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Fuzzy semi-Baire spaces

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Abstract

In this paper the concepts of fuzzy semi-Baireness in fuzzy topological spaces are introduced. For this purpose we define fuzzy semi-nowhere dense, fuzzy semi-first category and fuzzy semi-second category sets. Several characterizations of fuzzy semi-Baire spaces are also studied. Several examples are given to illustrate the concepts introduced in this paper.

Keywords: *Fuzzy semi-open set, Fuzzy semi-nowhere dense set, Fuzzy semi-first category, Fuzzy semi-second category and Fuzzy semi-Baire spaces.*

1. Introduction

The fuzzy concept has invaded almost all branches of Mathematics ever since the introduction of fuzzy set by L.A.ZADEH [14]. The theory of fuzzy topological spaces was introduced and developed by C.L.CHANG [3]. Since then much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting and thus a modern theory of Fuzzy Topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. It has been shown that the fuzzy Kahler manifolds which are based on a topology play the important role in ε^{∞} theory [4]. The semi-open sets were introduced by N. Levine [7] in 1963 and the above concept was introduced and studied in fuzzy setting by K.K.Azad [1].

The concepts of Baire spaces have been studied extensively in classical topology in [5], [6], [8] and [15]. The concept of Baire space in fuzzy setting was introduced and studied by G.Thangaraj and S.Anjalmose in [9]. The aim of this paper is to introduce the concepts of fuzzy semi-Baireness in fuzzy topological spaces. For this purpose in section 3, we introduce fuzzy semi-nowhere dense sets and we study some of their properties. Also we discuss the relationship between fuzzy semi-nowhere dense sets and fuzzy nowhere dense sets.

Using the fuzzy semi-nowhere dense sets, in section 4, we introduce fuzzy semi-Baire spaces and study several characterizations of fuzzy semi-Baire spaces. Several examples are given to illustrate the concepts introduced in this paper. In section 5, the inter-relationships between fuzzy semi-Baire spaces, fuzzy Baire spaces, fuzzy pre-Baire spaces and fuzzy D-Baire spaces are also investigated. Several examples are given to illustrate the concepts introduced in this paper.

2. Preliminaries

Now we introduce some basic notions and results that are used in the sequel. In this work by a fuzzy topological space we shall mean a non-empty set X together with a fuzzy topology T (in the sense of Chang) and denote it by (X, T). The interior, closure and the complement of a fuzzy set λ will be denoted by int(λ), cl(λ) and 1-λ respectively.

Definition 2.1[1]: Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T). We define $cl(\lambda) = \wedge\{\mu / \lambda \leq \mu, 1 - \mu \, \varepsilon \, T\}$ and $int(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu \, \varepsilon \, T\}$. For any fuzzy set in a fuzzy topological space (X, T), it is easy to see that $1 - cl(\lambda) = int(1-\lambda)$ and $1 - int(\lambda) = cl(1- \lambda)$. [1]

Definition 2.2[2]: A fuzzy set λ in a fuzzy topological space X is called fuzzy semi-open if $\lambda \leq cl\, int(\lambda)$ and fuzzy semi-closed if $int\, cl(\lambda) \leq \lambda$.

Definition 2.3[1]: Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T). We define the fuzzy semi-closure and the fuzzy semi-interior of λ as follows:

(1) $scl(\lambda) = \wedge\{\mu / \lambda \leq \mu, 1 - \mu \in T\}$ is fuzzy semi-closed set of X.

(2) $sint(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu \in T\}$ is fuzzy semi-open set of X.

Definition 2.4[1]: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy semi-closed set if $\lambda = scl(\lambda)$ and fuzzy semi-open set if $\lambda = sint(\lambda)$.

Lemma 2.1[1]: Let λ be a fuzzy set of a fuzzy topological space (X,T). Then

(1) $1\text{-}scl(\lambda) = sint(1\text{-}\lambda)$

(2) $1\text{-}sint(\lambda) = scl(1\text{-}\lambda)$

Definition 2.5[12]: A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$.

Definition 2.6[12]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\, cl(\lambda) = 0$.

Definition 2.7[12]: A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy first category if $= \vee_{i=1}^{\infty} (\lambda_i)$, where λ_i 's are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.

Definition 2.8[9]: Let λ be a fuzzy first category set in (X, T). Then $1\text{-}\lambda$ is called a fuzzy residual set in (X, T).

Definition 2.9[12]: A fuzzy topological space (X, T) is called fuzzy first category if $1 = \vee_{i=1}^{\infty} (\lambda_i)$, where λ_i 's are fuzzy nowhere dense sets in (X,T).

A topological space which is not of fuzzy first category, is said to be of fuzzy second category.

