ is faintly M-continuous, then g is faintly M-continuous,

(ii) If, f is a surjective pre-M-closed and $g \circ f : X \rightarrow Z$ is faintly M-continuous, then g is faintly M-continuous.

Proof. (i) Let $V \subseteq Z$ be a θ-open set. Since, $g \circ f$ is faintly M-continuous, then $(g \circ f)^{-1}(V) \in MO(X)$. But, f is surjective pre-M-open, then $g^{-1}(V) \in MO(Y)$. Therefore, g is faintly M-continuous.

(ii) Obvious.

Theorem 3.7. Let $f : X \rightarrow Y$ be a surjective pre-M-open and M-irresolute mapping. Then $g \circ f : X \rightarrow Z$ is faintly M-continuous if and only if g is faintly M-continuous.

Proof. Necessity. Obvious from Theorem 3.5.

Sufficiency. Let $g \circ f : X \rightarrow Z$ be a faintly M-continuous mapping and V be a θ-open set of Z. Then $(g \circ f)^{-1}(V) \in MO(X)$. Since f is surjective pre-M-open, then $g^{-1}(V) \in MO(Y)$. Therefore, g is faintly M-continuous.

Definition 3.2. A topological space (X, τ) is called:

(i) M-compact [18] (resp. θ-compact [22]) if every M-open (resp. θ-open) cover of X has a finite sub cover,

(ii) countably M-compact [18] (resp. countably θ-compact) if every countable cover of X by M-open (resp. θ-open) sets has a finite subcover,

(iii) M-connected [18] (resp. θ-connected) if X can not be expressed as the union of two disjoint non-empty M-open (resp. θ-open) sets of X,

(iv) M-Lindelöff [18] (resp. θ-Lindelöff) if every M-open(resp. θ-open) cover of X has a countable subcover.

Theorem 3.8. If, $f : (X, \tau) \rightarrow (Y, \sigma)$ is a surjective faintly M-continuous mapping and X is M-connected space, then Y is θ-connected.

Proof. Suppose that Y is θ-disconnected and U, V be two disjoint non-empty θ-open sets such that $Y = U \cup V$. Since f is a surjective faintly M-continuous, then $X = f^{-1}(U) \cup f^{-1}(V)$ which is the union of two M-open sets. Therefore, X is M-disconnected. This is a contradiction with the fact X is M-connected. Hence, Y is θ-connected.

Theorem 3.9. If, $f : (X, \tau) \to (Y, \sigma)$ is a surjective faintly M-continuous mapping. Then the following statements are hold:

(i) If X is M-compact, then Y is θ-compact,

(ii) If X is countably M-compact, then Y is countably θ-compact,

(iii) If X is M-Lindelöff, then Y is θ-Lindelöff.

Proof. (i) Let $\{U_i : i \in I\}$ be a θ-open cover of Y. Since f is faintly M-continuous, then $\{f^{-1}(U_i) : i \in I\}$ is an M-open cover of X. But X is M-compact, then there exists a finite subcover I_o of I such that $X = \cup\{f^{-1}(U_i) : i \in I_o\}$. By surjective f, we have $Y = \cup\{U_i : i \in I_o\}$ of Y. Hence, Y is θ-compact.

(ii) Similar to (i).

(iii) Similar to (i).

Definition 3.3. A topological space (X, τ) is called:

(i) M-T_1 [16] (resp. θ-T_1) space if for every two distinct points x , y of X , there exist two M-open (resp.θ-open) sets U , V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.

(ii) M-T_2 or M-Hausdorff [16] (resp.θ-T_2 [38]) space if for every two distinct points x, y of X , there exist two disjoint M-open (resp.θ-open) sets U, V such that $x \in U$ and $y \in V$,

(iii) strongly M-regular (resp. strongly θ-regular) if for each M-closed (resp. θ-closed) set F and each point $x \notin F$, there exist two disjoint M-open (resp. θ-open) sets U, V such that $F \subseteq U$ and $x \in V$,

(iv) strongly M-normal (resp. strongly θ-normal) if for any pair of disjoint M-closed (resp. θ-closed) subsets F_1 , F_2 of X, there exist two disjoint M-open (resp. θ-open) sets U , V such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

Theorem 3.10. If, $f : (X, \tau) \to (Y, \sigma)$ is an injective faintly M-continuous mapping and Y is a θ-T_1 space, then X is M-T_1.

Proof. Let x, $y \in X$ and $x \neq y$. By hypothesis, $f(x) \neq f(y)$. Since Y is a θ-T_1 space, then there exist two θ-open sets U, V such that $f(x) \in U$, $f(y) \notin U$ and $f(x) \notin V$, $f(y) \in V$. Since f is faintly M-continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are M-open subsets of X such that

$x \in f^{-1}(U)$, $y \notin f^{-1}(U)$ and $x \notin f^{-1}(V)$, $y \in f^{-1}(V)$. Therefore, X is M-T_1.

Theorem 3.11. If, $f : (X, \tau) \to (Y, \sigma)$ is an injective faintly M-continuous mapping and Y is a θ-T_2 space, then X is M-T_2.

Proof. Let x, $y \in X$ and $x \neq y$. By hypothesis, $f(x) \neq f(y)$. Since Y is a θ-T_2 space, then there exist two disjoint θ-open sets U, V such that $f(x) \in U$ and $f(y) \in V$. Since f is an injective faintly M-continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are two disjoint M-open subsets of X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Therefore, X is M-T_2.

Definition 3.4. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called :

(i) Mθ-open if $f(V) \in \sigma_\theta$ for each $V \in MO(X)$,

(ii) Mθ-closed if $f(V)$ is θ-closed in Y for each $V \in MC(X)$.

Theorem 3.12. If $f : X \to Y$ is a bijective faintly M-continuous and Mθ-open mapping from strongly M-regular space X onto a space Y, then Y is strongly θ-regular.

Proof. Let $F \subseteq Y$ be θ-closed and $y \notin F$. Since f is faintly M-continuous, then $f^{-1}(F) \in MC(X)$ and $f^{-1}(y) = x \notin f^{-1}(F)$. Since X is strongly M-regular, then there exist two disjoint M-open sets U, V such that $f^{-1}(F) \subseteq U$ and $x \in V$. Since f is a bijective Mθ-open, then f(U) and f(V) are two disjoint θ-open subset of Y such that $f f^{-1}(F) = F \subseteq f(U)$ and $y \in f(V)$. Therefore, Y is strongly θ-regular.

Theorem 3.13. If $f : X \to Y$ is an injective faintly M-continuous and Mθ-open mapping from strongly M-normal space X onto a space Y, then Y is strongly θ-normal.

Proof. Let F_1 and F_2 be two disjoint θ-closed subsets of Y. Since f is an injective faintly M-continuous, then $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are two disjoint M-closed sets of X. Since X is strongly M-normal, then there exist two disjoint M-open sets U, V such that $f^{-1}(F_1) \subseteq U$ and $f^{-1}(F_2) \subseteq V$ and by Mθ-open mapping, we have $F_1 \subseteq f(U)$ and $F_2 \subseteq f(V)$. Therefore, Y is strongly θ-normal.

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