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Application of Seasonal Box-Jenkins Techniques for Modelling Monthly Internally Generated Revenue of Rivers State of Nigeria

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Abstract

This work is focused on the modeling of the monthly internally generated revenue of Rivers State of Nigeria. The particular realization analyzed and herein called RIGR spans from 2007 to 2012. Examination of the series reveals a seasonal nature of annual periodicity. It has a positive outlier in December 2008. Augmented Dickey-Fuller (ADF) Test adjudges RIGR as stationary. However the existence of an outlier invalidates this stationarity claim. Twelve-monthly differencing of RIGR yields the series SDRIGR which may be said to have a generally horizontal trend with an upper spike counterbalanced by a comparably lower one. The correlogram of SDRIGR has significant negative spikes at lag 12 on both the autocorrelation function (ACF) and the partial autocorrelation function (PACF). With these spikes as the only significant ones, the autocorrelation structure is that of a SARIMA$(0,0,0)\text{x}(1,1,1)_{12}$ model; hence its proposal and estimation. However, the residuals of this proposed model are correlated; this invalidates the model. A non-seasonal differencing of SDRIGR yields the series DSDRIGR which might be considered stationary; two upward spikes are counterbalanced by two comparable downward spikes. The ADF test on the series is highly significant, showing that it is stationary. The autocorrelation structure of the series is suggestive of a SARIMA$(0, 1, 1)\text{x}(1, 1, 1)_{12}$ model. Hence, its proposal and fitting. This model is found to be adequate.

Keywords: *Internally generated revenue, Sarima Models, Time Series, Seasonal Series, Rivers State, Nigeria.*

1. Introduction

Every institution is encouraged to augment its finances by generating revenue internally. The forecasting and control of such internally generated revenue could help in management and governance. Rivers State of Nigeria just as any other institution generates revenue internally to complement the efforts of the Federal Government in its funding. The aim of this write-up is to model the monthly internally generated revenue of the state in order to provide a model for possible forecasting of the series.

Like many other economic and financial time series, internally generated revenue could be seasonal. See, for example, Nwogu and Nwosu [1]. The time plot of the internally generated revenue of Ikot Ekpene Local Government Area of Akwa Ibom State of Nigeria shows some seasonal tendencies [2]. Such seasonal series may be modeled by seasonal Box-Jenkins methods otherwise called *seasonal autoregressive integrated moving average (SARIMA) models.*

Seasonal Box-Jenkins methods are gaining popularity for the modeling of seasonal time series. A few of such series which have been thus modeled are: inflation rates [3], savings deposit rates [4], water demand [5], traffic flow [6], crude oil prices [7], unemployment rates [8], foreign exchange rates [9], dengue numbers [10], temperature [11] and tuberculosis incidence[12]. It has been observed that SARIMA models often outdo other types of models in explaining the variation of seasonal series ([13])

Section 2 of this paper considers the materials and methods used herein. Section 3 discusses the findings. Section 4 is the concluding remarks.

2. Materials and Methods

2.1 Data

The data for this work are monthly internally generated revenues of Rivers State of Nigeria from 2007 to 2012. The figures used are corrected to the nearest hundred thousand naira. This realization shall be called RIGR.

2.2 Seasonal Box-Jenkins Models

A stationary time series $\{X_t\}$ is said to follow an *autoregressive moving average model of orders p and q,* denoted by ARMA(p, q), if it satisfies the following equation

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \dots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \dots + \beta_q \varepsilon_{t-q} \quad (1)$$

where the α's and the β's are constants such that the model is both stationary and invertible. $\{\varepsilon_t\}$ is a white noise process.

Suppose model (1) is written as $A(L)X_t = B(L)\varepsilon_t$ where $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_p L^p$ and $B(L) = 1 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q$ and $L^k X_t = X_{t-k}$. For stationarity the zeros of A(L) must lie outside the unit circle and for invertibility the zeros of B(L) must lie outside the unit circle.