3. Fuzzy semi-nowhere dense sets.

Definition 3.1: Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called a fuzzy semi-nowhere dense set if there exists no non-zero fuzzy semi-open set μ in (X,T) such that $\mu < scl(\lambda)$. That is, $sint\, scl(\lambda) = 0$.

Example 3.1: Let X = {a, b, c}. The fuzzy sets λ, μ, γ, α and β are defined on X as follows:

$\lambda : X \to [0,1]$ defined as $\lambda(a) = 0.8$; $\lambda(b) = 0.2$; $\lambda(c) = 0.6$, $\mu : X \to [0,1]$ defined as $\mu(a) = 0.1$; $\mu(b) = 0.9$; $\mu(c) = 0.5$, $\gamma : X \to [0,1]$ defined as $\gamma(a) = 0.4$; $\gamma(b) = 0.6$; $\gamma(c) = 0.7$, $\alpha : X \to [0,1]$ defined as $\alpha(a) = 0.9$; $\alpha(b) = 0.9$; $\alpha(c) = 0.6$, $\beta : X \to [0,1]$ defined as $\beta(a) = 0.9$; $\beta(b) = 0.9$; $\beta c) = 0.5$.

Then T = {0, λ, μ, γ, $(\lambda\vee\mu)$, $(\lambda\vee\gamma)$, $(\mu\vee\gamma)$, $(\lambda\wedge\mu)$, $(\lambda\wedge\gamma)$, $(\mu\wedge\gamma)$, $\lambda\vee\mu\vee\lambda$, $\mu\vee(\lambda\wedge\gamma)$, $\lambda\vee(\mu\vee\gamma)$, $\gamma\wedge(\lambda\vee\mu)$, 1} is a fuzzy topology on X. The non-zero fuzzy semi-open sets in (X, T) are λ, μ, γ, $(\lambda\vee\mu)$, $(\lambda\vee\gamma)$, $(\mu\vee\gamma)$, $(\lambda\wedge\mu)$, $(\lambda\wedge\gamma)$, $(\mu\wedge\gamma)$, $\lambda\vee\mu\vee\lambda$, $\mu\vee(\lambda\wedge\gamma)$, $\lambda\vee(\mu\vee\gamma)$, $\gamma\wedge(\lambda\vee\mu)$,

Now the fuzzy sets $1 - \lambda$, $1 - \mu$, $1 - \gamma$, $1\text{-}(\lambda\vee\mu)$, $1 - (\lambda\vee\gamma)$, $1\text{- } \lambda\wedge\gamma$, $1\text{- }(\mu\vee\gamma)$, $1\text{- } \lambda\vee(\mu\wedge\gamma)$, $1\text{- }(\mu\vee(\lambda\wedge\gamma))$, $1\text{-}\lambda\vee\mu\vee\gamma$, $1\text{-}\gamma\wedge\lambda\vee\mu$, $1\text{- } \alpha$, $1\text{- } \beta$ are fuzzy semi-nowhere dense sets in (X,T).

Proposition 3.1: The complement of a fuzzy semi-nowhere dense set in a fuzzy topological space (X, T) need not be fuzzy semi-nowhere dense set.

Proof: For, in example 3.1, $(1 - \alpha)$ is a fuzzy semi-nowhere dense set in (X, T) whereas $\alpha = 1 - (1 - \alpha)$ is not a fuzzy semi-nowhere dense set in (X,T).

Proposition 3.2: If λ and μ are fuzzy semi-nowhere dense sets in a fuzzy topological space (X, T), then $\lambda\vee\mu$ need not be fuzzy semi-nowhere dense set in (X, T).

Proof: For, in example 3.1, $1 - \alpha$, $1 - \lambda$ are fuzzy semi-nowhere dense sets in (X, T). But $(1 - \alpha) \vee (1 - \lambda) = \delta$ implies that $sint\, scl(\delta) \neq 0$. Therefore union of fuzzy semi-nowhere dense sets need not be fuzzy semi-nowhere dense set in (X, T).

Proposition 3.3: If the fuzzy sets λ and μ are fuzzy semi-nowhere dense sets in a fuzzy topological space (X, T) then $\lambda \wedge \mu$ is a fuzzy semi-nowhere dense set in (X, T).

Proof: Let the fuzzy sets λ and μ be fuzzy semi-nowhere dense sets in (X, T). Now $\text{sint scl}(\lambda \wedge \mu) \leq \text{sint scl}(\lambda) \wedge \text{sint scl}(\mu) \leq 0 \wedge 0$ (since $\text{sint scl}(\lambda) = 0$ and $\text{sint scl}(\mu) = 0$). That is, $\text{sint scl}(\lambda \wedge \mu) = 0$. Hence $(\lambda \wedge \mu)$ is a fuzzy semi-nowhere dense set in (X, T).

