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Integral Solutions Of $61X^2 + Y^2 = Z^2$

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Abstract

The ternary quadratic equation $61x^2 + y^2 = z^2$ *is analyzed for its non-zero distinct integral points on it. Employing the integral solutions of the above equation, a few interesting relations between special polygonal and pyramidal numbers are exhibited*

Keywords: *ternary quadratic,integer solutions*

1. *Introduction*

The ternary homogeneous quadratic Diophantine equation offers an unlimited field for research because of their variety [1-2]. For an extensive review of various problems one may refer [3-11,15-17]. In [12] infinitely many non-zero integral points on the hyperboloid of two sheets is given by $x^2 - 6xy + y^2 + 6x - 2y + 5 = z^2 + 4$ are obtained. In [13] integral points on the ternary quadratic equation $z^2 = 2x^2 - 7y^2$ representing homogeneous cone are obtained In [14], ternary quadratic equation $x^2 + 2y^2 - z^2 = 2$ representing hyperboloid of one sheet is analyzed for its integer solutions. This communication concerns with ternary quadratic equation $61x^2 + y^2 = z^2$ for determining its infinitely many integral solutions. Employing the integral solution ,a few interesting relations among the special polygonal and pyramidal numbers are given.

Notations:

Polygonal Number of rank n with size m

$$t_{(m,n)} = n\{1 + \frac{(n-1)(m-2)}{2}\}$$

Pentagonal pyramidal number of rank n

$$P_n^5 = \frac{n^2(n+1)}{2}$$

Pronic number of rank n

$$\Pr_n = n(n+1)$$

Star number of rank n

$$S_n = 6n(n-1) + 1$$

2. **Method Of Analysis:**

The ternary quadratic equation under consideration is

$$61x^2 + y^2 = z^2 \qquad (1)$$

To start with it is seen that the triples

$$(k, 30k, 31k), (2k+1, 2k^2 + 2k - 31, 2k^2 + 2k + 31)$$

and $(2rs, r^2 - 61s^2, r^2 + 61s^2)$ satisfy (1).

However, we have other choices of solutions to (1) which are illustrated below:

Consider (1) as

$$61x^2 + y^2 = z^2 * 1 \qquad (2)$$

Assume

$$z = a^2 + 61b^2 \qquad (3)$$

Write 1 as

$$1 = \frac{[(30+2n-2n^2) + i(2n-1)\sqrt{61}][(30+2n-2n^2) - i(2n-1)\sqrt{61}]}{(31-2n+2n^2)^2} \qquad (4)$$

Substituting (3) and (4) in (2) and employing the method of factorization. define

$$y + i\sqrt{61}x = \frac{(a^2 - 61b^2 + i2ab\sqrt{61})[(30+2n-2n^2) + i(2n-1)\sqrt{61}]}{(31-2n+2n^2)}$$

Equating the real and imaginary parts in the above equation, we get

$$x = \frac{[2(30+2n-2n^2)ab + (a^2 - 61b^2)(2n-1)]}{31-2n+2n^2}$$

$$y = \frac{[(30+2n-2n^2)(a^2 - 61b^2) - 122ab(2n-1)]}{31-2n+2n^2}$$

Replacing a by $(31-2n+2n^2)A$, b by $(31-2n+2n^2)B$ in the above equation, corresponding integer solutions to (1) are given by

$$x = (31-2n+2n^2)[(A^2 - 61B^2)(2n-1) + \{2AB(30+2n-2n^2)\}]$$

$y=(31-2n+2n^2)[(A^2-61B^2)(30+2n-2n^2)-\{122AB(2n-1)\}]$

$z=(31-2n+2n^2)^2(A^2+61B^2)$

For simplicity and clear understanding, taking n=1 in above the corresponding integer solutions of (1) are given by