Most natural time series are non-stationary. Box and Jenkins [14] proposed that for such a series differencing of an appropriate order may make it stationary. Let d be the minimum order for stationarity. Then the resultant stationary series is denoted by $\{\nabla^d X_t\}$ where $\nabla = 1 - L$. If $\{\nabla^d X_t\}$ follows an ARMA(p, q) model the original series $\{X_t\}$ is said to follow an *autoregressive integrated moving average model of orders p, d and q,*denoted by ARIMA(p, d, q).

If a series is seasonal of period s, Box and Jenkins [14] proposed further that it could be modelled by

$$A(L)\Phi(L^s)\nabla^d \nabla_s^D X_t = B(L)\Theta(L^s)\varepsilon_t \quad (2)$$

where $\nabla_s = 1 - L^s$ is the seasonal difference operator. $\Phi(L)$ is the seasonal autoregressive (AR) operator and $\Theta(L)$ is its moving average (MA) counterpart. They both are polynomials in L. Suppose they are of degrees P and Q respectively. Then the model (2) is called a *(multiplicative) seasonal autoregressive integrated moving average model of orders p, d, q, P, D, Q and s,* denoted by SARIMA(p, d, q)x(P, D, Q)$_s$.

2.3 SARIMA Model Estimation

For the model (2) to be fitted its orders p, d, q, P, D, Q and s must be determined. Seasonality period s may be obvious from the nature of the series. Other avenues for determination of s are the time-plot and the correlogram. The correlogram of an s-period seasonal series exhibits fluctuating movements of the same periodicity as the series. The difference orders d and D are often chosen so that they sum up to at most 2. At each stage of the differencing process, the series is tested for stationarity until it is attained. Here, the Augmented Dickey Fuller (ADF) test shall be used to test for stationarity after each stage of differencing. The AR orders p and P are estimated as the non-seasonal and the seasonal cut-off lags of the autocorrelation function (ACF) respectively. Similarly the MA orders q and Q are estimated as the non-seasonal and the seasonal cut-off lags of the partial autocorrelation function (PACF) respectively.

The parameters are thereafter estimated by the use of non-linear optimization techniques because of the involvement of white noise process items in the model. In this work the least error sum of squares technique shall be used in model estimation.

After model fitting the fitted model is usually subjected to residual analysis for validation. All analysis in this work was done using the statistical package Eviews 7.

3. Results and Discussion

The time plot of RIGR in Figure 1 shows a generally horizontal trend except for a sharp positive outlier in December 2008. Its ADF test statistics has a value of -7.2. With the 1%, 5% and 10% critical values given respectively by -3.5, -2.9 and -2.6, it is adjudged to be stationary. However with the sharp outlier its time plot gives a contrary impression. An examination of RIGR shows that the yearly minimums fall in the first four months of the year: In January in 2007; in February in 2008; in April in 2009 and 2010; in February in 2011 and in March in 2012. The respective maximums are in May, June, March, May, July and February. Therefore it may be said that the maximums are mostly between May and July. These tendencies indicate the presence of a seasonal component of 12-monthly periodicity

A seasonal (i.e. 12-point) differencing of RIGR yields SDRIGR. Its time-plot in Figure 2 shows a generally horizontal trend, a positive spike in 2008 being counterbalanced by a comparably negative one in 2009. The ADF test statistic value is equal to -6.8. Hence SDRIGR is stationary. Its correlogram in Figure 3 shows a negative spike at lag 12 on the ACF as well as on the PACF. This means that the series is seasonal of period 12 months and involved is a seasonal AR component and a seasonal MA one, each of order one. That leads to the proposal of a SARIMA(0, 0, 0)x(1, 1, 1)$_{12}$ model. Estimation of this model is done in Table 1. However