Proposition 3.4: If λ is a fuzzy semi-nowhere dense set in a fuzzy topological space (X, T) then $\text{sint}(\lambda) = 0$.

Proof: Let λ be a fuzzy semi-nowhere dense set in (X, T). Then, we have $\text{sint scl}(\lambda) = 0$. Now $\lambda \leq \text{scl}(\lambda)$ we have $\text{sint}(\lambda) \leq \text{sint scl}(\lambda) = 0$. Hence $\text{sint}(\lambda) = 0$.

Proposition 3.5: If λ is a fuzzy nowhere dense set in a fuzzy topological space (X,T) then $\text{int scl}(\lambda) = 0$.

Proof: Let λ be a fuzzy nowhere dense sets in (X, T). Then, we have $\text{int cl}(\lambda) = 0$ and $\text{int}(\lambda) = 0$. Now $\text{scl}(\lambda) = \lambda$, since λ is fuzzy semi-closed set in (X, T)[9] implies that $\text{int scl}(\lambda) = \text{int}(\lambda) = 0$. Hence $\text{int scl}(\lambda) = 0$.

Proposition 3.6: If λ is a fuzzy semi-nowhere dense set and μ is any fuzzy set in a fuzzy topological space (X, T), then $(\lambda \wedge \mu)$ is a fuzzy semi-nowhere dense set in (X, T).

Proof: Let λ be a fuzzy semi-nowhere dense set in (X, T). Then, $\text{sint scl}(\lambda) = 0$. Now $\text{sint scl}(\lambda \wedge \mu) \leq \text{sint scl}(\lambda) \wedge \text{sint scl}(\mu) \leq 0 \wedge \text{sint scl}(\mu) = 0$. That is, $\text{sint scl}(\lambda \wedge \mu) = 0$. Hence $(\lambda \wedge \mu)$ is a fuzzy semi nowhere dense set in (X, T).

Definition 3.2[13]: A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy semi-dense if there exists no fuzzy semi-closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $\text{scl}(\lambda) = 1$.

Proposition 3.7: If λ is a fuzzy semi-dense and fuzzy semi-open set in a fuzzy topological space (X, T) and if $\mu \leq 1 - \lambda$, then μ is a fuzzy semi-nowhere dense set in (X, T).

Proof: Let λ be a fuzzy semi-dense set in (X, T). Then we have $\text{scl}(\lambda) = 1$ and $\text{sint}(\lambda) = \lambda$. Now $\mu \leq 1 - \lambda$, implies that $\text{scl}(\mu) \leq \text{scl}(1 - \lambda)$. Then $\text{scl}(\mu) \leq 1 - \text{sint}(\lambda) = 1 - \lambda$. Hence $\text{scl}(\mu \leq (1 - \lambda)$, which implies that $\text{sint scl}(\mu) \leq \text{sint}(1-\lambda) = 1-\text{scl}(\lambda) = 1 – 1 = 0$. That is, $\text{sint scl}(\mu) = 0$. Hence μ is a fuzzy semi-nowhere dense set in (X, T).

Proposition 3.8: If λ is a fuzzy semi-nowhere dense set in a fuzzy topological space (X, T), then $1 - \lambda$ is a fuzzy semi-dense set in (X, T).

Proof: Let λ be a fuzzy semi-nowhere dense set in (X, T). Then, $\text{sint scl}(\lambda) = 0$. Now $\lambda \leq \text{scl}(\lambda)$ implies that $\text{sint}(\lambda) \leq \text{sint scl}(\lambda) = 0$. Then $\text{sint}(\lambda) = 0$ and $\text{scl}(1 - \lambda) = 1 - \text{sint}(\lambda) = 1 – 0 = 1$and hence $1 - \lambda$ is a fuzzy semi-dense set in (X, T).

Proposition 3.9: If λ is a fuzzy semi-nowhere dense set in a fuzzy topological space (X, T), then $\text{scl}(\lambda)$ is also a fuzzy semi-nowhere dense set in (X, T).

Proof: Let λ be a fuzzy semi-nowhere dense set in (X, T). Then, $\text{sint scl}(\lambda) = 0$. Now $\text{scl scl}(\lambda) = \text{scl}(\lambda)$. Hence $\text{sint scl}(\text{scl}(\lambda)) = \text{sint scl}(\lambda) = 0$. Therefore $\text{scl}(\lambda)$ is also a fuzzy semi-nowhere dense set in (X, T).

