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Reliability and Cost Analysis of a Series System Model Using Fuzzy Parametric Geometric Programming

Medhat El-Damcese [1], Faheem Abbas [2] and Eman El-Ghamry [2]

[1] Department of Mathematics (Section of Mathematical Statistics), Faculty of Science, Tanta University, Tanta, 31521, Egypt

[2] Department of Mathematics and Engineering physics, Faculty of Engineering, Tanta University, Tanta, 31521, Egypt

Abstract
In this paper we present two types of reliability optimization models of a series system; the first model is to find out the optimum reliability of the system with cost constraint and the other is to minimize the system cost subject to a targeted system reliability goal in fuzzy environment. A fuzzy parametric geometric programming technique is developed for the solution of the two resulting reliability optimization problems. Two numerical examples are given for the illustration of computational details of the procedure.

Keywords: *Series system, Reliability optimization, Cost optimization, Fuzzy sets, Fuzzy parametric geometric programming.*

1. Introduction

The reliability optimization is an important research topic in engineering and operations research. In practical, the problem of system reliability may be formed as a typical non-linear programming problem. Geometric programming GP is a technique developed for solving a special type of nonlinear programming and provides a powerful tool for solving a variety of engineering optimization problems. GP is introduced by Zener [1] and many researchers as Rao [2] and Chong [3] developed efficient algorithms for solving GP problems when the cost and constraint coefficients are known exactly. However, in many applications of GP, some parameters are imprecise estimated of actual values. The fuzzy set theory can specify this imprecise data using membership function. Zimmermann [4] first introduced fuzzy geometric programming problem FPGP and other researchers as Liu [5] and Yang [6] studied it well. Leung [7], Benjamin [8], Vicki [9], Sung [10] and Sun [11] applied GP to solve different reliability optimization problems of series and parallel systems. Cao [12] used FPGP to develop and solve different fuzzy reliability optimization models. different situations and solutions techniques on series systems have presented. Ruan [13] and Yuan [14] concentrated on solving reliability optimization problems of series system models in different environments by using FPGP. Mahapatra [15] used FPGP with cost constraint to find optimal reliability for a series system with cost constraint by using fuzzy geometric programming and he analyzed different fuzzy series system models to optimize the reliability or the cost of these systems as in [16].

In this paper two types of series system models are formulated to find out (i) the optimum system reliability with cost constraint and (ii) the minimum system cost with a targeted system reliability goal if objective and constraint are posynomial functions having fuzzy parameters modeled by triangular membership functions, which are solved through FPGP. Finally, two numerical examples are given to show the utility of FPGP on series system reliability models in fuzzy environment.

2. Formulation of Series System Optimization Model

For a series system consists of n components each with a reliability $R_i, i = 1, 2, \dots, n$ and cost, is monotonically increasing non-linear function of reliability, $C_i(R_i) = C_i R_i^{a_i}$ where $a_i < 1$ and $C_i > 0$ are the shape parameters of the i^{th} component. The system reliability is $R_s(R_1, R_2, \dots, R_n) = R_1 R_2 \dots R_n$ and the total system cost is $C_s(R_1, R_2, \dots, R_n) = \sum_{i=1}^{n} C_i(R_i) = \sum_{i=1}^{n} C_i R_i^{a_i}$.

2.1 Crisp Optimization Model

In the following, we consider two types of problem formulations of our series system. In first one, we have to find the maximization of $R_s(R_1, R_2, \dots, R_n)$ subject to the limited available cost C. In the second model, we have to find out minimum of $C_s(R_1, R_2, \dots, R_n)$ with targeted goal of system reliability at least R. The mathematical forms of the two models are as follow:

Type-I

$$\text{Max} \quad R_s(R_1, R_2, \dots, R_n) = R_1\, R_2\, \dots\, R_n\,, \tag{1}$$

$$\text{s.t.} \quad C_s(R_1, R_2, \dots, R_n) = \sum_{i=1}^{n} C_i\, {R_i}^{a_i} \le C,$$

$$0 < R_i \le 1, \; i = 1,2, \dots, n.$$

Type-II

$$\text{Min} \quad C_s(R_1, R_2, \dots, R_n) = \textstyle\sum_{i=1}^{n} C_i\, {R_i}^{a_i} \tag{2}$$

$$\text{s.t.} \quad R_s(R_1, R_2, \dots, R_n) = R_1\, R_2\, \dots\, R_n \ge R,$$

$$0 < R_i \le 1, \; i = 1,2, \dots, n.$$

2.2 Fuzzy Optimization Model

In many practical situations, the objective as well as constraint goal in the reliability optimization model may involve uncertain parameters which can be represented by fuzzy numbers so we have to change the standard form of our series system optimization problem in fuzzy environment as follows:

Type-I

We have to change the objective function in minimization type as follows:

$$\text{Min} \quad \acute{R}_s(R_1, R_2, \dots, R_n) = {R_1}^{-1}\, {R_2}^{-1}\, \dots\, {R_n}^{-1} \tag{3}$$

$$\text{s.t.} \quad C_s(R_1, R_2, \dots, R_n) = \sum_{i=1}^{n} \widetilde{C_i}\, {R_i}^{a_i} \le \widetilde{C},$$

$$0 < R_i \le 1, \; i = 1,2, \dots, n.$$

Type-II

$$\text{Min} \quad C_s(R_1, R_2, \dots, R_n) = \textstyle\sum_{i=1}^{n} \widetilde{C_i}\, {R_i}^{a_i} \tag{4}$$

$$\text{s.t.} \quad \acute{R}_s(R_1, R_2, \dots, R_n) = \tilde{R}\, {R_1}^{-1}\, {R_2}^{-1}\, \dots\, {R_n}^{-1} \le 1,$$

$$0 < R_i \le 1, \; i = 1,2, \dots, n.$$

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3. Fuzzy Parametric Geometric Programming

3.1 Geometric Programming

Suppose the constrained minimization geometric programming is:

$$Z = \text{Min} \quad \textstyle\sum_{t=1}^{s_0} c_{0t} \prod_{j=1}^{n} {x_j}^{a_{0tj}}, \tag{5}$$

$$\text{s.t.} \quad \sum_{t=1}^{s_i} c_{it} \prod_{j=1}^{n} {x_j}^{\gamma_{itj}} \le b_i, \qquad i = 1, 2, \dots, m,$$

$$x_j > 0, \qquad j = 1,2, \dots, n.$$

Where, the coefficients c_{0t}, c_{it} are positive constants, the exponents a_{0tj}, γ_{itj} are any real constants, the design variables x_j are assumed to take positive values, and all b_i are positive numbers.

3.2 Fuzzy Parametric Geometric Programming

Intuitively, if any of the parameters a_{0tj}, b_i, c_{0t}, and c_{it} are approximately known. It can be represented by fuzzy numbers $\tilde{A}_{0tj}$, $\tilde{B}_i$, $\tilde{C}_{0t}$, and $\tilde{C}_{it}$ with membership functions $\mu_{\tilde{A}_{0tj}}$, $\mu_{\tilde{B}_i}$, $\mu_{\tilde{C}_{0t}}$, and $\mu_{\tilde{C}_{it}}$. The conventional geometric programming problem defined in (5) then turns into fuzzy parametric geometric programming problem as follows:

$$\tilde{Z} = \text{Min} \quad \textstyle\sum_{t=1}^{s_0} \tilde{C}_{0t} \prod_{j=1}^{n} {x_j}^{\tilde{A}_{0tj}} \tag{6}$$

$$\text{s.t.} \quad \sum_{t=1}^{s_i} \tilde{C}_{it} \prod_{j=1}^{n} {x_j}^{\gamma_{itj}} \le \tilde{B}_i, \qquad i = 1, 2, \dots, m,$$

$$x_j > 0, \qquad j = 1,2, \dots, n.$$

Based on Zadeh's extension principle, we can derive the membership function $\mu_{\tilde{Z}}$ of the objective value $\tilde{Z}$ as follows:

$$\mu_{\tilde{Z}}(z) = \mathit{Sup\ Min} \left\{ \mu_{\tilde{A}_{0tj}}(a_{0tj}), \mu_{\tilde{B}_i}(b_i), \mu_{\tilde{C}_{0t}}(c_{0t}), \mu_{\tilde{C}_{it}}(c_{it}), \forall i, j, t \;\middle|\; z = Z(a, b, c) \right\} \tag{7}$$

Where $Z(a, b, c)$ is the function of the conventional geometric program that is defined in model (6).