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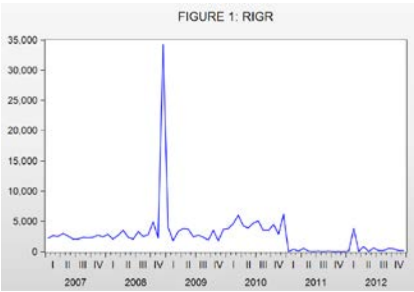
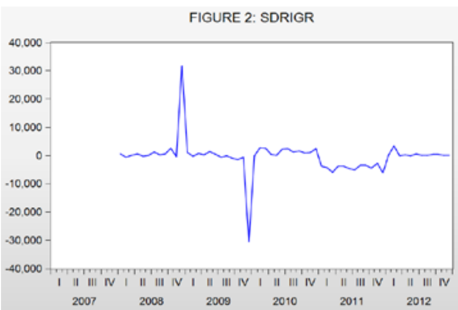
residual analysis done on the model shows that the residuals are correlated (See Figure 4). This invalidates the model.





Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.111	0.111	0.7716	0.380
		2	0.084	0.072	1.2192	0.544
		3	0.058	0.042	1.4404	0.696
		4	0.053	0.038	1.6269	0.804
		5	0.086	0.071	2.1268	0.831
		6	-0.006	-0.031	2.1295	0.907
		7	-0.056	-0.069	2.3518	0.938
		8	-0.020	-0.014	2.3794	0.967
		9	-0.048	-0.042	2.5465	0.980
		10	-0.060	-0.050	2.8182	0.985
		11	-0.071	-0.046	3.1999	0.988
		12	-0.530	-0.516	24.954	0.015
		13	0.004	0.144	24.956	0.023
		14	0.030	0.124	25.029	0.034
		15	0.076	0.140	25.511	0.043
		16	0.030	0.057	25.588	0.060
		17	0.018	0.073	25.616	0.082
		18	0.082	0.034	26.216	0.095
		19	0.099	0.024	27.100	0.102
		20	0.053	0.006	27.356	0.126
		21	0.059	-0.011	27.685	0.149
		22	0.074	0.015	28.224	0.168
		23	0.053	-0.014	28.509	0.197
		24	0.128	-0.241	30.191	0.179
		25	-0.051	0.017	30.471	0.207
		26	-0.101	-0.039	31.594	0.207
		27	-0.087	0.056	32.457	0.216
		28	-0.068	-0.015	32.989	0.236

Figure 3: Correlogram of SDRIGR

Table 1: Estimation of the SARIMA$(0,0,0)x(1,1,1)_{12}$ Model

Dependent Variable: SDRIGR
Method: Least Squares
Date: 08/25/14 Time: 08:51
Sample (adjusted): 2009M01 2012M12
Included observations: 48 after adjustments
Convergence achieved after 17 iterations
MA Backcast: 2008M01 2008M12

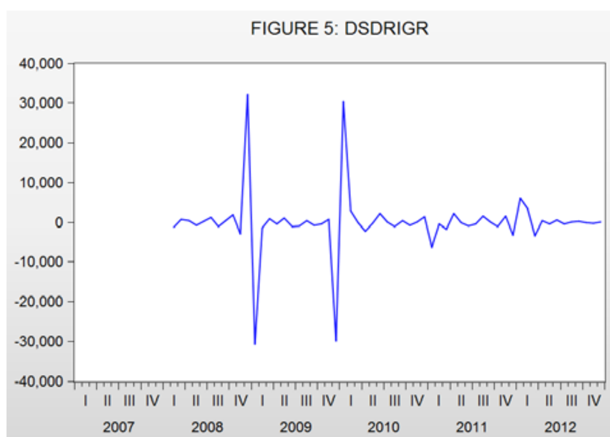
Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(12)	-0.383789	0.071914	-5.336749	0.0000
MA(12)	0.926695	0.030048	30.84079	0.0000

R-squared	0.746461	Mean dependent var	-1220.981
Adjusted R-squared	0.740949	S.D. dependent var	4939.901
S.E. of regression	2514.264	Akaike info criterion	18.53812
Sum squared resid	2.91E+08	Schwarz criterion	18.61609
Log likelihood	-442.9149	Hannan-Quinn criter.	18.56759
Durbin-Watson stat	0.641867		