Proposition 3.10: If λ is a fuzzy semi-nowhere dense set in a fuzzy topological space (X, T), then $1 - \text{scl}(\lambda)$ is a fuzzy semi-dense set in (X, T).

Proof: Let λ be a fuzzy semi-nowhere dense set in (X, T). Then, by proposition 3.9, $\text{scl}(\lambda)$ is a fuzzy semi-nowhere dense set in (X, T). Also by proposition 3.8, $1 - \text{scl}(\lambda)$ is a fuzzy semi-dense set in (X, T).

Proposition 3.11: Let λ be a fuzzy semi-dense set in a fuzzy topological space (X, T). If μ is any fuzzy set in (X, T), then μ is a fuzzy semi-nowhere dense set in (X, T) if and only if $\lambda \wedge \mu$ is a fuzzy semi-nowhere dense set in (X, T).

Proof: Let μ be a fuzzy semi-nowhere dense set in (X, T). Then, sint scl(μ) = 0. Now sint scl($\lambda \wedge \mu$) $\leq$ sint scl(λ) $\wedge$ sint scl(μ) $\leq$ sint scl(λ) $\wedge$ 0 = 0. That is, sint scl($\lambda \wedge \mu$) = 0. Hence ($\lambda \wedge \mu$) is a fuzzy semi-nowhere dense set in (X, T).

Conversely, let ($\lambda \wedge \mu$) be a fuzzy semi-nowhere dense set in (X, T). Then sint scl($\lambda \wedge \mu$) = 0. Then, sint scl(λ) $\wedge$ sint scl(μ) = 0. Since λ is a fuzzy semi-dense set in (X, T), scl(λ) = 1. Then, sint(1) $\wedge$ sint scl(μ) = 0. That is, (1) $\wedge$ sint scl(μ) = 0. Hence sint scl(μ) = 0, which means that μ is a fuzzy semi-nowhere dense set in (X, T).

Definition 3.3: Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called fuzzy semi-first category if $\lambda = \vee_{1=1}^{\infty} (\lambda_i)$, where λ_i's are fuzzy semi-nowhere dense sets in (X,T). Any other fuzzy set in (X, T) is said to be of fuzzy semi-second category.

Example 3.2: Let X = {a, b}. Consider the fuzzy sets λ, μ and α, β defined on X as follows:
$\lambda : X \to [0,1]$ defined as $\lambda(a) = 0.5; \lambda(b) = 0.7$,
$\mu : X \to [0,1]$ defined as $\mu(a) = 0.8; \mu(b) = 0.9$,
$\alpha : X \to [0,1]$ defined as $\alpha(a) = 0.6; \alpha(b) = 0.9$,
$\beta : X \to [0,1]$ defined as $\beta(a) = 0.7; \beta(b) = 0.8$.

Then T = {0, λ, μ, 1} is a fuzzy topology on X. Now λ, μ, α, β, ($\alpha \vee \beta$), ($\alpha \wedge \beta$) and 1 are the non-zero fuzzy semi-open sets in (X, T). Then 1- λ, 1- μ, 1- α, 1- β, 1-($\alpha \vee \beta$), 1-($\alpha \wedge \beta$) are fuzzy semi-nowhere dense sets in (X,T) and $[(1-\alpha) \vee (1-\beta) \vee 1-(\alpha \vee \beta) \vee (1-\mu)] = 1-\lambda$ and 1– λ is a fuzzy semi-first category set in (X, T).

Definition 3.4: Let λ be a fuzzy semi-first category set in a fuzzy topological space (X, T). Then 1 - λ is called a fuzzy semi-residual set in (X, T).

Proposition 3.12: If λ is a fuzzy semi-first category set in a fuzzy topological space (X, T), then $1-\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where scl($\mu_i$) = 1.

Proof: Let λ be a fuzzy semi-first category set in (X, T). Then we have $\lambda = \vee_{1=1}^{\infty} (\lambda_i)$, where λ_i's are fuzzy semi-nowhere dense sets in (X,T). Now $1-\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, $= \wedge_{i=1}^{\infty} (\lambda_i)$. Let $\mu_i = 1 - \lambda_i$. Then $1-\lambda = \wedge_{i=1}^{\infty} (\mu_i)$. Since λ_i's are fuzzy semi-nowhere dense sets in (X,T), by proposition 3.8, we have 1-λ 's are fuzzy semi-dense sets in (X, T). Hence scl(μ_i) = scl(1-λ_i) =1. Therefore we have $1-\lambda = \wedge_{i=1}^{\infty} (\mu_i)$. where scl($\mu_i$) = 1.