It is hardly possible to get $\mu_{\tilde{Z}}$ in closed form so it suffices to find the upper bound $Z^U(\propto)$ and the lower bound $Z^L(\propto)$ of the objective value $\tilde{Z}$ at specific $\propto$ level which they can be expressed as

$$Z^U(\propto) = \mathit{Max} \left\{ Z(a, b, c) \;\middle|\; \begin{array}{l} {A_{0tj}}^L(\propto) \le a_{0tj} \le {A_{0tj}}^U(\propto), {B_i}^L(\propto) \le b_i \le {B_i}^U(\propto), \\ {C_{0t}}^L(\propto) \le c_{0t} \le {C_{0t}}^U(\propto), {C_{it}}^L(\propto) \le c_{it} \le {C_{it}}^U(\propto), \forall i, j, t \end{array} \right\},$$

$$Z^L(\propto) =$$

$$\text{Min}\left\{Z(a,b,c)\,\middle|\, \begin{array}{l} A_{0tj}{}^{L}(\propto) \le a_{0tj} \le A_{0tj}{}^{U}(\propto), B_i{}^{L}(\propto) \le b_i \le B_i{}^{U}(\propto), \\ C_{0t}{}^{L}(\propto) \le c_{0t} \le C_{0t}{}^{U}(\propto), C_{it}{}^{L}(\propto) \le c_{it} \le C_{it}{}^{U}(\propto), \forall i,j,t \end{array}\right\}.$$

$Z^U(\propto)$ and $Z^L(\propto)$ can be reformulated to be a pair of two-level mathematical programs as follow:

$$Z^U(\propto) = \tag{8.a}$$

$$\underset{\substack{A_{0tj}{}^{L}(\propto) \le a_{0tj} \le A_{0tj}{}^{U}(\propto) \\ B_i{}^{L}(\propto) \le b_i \le B_i{}^{U}(\propto) \\ C_{0t}{}^{L}(\propto) \le c_{0t} \le C_{0t}{}^{U}(\propto) \\ C_{it}{}^{L}(\propto) \le c_{it} \le C_{it}{}^{U}(\propto) \\ \forall i,j,t}}{\text{Max}} \left\{ \begin{array}{ll} \text{Min} & \sum_{t=1}^{s_0} c_{0t} \prod_{j=1}^{n} x_j^{a_{0tj}} \\ \text{s.t.} & \sum_{t=1}^{s_i} \frac{c_{it}}{b_i} \prod_{j=1}^{n} x_j^{\gamma_{itj}} \le 1, \quad i = 1,2,\dots,m, \\ & x_j > 0, \quad j = 1,2,\dots,n. \end{array} \right.,$$

$$Z^L(\propto) = \tag{8.b}$$

$$\underset{\substack{A_{0tj}{}^{L}(\propto) \le a_{0tj} \le A_{0tj}{}^{U}(\propto) \\ B_i{}^{L}(\propto) \le b_i \le B_i{}^{U}(\propto) \\ C_{0t}{}^{L}(\propto) \le c_{0t} \le C_{0t}{}^{U}(\propto) \\ C_{it}{}^{L}(\propto) \le c_{it} \le C_{it}{}^{U}(\propto) \\ \forall i,j,t}}{\text{Min}} \left\{ \begin{array}{ll} \text{Min} & \sum_{t=1}^{s_0} c_{0t} \prod_{j=1}^{n} x_j^{a_{0tj}} \\ \text{s.t.} & \sum_{t=1}^{s_i} \frac{c_{it}}{b_i} \prod_{j=1}^{n} x_j^{\gamma_{itj}} \le 1, \quad i = 1,2,\dots,m, \\ & x_j > 0, \quad j = 1,2,\dots,n. \end{array} \right..$$

Based on duality algorithm, the two-level mathematical programs (8.a) and (8.b) are transformed into a pair of conventional geometric programs as follow:

- *Upper bound*

$$Z^U(\propto) = \text{Max} \prod_{t=1}^{s_0} \left(\frac{C_{0t}{}^{U}(\propto)}{w_{0t}}\right)^{w_{0t}} \prod_{i=1}^{m} \prod_{t=1}^{s_i} \left(\frac{C_{it}{}^{U}(\propto)\, w_{i0}}{B_i{}^{L}(\propto)\, w_{it}}\right)^{w_{it}} \tag{10.a}$$

$$\begin{aligned} \text{s.t.}\quad & \textstyle\sum_{t=1}^{s_0} w_{0t} = 1, \\ & \sum_{t=1}^{s_0} a_{0tj} w_{0t} + \sum_{i=1}^{m} \sum_{t=1}^{s_i} \gamma_{itj} w_{it} = 0, \quad j = 1,2,\dots,n, \\ & A_{0tj}{}^{L}(\propto) \le a_{0tj} \le A_{0tj}{}^{U}(\propto), \quad \forall j,t, \\ & w_{it} \ge 0, \quad \forall i,t, \quad i = 1,2,\dots,m, \end{aligned}$$

- *Lower bound*

$$Z^L(\propto) = \text{Max} \prod_{t=1}^{s_0} \left(\frac{C_{0t}{}^{L}(\propto)}{w_{0t}}\right)^{w_{0t}} \prod_{i=1}^{m} \prod_{t=1}^{s_i} \left(\frac{C_{it}{}^{L}(\propto)\, w_{i0}}{B_i{}^{U}(\propto)\, w_{it}}\right)^{w_{it}} \tag{10.b}$$

$$\begin{aligned} \text{s.t.}\quad & \textstyle\sum_{t=1}^{s_0} w_{0t} = 1, \\ & \sum_{t=1}^{s_0} A_{0tj}{}^{L}(\propto)\, w_{0t} + \sum_{i=1}^{m} \sum_{t=1}^{s_i} \gamma_{itj} w_{it} = 0, \quad \forall j \in P, \\ & \sum_{t=1}^{s_0} A_{0tj}{}^{U}(\propto)\, w_{0t} + \sum_{i=1}^{m} \sum_{t=1}^{s_i} \gamma_{itj} w_{it} = 0, \quad \forall \\ & w_{it} \ge 0, \quad \forall i,t, \quad i = 1,2,\dots,m, \end{aligned}$$

Where $\sum_{t=1}^{s_i} w_{it} = w_{i0}, \forall i$ and $P = \{j | x_j{}^* \ge 1\}, Q = \{j | 0 < x_j{}^* < 1\}$, and $P \cup Q = J; J = \{1,2,\dots,n$.

We can derive the upper and lower bounds of the objective value by solving Model (10), respectively. By collecting numerical solutions of $Z^L(\propto)$ and $Z^U(\propto)$ at different possibility levels of the left shape $L(z)$ and the right shape $R(z)$ of the membership function of $\mu_{\tilde{Z}}$ can be constructed as:

$$\mu_{\tilde{Z}} = \begin{cases} L(z), & Z^L(\propto)|_{\propto=0} \le z \le Z^L(\propto)|_{\propto=1}, \\ 1, & Z^L(\propto)|_{\propto=1} \le z \le Z^U(\propto)|_{\propto=1}, \\ R(z), & Z^U(\propto)|_{\propto=1} \le z \le Z^U(\propto)|_{\propto=0}. \end{cases}$$

Once the optimum values of w_{0t} and w_{it}, the values of the lower bound $Z^L(\propto)$ and upper bound $Z^U(\propto)$ of the objective function at different possibility level $\propto$ can be obtained then we can determine the optimum values of the design variables x_j, $j = 1,2,\dots,n$ by solving simultaneously the following equations:

$$w_{0t} = \frac{c_{0t}}{Z} \prod_{j=1}^{n} (x_j)^{a_{0tj}}, \quad t = 1,2,\dots,s_0 \tag{11.a}$$

$$\frac{w_{it}}{\sum_{t=1}^{s_i} w_{it}} = \frac{c_{it}}{b_i} \prod_{j=1}^{n} (x_j)^{\gamma_{itj}}, \quad \forall i, \quad \begin{array}{l} t = 1,2,\dots,s_i \\ i = 1,2,\dots,m \end{array} \tag{11.b}$$