Inverted AR Roots	.89+.24i	.89-.24i	.65-.65i	.65-.65i
	.24+.89i	.24-.89i	-.24-.89i	-.24+.89i
	-.65-.65i	-.65+.65i	-.89-.24i	-.89+.24i
Inverted MA Roots	.96+.26i	.96-.26i	.70-.70i	.70+.70i
	.26-.96i	.26+.96i	-.26+.96i	-.26-.96i
	-.70-.70i	-.70-.70i	-.96-.26i	-.96+.26i

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.666	0.666	22.627	
		2	0.539	0.171	37.762	
		3	0.446	0.065	48.384	0.000
		4	0.402	0.082	57.186	0.000
		5	0.300	-0.067	62.221	0.000
		6	0.160	-0.160	63.677	0.000
		7	0.110	0.005	64.381	0.000
		8	0.053	-0.034	64.550	0.000
		9	-0.060	-0.152	64.770	0.000
		10	-0.175	-0.139	66.708	0.000
		11	-0.183	0.039	68.871	0.000
		12	-0.095	0.189	69.469	0.000
		13	-0.091	0.035	70.043	0.000
		14	-0.120	-0.035	71.058	0.000
		15	-0.060	0.094	71.319	0.000
		16	-0.113	-0.210	72.274	0.000
		17	-0.108	-0.057	73.177	0.000
		18	-0.121	-0.002	74.348	0.000
		19	-0.127	-0.100	75.690	0.000
		20	-0.182	-0.203	78.523	0.000

Figure 4: Correlogram of SARIMA$(0,0,0)x(1,1,1)_{12}$ Residuals



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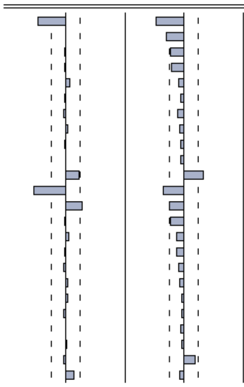


	AC	PAC	Q-Stat	Prob
1	-0.485	-0.485	14.572	0.000
2	-0.001	-0.309	14.572	0.001
3	-0.011	-0.234	14.580	0.002
4	-0.021	-0.219	14.610	0.006
5	0.070	-0.094	14.940	0.011
6	-0.024	-0.053	14.979	0.020
7	-0.048	-0.104	15.141	0.034
8	0.036	-0.070	15.235	0.055
9	-0.009	-0.060	15.241	0.085
10	-0.001	-0.063	15.241	0.124
11	0.244	0.348	19.715	0.049
12	-0.550	-0.355	42.863	0.000
13	0.286	-0.260	49.257	0.000
14	-0.012	-0.233	49.268	0.000
15	0.052	-0.130	49.489	0.000
16	-0.019	-0.132	49.520	0.000
17	-0.043	-0.085	49.677	0.000
18	0.027	-0.071	49.742	0.000
19	0.035	-0.049	49.852	0.000
20	-0.029	-0.031	49.930	0.000
21	-0.005	-0.054	49.933	0.000
22	0.020	-0.025	49.971	0.001
23	-0.046	0.194	50.181	0.001
24	0.134	-0.066	52.020	0.001

Figure 6: Correlogram of DSDRIGR

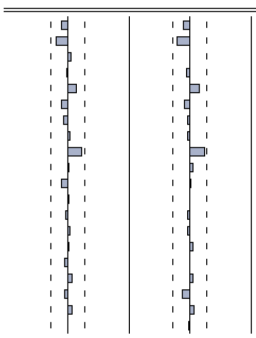
Table 2: Estimation of SARIMA$(0,1,1)x(1,1,1)_{12}$ Model

Dependent Variable: DSDRIGR
Method: Least Squares
Date: 08/25/14 Time: 09:16
Sample (adjusted): 2009M02 2012M12
Included observations: 47 after adjustments
Convergence achieved after 11 iterations
MA Backcast: 2008M01 2009M01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(12)	-0.206583	0.031026	-6.658481	0.0000
MA(1)	0.160127	0.041128	3.893360	0.0003
MA(12)	0.952820	0.011040	86.30508	0.0000
MA(13)	0.125668	0.043267	2.904501	0.0058