Definition 3.5: A fuzzy topological space (X, T) is called a fuzzy semi-first category space if the fuzzy set 1_X is a fuzzy semi-first category set in (X, T). That is, $1_X = \vee_{i=1}^{\infty} (\lambda_i)$. where λ_i's are fuzzy semi-nowhere dense sets in (X,T). Otherwise (X, T) will be called a fuzzy semi-second category space.

Proposition 3.13: If λ is a fuzzy semi-closed set in a fuzzy topological space (X, T) and if sint(λ) = 0, then λ is a fuzzy semi-nowhere dense set in (X, T).

Proof: Let λ be a fuzzy semi-closed set in (X,T). Then we have scl(λ) = λ. Now Sint scl(λ) = sint(λ) and sint(λ) = 0, implies that sint scl(λ) = 0. Hence λ is a fuzzy semi-nowhere dense set in (X, T).

Remarks 3.1: If λ is a fuzzy set in a fuzzy topological space (X, T) such that sint(λ) = 0, then λ need not be a fuzzy semi-nowhere dense set in (X, T). For, consider the following example:

Example 3.3: Let X = {a, b}. Consider the fuzzy sets λ, μ, α, β defined on X as follows:
$\lambda : X \to [0,1]$ defined as $\lambda(a) = 0.5; \lambda(b) = 0.7$,
$\mu : X \to [0,1]$ defined as $\mu(a) = 0.8; \mu(b) = 0.9$,
$\alpha : X \to [0,1]$ defined as $\alpha(a) = 0.6; \alpha(b) = 0.9$,
$\beta : X \to [0,1]$ defined as $\beta(a) = 0.7; \beta(b) = 0.8$,

Then T = {0, λ, μ, 1} is a fuzzy topology on X. Now λ, μ, α, β, $\alpha \vee \beta$, $\alpha \wedge \beta$ and 1 are the non-zero fuzzy pre-open sets in (X, T). Now the fuzzy set $\eta : X \to [0,1]$ defined as $\eta(a) = 0.4$; η (b) = 0.5 is not a fuzzy pre-nowhere dense set in (X, T), even though sint(η) = 0.

4. Fuzzy semi-Baire spaces.

Motivated by the classical concept introduced in [6] we shall now define:

Definition 4.1: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy semi-Baire space if sint$[\vee_{i=1}^{\infty} (\lambda_i)] = 0$, where λ_i's are fuzzy semi-nowhere dense sets in (X, T).

Example 4.1: Let X = {a, b}. Consider the fuzzy sets λ, μ, α and β defined on X as follows:
$\lambda : X \to [0,1]$ defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.7$,
$\mu : X \to [0,1]$ defined as $\mu(a) = 0.8$; $\mu(b) = 0.9$,
$\alpha : X \to [0,1]$ defined as $\alpha(a) = 0.6$; $\alpha(b) = 0.9$,
$\beta : X \to [0,1]$ defined as $\beta(a) = 0.7$; $\beta(b) = 0.8$,

Then T = {0, λ, μ, α, β, $\alpha\vee\beta$, $\alpha\wedge\beta$, 1} is a fuzzy topology on X. Now λ, μ, α, β, $\alpha\vee\beta$, $\alpha\wedge\beta$ are the non-zero fuzzy semi-open sets in (X,T). Then 1- λ, 1-μ, 1-α, 1- β, 1-($\alpha\vee\beta$), 1- ($\alpha\wedge\beta$) are fuzzy semi -nowhere dense sets in (X,T) and sint[(1- λ) $\vee$ (1-μ) $\vee$ (1- α) $\vee$ (1- β) $\vee$ (1- $\alpha\vee\beta$) $\vee$ (1- $\alpha\wedge\beta$)] = 0. Hence the fuzzy topological space (X, T) is a fuzzy semi-Baire space.

Example 4.2: Let X = {a, b, c}. Consider the fuzzy sets λ, μ and α defined on X as follows:
$\lambda : X \to [0,1]$ defined as $\lambda(a) = 0.3$; $\lambda(b) = 0.2$; $\lambda(c) = 0.7$;
$\mu : X \to [0,1]$ defined as $\mu(a) = 0.8$; $\mu(b) = 0.8$; $\mu(c) = 0.4$,
$\alpha : X \to [0,1]$ defined as $\alpha(a) = 0.7$; $\alpha(b) = 0.7$; $\alpha(c) = 0.7$.