$x(A,B)=31A^2-1891`B^2+1860AB$

$y(A,B)=930A^2-56730B^2-3782AB$

$z(A,B)=31^2(A^2+61B^2)$

2.1 Properties:

1) $x(A,1)-t_{(64,A)}\equiv -1(\mathrm{mod}1890)$

2) $x(n+1,n^2)-t_{(64,n)}+1891t_{(4,n^2)}-3720p_n^5\equiv 31(\mathrm{mod}92)$

3) $x(n+1,n)-t_{(3722,n)}-1860pr_n\equiv 31(\mathrm{mod}1797)$

4) $y(A,1)-t_{(1862,A)}\equiv -2523(\mathrm{mod}2853)$

5) $y(n+1,n)-t_{(111602,n)}+3782pr_n\equiv 930(\mathrm{mod}53939)$

6) $z(A,1)-t_{(1924,A)}\equiv 61(\mathrm{mod}61)$

7) $y(n+1,n)+3782\mathrm{Pr}_n+310S_n+53940t_{(4,n)}=1240.$

It is worth to note that 1 in (2) may also be represented as

$$1=\frac{[(61-4n^2)+i(4n)\sqrt{61}][(61-4n^2)-i(4n)\sqrt{61}]}{(61+4n^2)^2}$$

Following the analysis presented above, the corresponding integer solutions to (1) are found to be

$x(A,B)=(61+4n^2)[(A^2-61B^2)(4n)+\{2AB(61-4n^2)\}]$

$y(A,B)=(61+4n^2)[(A^2-61B^2)(61-4n^2)-(488AB)]$

$z(A,B)=(61+4n^2)^2(A^2+61B^2)$

For the sake of simplicity, taking n=1 in the above, the corresponding integer solution of (1) are given by

$x(A,B)=260A^2-15860B^2+7410AB$

$y(A,B)=3705A^2-226005B^2-31720AB$

$z(A,B)=4225A^2+257725B^2$

2.2 Properties:

1) $x(A,1)-t_{(522,A)}\equiv -522(\mathrm{mod}7669)$

2) $x(n+1,n^2)-t_{(522,n)}+158560t_{(4,n^2)}-14820p_n^5\equiv 260(\mathrm{mod}779)$

3) $x(n+1,n)-t_{(31202,n)}-7410pr_n\equiv 260(\mathrm{mod}15079)$

4) $y(n+1,n^2)-t_{(7412,n)}+226005t_{(4,n^2)}+63440p_n^5\equiv 3705(\mathrm{mod}11114)$

5) $y(n+1,n)-t_{(444602,n)}+31720pr_n\equiv 3705(\mathrm{mod}214889)$

6) $z(A,1)-t_{(8452,A)}\equiv 3853(\mathrm{mod}4224)$

3. Generation of integer solutions

Let (x_0,y_0,z_0) be any given integer solution of (1) Then, each of the following triple of integers satisfies (1):

Triple 1 : $(x_{n_1},y_{n_1},z_{n_1})$ where

$$x_{n_1}=62x_0+2y_0-8z_0$$
$$y_{n_1}=61x_0+2y_0-8z_0$$
$$z_{n_1}=488x_0+8y_0-63z_0$$

Triple 2 : $(x_{n_2},y_{n_2},z_{n_2})$ where

$$x_{n_2}=\frac{1}{60}[\{(30)^n(-1)+61(-30)^n\}x_0+\{(30)^n(1)-(-30)(1)^n\}z_0]$$
$$y_{n_2}=30^n y_0$$
$$z_{n_2}=\frac{1}{60}[\{-61(30)^n+61(-30)^n\}x_0+\{61(30)^n+(-30)^n(-1)\}z_0]$$

Triple 3: (x_{n_3},y_{n_3},z_0) where

$$x_{n_3}=\frac{1}{62}[\{(31)^n+61(-31)^n\}x_0+\{(-3`1)^n+(-31)^n(1)\}y_0]$$
$$y_{n_3}=\frac{1}{62}[\{-61(31)^n+61(-31)^n\}x_0+\{(31)^n(61)+(-31)^n(1)\}y_0]$$

Triple 4: $(x_{n_4},y_{n_4},z_{n_4})$ where

$$x_{n_4}=4^n x_0$$
$$y_{n_4}=\frac{1}{8}[\{(4)^n-9(-4)^n\}y_0+\{3(-4)^n-3(4)^n\}z_0]$$
$$z_{n_4}=\frac{1}{8}[\{3(4)^n-3(-4)^n\}y_0+\{(-4)^n-9(4)^n\}z_0]$$

4. CONCLUSION:

In this paper,we have obtained infinitely many integer points satisfying the cone $61x^2+y^2=z^2$ As ternary quadratic Diophantine equations are rich in variety one may consider other choices of ternary quadratic Diophantine equations and search for their integer solutions along with their properties.