4. Fuzzy Parametric Geometric Programming on Series System Model

The cost constraint goal $\tilde{C}$ in Model (3) and the system reliability constraint goal $\tilde{R}$ in Model (4), and the shape parameters $\tilde{C_i}$ are taken as triangular fuzzy numbers $\tilde{C} = (C_1, C_2, C_3)$, $\tilde{R} = (R_1, R_2, R_3)$, and $\tilde{C_i} = (C_{i1}, C_{i2}, C_{i3})$, respectively. By considering $\propto -cut$ of these parameters and using dual relation, our two models will be as follow:

Type-I

$$\acute{R}_s{}^{U}(\propto) = \text{Max} \left(\frac{1}{w_{01}}\right)^{w_{01}} \prod_{i=1}^{n} \left(\frac{C_i{}^{U}(\propto) \sum_{i=1}^{n} w_{1i}}{C^L(\propto)\, w_{1i}}\right)^{w_{1i}} \tag{12.a}$$

$$\begin{aligned} \text{s.t.}\quad & w_{01} = 1, \\ & -w_{01} + a_i w_{1i} = 0, \ \forall i, \end{aligned}$$

$$w_{1i} \geq 0, \quad i = 1, 2, \dots, n.$$

$$\acute{R}_s{}^L(\propto) = \text{Max} \ \left(\frac{1}{w_{01}}\right)^{w_{01}} \prod_{i=1}^{n} \left(\frac{C_i{}^L(\propto) \sum_{i=1}^{n} w_{1i}}{C^U(\propto)\, w_{1i}}\right)^{w_{1i}} \quad (12.b)$$

$$\text{s.t.} \quad w_{01} = 1,$$
$$-w_{01} + a_i w_{1i} = 0, \ \forall i,$$
$$w_{1i} \geq 0, \quad i = 1, 2, \dots, n.$$

After solving Model (12) and using the relations (11.a) and (11.b), we get the optimal components reliability for different values of $\propto \in [0,1]$ as follows:

$$R_i{}^{*U}(\propto) = \left(\frac{C^U(\propto)\, w_{1i}}{C_i{}^L(\propto)\ \sum_{i=1}^{n} w_{1i}}\right)^{\frac{1}{a_i}},$$

$$R_i{}^{*L}(\propto) = \left(\frac{C^L(\propto)\, w_{1i}}{C_i{}^U(\propto)\ \sum_{i=1}^{n} w_{1i}}\right)^{\frac{1}{a_i}}, \quad i = 1, 2, \dots, n \quad (13)$$

Type-II

$$C_s{}^U(\propto) = \text{Max} \ \prod_{i=1}^{n} \left(\frac{C_i{}^U(\propto)}{w_{0i}}\right)^{w_{0i}} \left(R^U(\propto)\right)^{w_{11}} \quad (14.a)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} w_{0i} = 1,$$
$$a_i w_{0i} - w_{11} = 0, \quad \forall i,$$
$$w_{0i} \geq 0, \quad i = 1, 2, \dots, n.$$

$$C_s{}^L(\propto) = \text{Max} \ \prod_{i=1}^{n} \left(\frac{C_i{}^L(\propto)}{w_{0i}}\right)^{w_{0i}} \left(R^L(\propto)\right)^{w_{11}} \quad (14.b)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} w_{0i} = 1,$$
$$a_i w_{0i} - w_{11} = 0, \ \forall i,$$
$$w_{0i} \geq 0, \quad i = 1, 2, \dots, n.$$

After solving Model (14) and using the relations (11.a) and (11.b), we get the optimal components reliability for different values of $\propto \in [0,1]$ as follows:

$$R_i{}^{*U}(\propto) = \left(\frac{C_s{}^{*U}(\propto)\, w_{0i}}{C_i{}^L(\propto)}\right)^{\frac{1}{a_i}},$$

$$R_i{}^{*L}(\propto) = \left(\frac{C_s{}^{*L}(\propto)\, w_{0i}}{C_i{}^U(\propto)}\right)^{\frac{1}{a_i}}, i = 1, 2, \dots, n. \quad (15)$$

5. Numerical Example

We consider a device has four units in series. The input parameters for indices are $a_1 = 0.65, a_2 = 0.55, a_3 = 0.75,$ and $a_4 = 0.55$ and the cost coefficients are taken as triangular fuzzy numbers $\widetilde{c_1} = (6.5, 7, 7.5)$, $\widetilde{c_2} = (7.75, 8, 8.5)$, $\widetilde{c_3} = (5.65, 6, 6.25)$, and $\widetilde{c_4} = (8, 9, 9.25)$.