Then T = {0, λ, μ, $\lambda\vee\mu$, $\lambda\wedge\mu$, 1} is clearly a fuzzy topology on X. The non-zero fuzzy semi-open sets in (X, T) are λ, μ, $\lambda\vee\mu$, $\lambda\wedge\mu$, α. Now the fuzzy sets 1 - λ, 1 - μ, 1 - $\lambda\vee\mu$, 1- α are fuzzy semi-nowhere dense sets in (X,T) and (1- λ) $\vee$ (1- μ) $\vee$ (1-$\lambda\vee\mu$) $\vee$ (1- α) = 1- ($\lambda\wedge\mu$) $\neq$ 0. Hence the fuzzy topological space (X, T) is not a fuzzy semi-Baire space.

Proposition 4.1: If sint($\vee_{i=1}^{\infty}$ (λ_i)), = 0, where sint(λ_i) = 0 and λ_i's are fuzzy semi-closed sets in a fuzzy topological space (X,T), then (X,T) is a fuzzy semi-Baire space.

Proof: Let λ_i's be fuzzy semi-closed sets in (X, T). Since sint(λ_i) = 0, by theorem 3.13, the λ_i's are fuzzy semi-nowhere dense sets in (X,T). Therefore we have sint($\vee_{i=1}^{\infty}$ (λ_i)) = 0, where λ_i's are fuzzy semi-nowhere dense sets in (X,T). Hence (X, T) is a fuzzy semi-Baire space.

Proposition 4.2: If scl($\wedge_{i=1}^{\infty}$ (λ_i)) = 1, where λ_i's are fuzzy semi-dense and fuzzy semi-open sets in a fuzzy topological space (X, T), then (X, T) is a fuzzy semi-Baire space.

Proof: Now scl ($\wedge_{i=1}^{\infty}$ (λ_i)) = 1implies that 1-scl($\wedge_{i=1}^{\infty}$ (λ_i)) = 0. Then we have sint(1-$\wedge_{i=1}^{\infty}$ (λ_i)) = 0,which implies that sint ($\vee_{i=1}^{\infty}$ (1- λ_i)) = 0. Since λ_i's are fuzzy semi-dense sets in (X, T), scl(λ_i) = 1 and sint(1-λ_i) = 1-scl(λ_i) = 1-1 = 0. Hence we have sint($\vee_{i=1}^{\infty}$ (1-λ_i)) = 0, where sint(1-λ_i) = 0 and (1-λ_i) 's are fuzzy semi-closed sets in (X, T). Then, by proposition 4.1, (X, T) is a fuzzy semi-Baire space.

Proposition 4.3: Let (X, T) be a fuzzy topological space. Then the following are equivalent:

(1). (X, T) is a fuzzy semi-Baire space.

(2). sint(λ) = 0 for every fuzzy semi-first category set λ in (X, T).

(3). scl(μ) = 1 for every fuzzy semi-residual set in (X, T).

Proof: (1) $\to$ (2). Let λ be a fuzzy semi-first category set in (X,T). Then $\lambda = \vee_{i=1}^{\infty}$ (λ_i), where λ_i's are fuzzy semi-nowhere dense sets in (X, T). Now sint(λ) = sint($\vee_{i=1}^{\infty}$ (λ_i)) = 0 (since (X, T) is a fuzzy semi-Baire space). Therefore sint(λ) = 0.

(2) $\to$ (3). Let μ be a fuzzy semi-residual set in (X, T). Then 1-μ is a fuzzy semi-first category set in (X, T). By hypothesis, sint(1-μ) = 0 which implies that 1-scl(μ) = 0. Hence we have scl(μ) = 1.

(3)$\to$ (1). Let λ be a fuzzy semi-first category set in (X, T). Then $\lambda = \vee_{i=1}^{\infty}$ (λ_i) where λ_i's are fuzzy semi-nowhere dense sets in (X,T). 1- λ is a fuzzy semi-residual set in (X, T). Since λ is a fuzzy semi-first category set in (X, T), By hypothesis, we have scl(1-λ) = 1. Then 1-sint(λ) = 1, which implies that sint(λ) = 0. Hence sint($\vee_{i=1}^{\infty}$ (λ_i)) = 0 where λ_i's are fuzzy semi-nowhere dense sets in (X,T). Hence (X, T) is a fuzzy semi-Baire space.

Proposition 4.4: If a fuzzy topological space (X, T) is a fuzzy semi-Baire space, then (X, T) is a fuzzy semi-second category space.

Proof: Let (X, T) be a fuzzy semi-Baire space. Then sint($\vee_{i=1}^{\infty}$ (λ_i)) = 0 where λ_i's are fuzzy semi-nowhere dense sets in (X, T). Then $\vee_{i=1}^{\infty} (\lambda_i) \neq 1_X$. (Suppose, $\vee_{i=1}^{\infty} (\lambda_i) = 1_X$ implies that sint($\vee_{i=1}^{\infty} (\lambda_i)$) = sint($1_X$) which implies that 0 = 1, a contradiction). Hence (X, T) is a fuzzy semi-second category space.