R-squared	0.956899	Mean dependent var	-21.13830
Adjusted R-squared	0.953892	S.D. dependent var	6547.985
S.E. of regression	1406.041	Akaike info criterion	17.41621
Sum squared resid	85008848	Schwarz criterion	17.57367
Log likelihood	-405.2809	Hannan-Quinn criter.	17.47546
Durbin-Watson stat	2.206099		

Inverted AR Roots	.85-.23i	.85+.23i	.62+.62i	.62-.62i
	.23+.85i	.23-.85i	-.23-.85i	-.23+.85i
	-.62+.62i	-.62+.62i	-.85+.23i	-.85-.23i
Inverted MA Roots	.96+.26i	.96-.26i	.70-.70i	.70+.70i
	.26+.96i	.26-.96i	-.13	-.26+.96i
	-.26-.96i	-.71+.70i	-.71-.70i	-.96-.26i
	-.96+.26i			



	AC	PAC	Q-Stat	Prob
1	-0.112	-0.112	0.6322	
2	-0.192	-0.207	2.5145	
3	0.048	-0.001	2.6367	
4	-0.019	-0.055	2.6559	
5	0.155	0.165	3.9796	0.046
6	-0.112	-0.094	4.6867	0.096
7	-0.069	-0.027	4.9568	0.175
8	0.038	-0.031	5.0407	0.283
9	0.227	0.251	8.1751	0.147
10	0.023	0.057	8.2071	0.223
11	-0.112	0.023	9.0148	0.252
12	0.018	-0.005	9.0369	0.339
13	-0.028	-0.045	9.0914	0.429
14	0.035	-0.037	9.1762	0.515
15	0.017	0.042	9.1977	0.604
16	-0.057	-0.007	9.4373	0.665
17	0.067	0.056	9.7784	0.712
18	-0.055	-0.125	10.021	0.761
19	0.079	0.075	10.532	0.785
20	0.004	-0.017	10.533	0.837

Figure 7: Correlogram of SARIMA$(0,1,1)x(1,1,1)_{12}$ Residuals

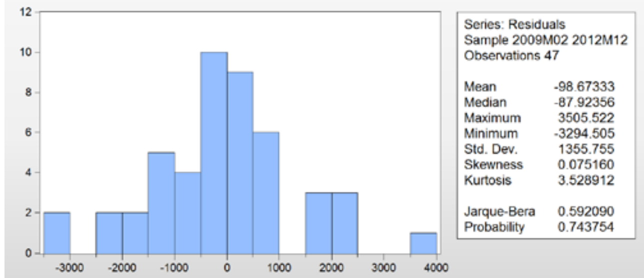


Figure 8: Histogram of SARIMA$(0,1,1)x(1,1,1)_{12}$ Residuals

Non-seasonal differencing of SDRIGR yields the series DSDRIGR which might be said to follow a generally horizontal trend, the two upward spikes counterbalanced by the same number of comparably negative ones (See Figure 5). The ADF test statistic is equal to -7.0 confirming the stationarity hypothesis. The correlogram of DSDRIGR in Figure 6 suggests a SARIMA$(0, 1, 1)x(1, 1, 1)_{12}$ model. The estimation of the model in Table 2 gives:

$$DSDRIGR_t + .2066DSDRIGR_{t-12} = \varepsilon_t + .1601\varepsilon_{t-1} + .9528\varepsilon_{t-12} + .1257\varepsilon_{t-13} \quad (3)$$

All coefficients of the model (3) are significant. The percentage of variation in the data accounted for by the model is very high at 96%. The residuals of the model are uncorrelated (See Figure 7) and normally distributed (see Figure 8). Hence the model is adequate.

4. Conclusion

It might be concluded that the monthly internally generated revenue of Rivers State of Nigeria follows a SARIMA$(0, 1, 1)x(1, 1, 1)_{12}$ model. It might be the basis of any forecasting of the series.

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