If the cost constraint goal of the model Type-I is taken as triangular fuzzy number $\tilde{C} = (27.5, 28, 28.25)$.
The optimal solution of the fuzzy Model (12) through FPGP is presented in Table.1 and the reliability of the system can be represented graphically as shown in Fig.1.

The fuzzy system reliability and the fuzzy optimal reliability of each component can be approximated to have triangular shaped membership functions which can be diffuzified to get their crisp values as follow

$$\hat{R}_S = \frac{1}{4}(0.379 + 2 * 0.588 + 0.993) = 0.637$$

$\hat{R}^*{}_1 = 0.9075$, $\hat{R}^*{}_2 = 0.9365$, $\hat{R}^*{}_3 = 0.9395$, $\hat{R}^*{}_4 = 0.74075$

If the fuzzy system reliability goal of the model Type-II is $\tilde{R} = (0.7, 0.72, 0.75)$.
The optimal solution of the fuzzy Model (14) through FPGP is presented in Table.2 and the system cost can be represented graphically as shown in Fig.2. Also, from the results, the system cost and the reliability of each component can be approximated to have triangular fuzzy membership functions with diffuzified values calculated as follow:

$$\hat{C}_S = \frac{1}{4}(26.414 + 2 * 28.498 + 30.126) = 28.384,$$

$\hat{R}^*{}_1 = 0.91$, $\hat{R}^*{}_2 = 0.93875$, $\hat{R}^*{}_3 = 0.93825$, $\hat{R}^*{}_4 = 0.81575$

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Table.1: Optimal solution of fuzzy model Type-I using FPGP

$\propto$	$[R_S^L(\propto), R_S^U(\propto)]$	$[R_1^{*L}(\propto), R_1^{*U}(\propto)]$	$[R_2^{*L}(\propto), R_2^{*U}(\propto)]$	$[R_3^{*L}(\propto), R_3^{*U}(\propto)]$	$[R_4^{*L}(\propto), R_4^{*U}(\propto)]$
0	[0.379, 0.993]	[0.800, 1]	[0.830, 1]	[0.872, 1]	[0.653, 0.892]
0.1	[0.396, 0.940]	[0.811, 1]	[0.842, 1]	[0.878, 1]	[0.658, 0.871]
0.2	[0.414, 0.891]	[0.822, 1]	[0.854, 1]	[0.885, 1]	[0.663, 0.850]
0.3	[0.432, 0.845]	[0.833, 0.999]	[0.866, 1]	[0.892, 1]	[0.669, 0.831]
0.4	[0.451, 0.801]	[0.844, 0.987]	[0.878, 1]	[0.899, 0.996]	[0.675, 0.811]
0.5	[0.471, 0.760]	[0.855, 0.974]	[0.891, 0.993]	[0.906, 0.986]	[0.680, 0.793]
0.6	[0.493, 0.722]	[0.867, 0.962]	[0.904, 0.986]	[0.913, 0.977]	[0.686, 0.775]
0.7	[0.515, 0.685]	[0.878, 0.950]	[0.917, 0.979]	[0.921, 0.969]	[0.691, 0.758]
0.8	[0.538, 0.651]	[0.890, 0.938]	[0.930, 0.972]	[0.928, 0.959]	[0.697, 0.741]
0.9	[0.562, 0.618]	[0.903, 0.927]	[0.944, 0.965]	[0.935, 0.951]	[0.703, 0.725]
1.0	[0.588, 0.588]	[0.915, 0.915]	[0.958, 0.958]	[0.943, 0.943]	[0.709, 0.709]