Remarks 4.1: The converse of the above proposition need not be true. A fuzzy semi-second category space need not be fuzzy semi-Baire space.

For, in example 4.2. The fuzzy sets 1-λ, 1-μ, 1-λ∨μ, 1-α are semi-nowhere dense sets in (X,T). Now (1-λ) ∨ (1-μ) ∨ (1-λ∨μ) ∨ (1-α) = 1-λ∧μ $\neq 1_X$ and int(1-λ∧μ) ≠ 0. Hence the fuzzy topological space (X, T) is a fuzzy semi-second category space but not a fuzzy semi-Baire space

Proposition 4.5: If a fuzzy topological space (X, T) is a fuzzy semi-Baire space, then no non-zero fuzzy semi-open set in (X, T) is a fuzzy semi-first category set in (X, T).

Proof: Suppose that λ is a non-zero fuzzy semi-open set in (X, T) such that $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where λ_i's are fuzzy semi-nowhere dense sets in (X,T). Then we have sint(λ) = sint($\vee_{i=1}^{\infty} (\lambda_i)$). Since λ is a non-zero fuzzy semi-open set in (X,T) sint(λ) = λ. Then sint($\vee_{i=1}^{\infty} (\lambda_i)$) = λ ≠ 0. But this is a contradiction to (X, T) being a fuzzy semi-Baire space, in which sint($\vee_{i=1}^{\infty} (\lambda_i)$) = 0, where λ_i's are fuzzy semi-nowhere dense sets in (X,T). Hence we must have $\lambda \neq (\vee_{i=1}^{\infty} (\lambda_i))$. Therefore no non-zero fuzzy semi-open set in (X, T) is a fuzzy semi-first category set in (X, T).

5. Fuzzy Baire spaces, fuzzy pre-Baire spaces, fuzzy D-Baire spaces and fuzzy semi-Baire spaces.

Definition 5.1[9]: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy Baire space if int ($\vee_{i=1}^{\infty} (\lambda_i)$) = 0, where λ_i's are fuzzy nowhere dense sets in (X, T).

Definition 5.2[10]: A fuzzy topological space (X, T) is called a fuzzy D-Baire space if every fuzzy first category set in (X, T) is a fuzzy nowhere dense set in (X, T). That is, (X, T) is a fuzzy D-Baire space if int cl(λ) = 0 for each fuzzy first category set λ in (X, T).

Definition 5.3[11]: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy pre-Baire space if pint($\vee_{i=1}^{\infty} (\lambda_i)$) = 0, where λ_i's are fuzzy pre-nowhere dense sets in (X, T).

A fuzzy semi-Baire space is a fuzzy Baire space. For consider the following example:

Example 5.1: Let X = {a, b,c}. Consider the fuzzy Sets λ, μ, γ, α, β and δ defined on X as follows:
λ : X → [0,1] defined as λ(a) = 0.5 ; λ(b) = 0.6; λ (c) = 0.7,
μ : X → [0,1] defined as μ(a) = 0.8; μ(b) = 0.4; μ(c) = 0.2,
γ : X → [0,1] defined as γ(a) = 0.7; γ(b) = 0.5; γ(c) = 0.8,
α : X →[0,1] defined as α(a) = 0.3; α(b) = 0.6; α(c) = 0.4,
β : X → [0,1] defined as β(a) = 0.6; β(b) = 0.2; β(c) = 0.7,
δ : X → [0,1] defined as δ(a) = 0.8; δ(b) = 0.3; δ(c) = 0.6.

Then T = {0, λ, μ, γ, λ∨μ, λ∨γ, μ∨γ, λ∧μ, λ∧γ, μ∧γ, λ∨μ∧γ, μ∨λ∧γ, γ∧λ∨μ, λ∨μ∨γ, 1} is clearly a fuzzy topology on X. The non-zero fuzzy semi-open sets in (X,T) are , λ, μ, γ, λ∨μ, λ∨γ, μ∨γ, λ∧μ, λ∧γ, μ∧γ, λ∨μ∧γ, μ∨λ∧γ, γ∧λ∨μ, λ∨μ∨γ, α∨δ, α∨β∨δ, β∨α∧δ, δ∧α∨β. Now the fuzzy sets 1- μ, 1- γ, 1- (λ∨μ), 1-(λ∨γ), 1-(μ∨γ), 1-[λ∨(μ∧γ)], 1-[μ∨(λ∧γ)], 1-[γ∧(λ∨μ)], 1-[λ∨(μ∨γ)], 1-(α∨δ) are fuzzy semi-nowhere dense sets in (X, T) and sint(1- μ) ∨ (1- γ) ∨ [1- (λ∨μ)] ∨ [1-(λ∨γ)] ∨ [1-(μ∨γ)] ∨ (1-[λ∨(μ∧γ)]) ∨ (1-[μ∨(λ∧γ)]) ∨ (1 [γ∧(λ∨ μ)]) ∨ (1-[λ∨(μ∨γ)]) ∨ [1-(α∨δ)] = sint(1-μ∧γ) = 0. Hence the fuzzy topological space (X,T) is a fuzzy semi-Baire space and (X, T) is a fuzzy Baire space.