References:

[**1**]. L. E. Dickson, *History of Theory of Numbers*, Vol. 2, Chelsea publishing Company, Newyork, 1952.

[**2**]. L. J. Mordel, *Diophantine Equations*, Academic press, Newyork, 1969.

[**3**]. M. A. Gopalan, V. Pandichelvi, *Integral solution of ternary quadratic equation* $z(x+y)=4xy$ Acta Ciencia, Indica, Vol. XXXIVM, No. 3, 1353-1358, (2008).

[**4**]. M. A Gopalan, J. Kaliga Rani, Observation on the Diophantine equation $y^2 = DX^2 + Z^2$, Impact J. Sci. Tech. , Vol. 2(2), 90-95, 2008.

[**5**]. M. A. Gopalan, V. Pandichelvi, *On ternary Quadratic equation* $x^2 + y^2 = z^2 + 1$, Impact J. Sci. Tech., Vol. 2(2), 55-58, 2008.

[**6**]. M. A. Gopalan. Manju Somanath and N. Vanitha, *Integral solution of ternary quadratic Diophantine equation* $x^2 + y^2 = (k^2+1)^2 z^2$, Impact J, Sci. Tech., Vol. 2(4), 175-178, 2008.

[**7**]. M. A. Gopalan. Manju Somanath, *Integral solution of ternary quadratic Diophantine equation* $xy + yz = zx$, Acta Ciencia, Indica, V M(1), 1-5, (2008).

[**8**]. M. A Gopalan V. Pandichelvi, *Integral solution of ternary quadratic equation* $z(x-y)=4xy$, Impact J. Sci. Tech., Vol 5 (1), 01-06, 2011.

[**9**]. M. A. Gopalan and A. Vijayasankar, *Observations on a Pythogorean problem*, Acta Ciencia Indica , Vol. XXXVIM, No. 4, 517-520, 2011.

[**10**]. M. A Gopalan, J. Kaliga Rani , *On ternary quadratic equation* $X^2 + Y^2 = Z^2 + 8$, Impact J. Sci. Tech., Vol5(1), 39-43, 2011.

[**11**]. M. A. Gopalan and A. Gnanam, *Pythogorean triangles and special polygonal numbers*, International Journal of Mathematical Sciences,Vol(9), No. 1-2, 211-215. jan-jun (2011).

[**12**]. M. A. Gopalan and D. Geetha, *Lattice points on the hyperboloid of one sheet* $x^2 - 6xy + y^2 + 6x - 2y + 5 = z^2 + 4$, Impact. J. Sci. Tech., Vol. 4, No. 1 23-32, 2010.

[**13**]. M. A. Gopalan, S. Vidhyalakshmi and A. Kavitha,*Integral points on the homogeneous cone* $z^2 = 2x^2 - 7y^2$, Diophantus Journal of Mathematics, **1**(2)(2012), 127-136.

[14] M. A. Gopalan, S. Vidhyalakshmi and S.Mallika, *Observatioms on hyperboloid of one sheet* $x^2 + 2y^2 - z^2 = 2$ Bessel Journal of Mathematics,Vol 2(3) (2012) 221-226.

[15] M. A. Gopalan, S. Vidhyalakshmi and J.Uma Rani *Integer points on the homogeneous cone* $x^2 + 4y^2 = 37z^2$ Cayley Journal of Mathematics,2(2),(2013),101-107.

[16] M. A. Gopalan, S. Vidhyalakshmi and D.Maheswari *Integer points on the homogeneous cone* $2x^2 + 3y^2 = 35z^2$ Indian Journal of Mathematics7(17),(2014),6-10.

[17] M. A. Gopalan, S. Vidhyalakshmi and T.Geetha *Integer points on the homogeneous cone* $7x^2 - 2y^2 = 10z^2$ Scholar Journal of Engineering and Technology,2(4C),(2014), 645-649.