Table.2: Optimal solution of fuzzy model Type-II using FPGP

$\propto$	$[C_S^L(\propto), C_S^U(\propto)]$	$[R_1^{*L}(\propto), R_1^{*U}(\propto)]$	$[R_2^{*L}(\propto), R_2^{*U}(\propto)]$	$[R_3^{*L}(\propto), R_3^{*U}(\propto)]$	$[R_4^{*L}(\propto), R_4^{*U}(\propto)]$
0	[26.414, 30.126]	[0.754, 1]	[0.773, 1]	[0.825, 1]	[0.663, 1]
0.1	[26.623, 29.963]	[0.771, 1]	[0.793, 1]	[0.839, 1]	[0.676, 1]
0.2	[26.832, 29.800]	[0.789, 1]	[0.813, 1]	[0.852, 1]	[0.689, 1]
0.3	[27.041, 29.474]	[0.807, 1]	[0.834, 1]	[0.865, 1]	[0.702, 1]
0.4	[27.25, 29.474]	[0.825, 1]	[0.855, 1]	[0.879, 1]	[0.716, 0.965]
0.5	[27.458, 29.312]	[0.844, 1]	[0.876, 1]	[0.893, 1]	[0.729, 0.935]
0.6	[27.666, 29.149]	[0.863, 1]	[0.898, 1]	[0.907, 1]	[0.743, 0.906]
0.7	[27.875, 28.986]	[0.882, 1]	[0.921, 1]	[0.921, 1]	[0.757, 0.8779]
0.8	[28.083, 28.824]	[0.902, 0.981]	[0.944, 1]	[0.935, 0.995]	[0.771, 0.851]
0.9	[28.291, 28.661]	[0.922, 0.962]	[0.967, 1]	[0.950, 0.979]	[0.786, 0.825]
1.0	[28.498, 0.588]	[0.943, 0.943]	[0.991, 0.991]	[0.964, 0.964]	[0.800, 0.800]

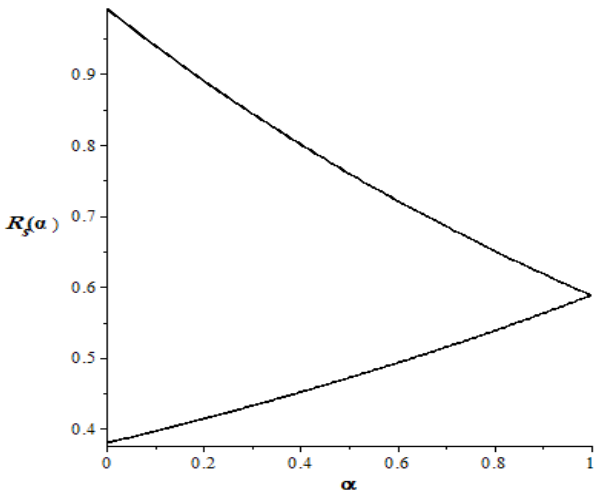

Fig.1: The fuzzy reliability of series system optimization model with four components Type-I

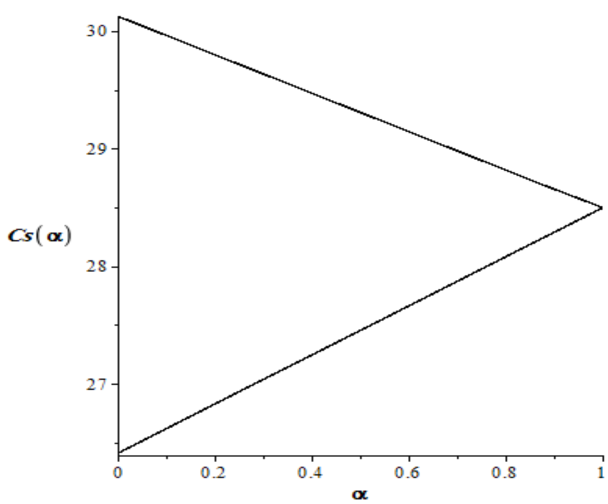

Fig.2: The fuzzy cost of series system optimization model with four components Type-II.

6. Conclusions

We introduced the technique for solving fuzzy parametric geometric programming problems FPGP and applied this technique to find the solution of two types of reliability optimization models of a series system; firstly optimized system reliability with cost constraint goal and secondly minimized system cost with a target of system reliability in fuzzy environment. Also, two numerical examples have been solved to show that the FPGP technique is effective in solving the two reliability optimization problems of the series system. As an extension to this paper, for other systems as parallel, parallel-series and series-parallel systems, fuzzy reliability optimization problems may be formulated and solved easily through FPGP.

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