Since, for the fuzzy nowhere dense sets 1-μ, 1-γ, 1-(λ∨μ), 1-(μ∨γ), 1-[λ∨(μ∧γ)], 1-[μ∨(λ∧γ)], 1-[γ∧(λ∨ μ)], 1-[λ∨(μ∨γ)] in (X,T) we have int[(1-μ) ∨ (1-γ) ∨ [1-(λ∨μ) ∨ 1-(μ∨γ) ∨ [1-(λ∨μ∧γ)] ∨ [1-(μ∨λ∧ γ)] ∨ [1- (γ∧λ∨μ) ∨ [1-(λ∨μ∨γ)] = 0. Hence a fuzzy semi-Baire space is a fuzzy Baire space.

There are some fuzzy topological spaces which are fuzzy semi-Baire is fuzzy Baire.

Proposition 5.1: A fuzzy semi-Baire space is a fuzzy

D-Baire space.

Proof: In Example 5.1, the space (X, T) is fuzzy semi-Baire and fuzzy Baire space but not a D-Baire space. Since, the fuzzy set $1-(\mu\wedge\gamma)$ is fuzzy first category, therefore int cl$[1-(\mu\wedge\gamma)] =0$. Hence a fuzzy semi-Baire space is a fuzzy D-Baire space.

There are some fuzzy topological spaces which are fuzzy semi-Baire is fuzzy D-Baire.

Proposition 5.2: A fuzzy semi-Baire space need not be fuzzy pre-Baire space.

Proof: In Example 5.1, the space (X, T) is fuzzy semi-Baire but not a pre-Baire space. Since, the non-zero fuzzy pre-open sets are λ, μ, γ, $\lambda\vee\mu$, $\lambda\vee\gamma$, $\mu\vee\gamma$, $\lambda\wedge\mu$, $\lambda\wedge\gamma$, $\mu\wedge\gamma$, $\lambda\vee\mu\wedge\gamma$, $\mu\vee\lambda\wedge\gamma$, $\gamma\wedge\lambda\vee\mu$, $\lambda\vee\mu\vee\gamma$, α, β, δ, $\alpha\vee\beta$, $\alpha\vee\delta$, $\beta\vee\delta$, $\alpha\wedge\beta$, $\alpha\wedge\delta$, $\beta\wedge\delta$, $\alpha\vee\beta\wedge\delta$, $\beta\vee\alpha\wedge\delta$, $\delta\wedge\alpha\vee\beta$. Now the fuzzy sets $1-\mu$, $1-\gamma$, $1-(\lambda\vee\mu)$, $1-(\lambda\vee\gamma)$, $1-(\mu\vee\gamma)$, $1-[\lambda\vee(\mu\wedge\gamma)]$, $1-[\mu\vee(\lambda\wedge\gamma)]$, $1-[\gamma\wedge(\lambda\vee\mu)]$, $1-[\lambda\vee(\mu\vee\gamma)]$, $1-\beta$, $1-\delta$, $1-(\alpha\vee\beta)$, $1-(\alpha\vee\delta)$, $1-(\beta\vee\delta)$, $1-[\beta\vee(\alpha\wedge\delta)]$ are fuzzy pre-nowhere dese sets in (X,T) and pint$[(1-\mu) \vee (1-\gamma) \vee [1-(\lambda\vee\mu)] \vee [1-(\lambda\vee\gamma)] \vee [1-(\mu\vee\gamma)] \vee (1-[\lambda\vee(\mu\wedge\gamma)]) \vee (1-[\mu\vee(\lambda\wedge\gamma)]) \vee (1-[\gamma\wedge(\lambda\vee\mu)]) \vee (1-[\lambda\vee(\mu\vee\gamma)]) \vee (1-\beta) \vee (1-\delta) \vee [1-(\alpha\vee\beta)] \vee [1-(\alpha\vee\delta)] \vee [1-(\beta\vee\delta)] \vee [1-\beta\vee(\alpha\wedge\delta)] = \alpha \neq 0$. Hence the fuzzy topological space is not a fuzzy pre-Baire space.

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