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with

$$A = -(a_{11} + a_{12} + a_{13}),$$
$$B = a_{11}a_{33} + a_{22}a_{33} + a_{11}a_{22} - a_{12}a_{21} - a_{23}a_{32},$$
$$C = (a_{12}a_{21} - a_{11}a_{22})a_{33} + a_{11}a_{23}a_{32}.$$

By the Routh-Hurwitz criterion, $E_3 = (x^*, y^*, z^*)$ is locally asymptotically stable if and only if $A, C,$ and $AB - C$ are positive.

6. NUMERICAL SIMULATIONS

In this section, we present the time plots, phase portraits and bifurcation diagrams to illustrate the theoretical analysis and show the interesting complex dynamical behaviors of the system (1).

Example1: We shall consider $r = 0.15, b = 0.15, c = 0.1,$ $d = 0.15, e = 0.04,$ $f = 0.95$ and $g = 0.94$. At equilibrium point E_0, the eigenvalues are $\lambda_1 = 0.15,$ $\lambda_2 = 0.9$ and $\lambda_3 = 0.05$ so that $|\lambda_{1,2,3}| < 1$. Hence the trivial equilibrium point is stable (see Figure-1).

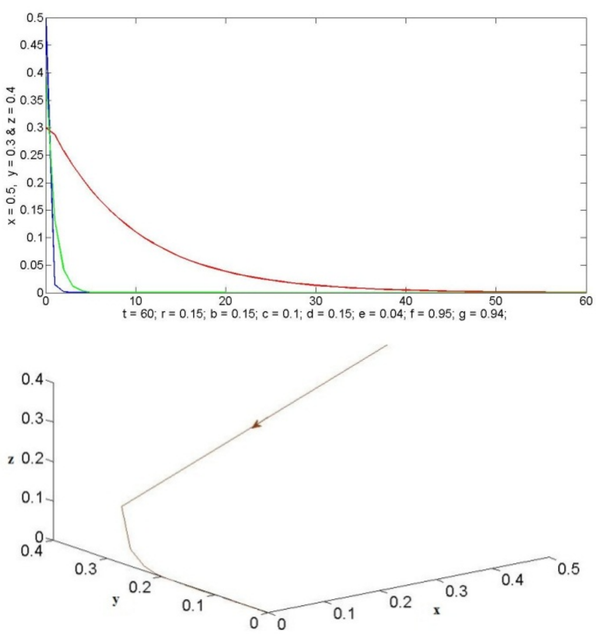


Figure 1: Time Series Plot and Phase Portrait at E_0.

Example2: We shall consider the parameter values $r = 1.1, b = 0.65, c = 0.1, d = 0.5, e = 1.14,$ $f = 0.5$ and $g = 0.4$. The equilibrium point $E_1 = (0.091, 0, 0)$ and the eigenvalues are $\lambda_1 = 0.9$ $\lambda_2 = 0.945$ and $\lambda_3 = 0.5$ so that $|\lambda_{1,2,3}| < 1$. Hence system (1) is stable (see Figure-2).

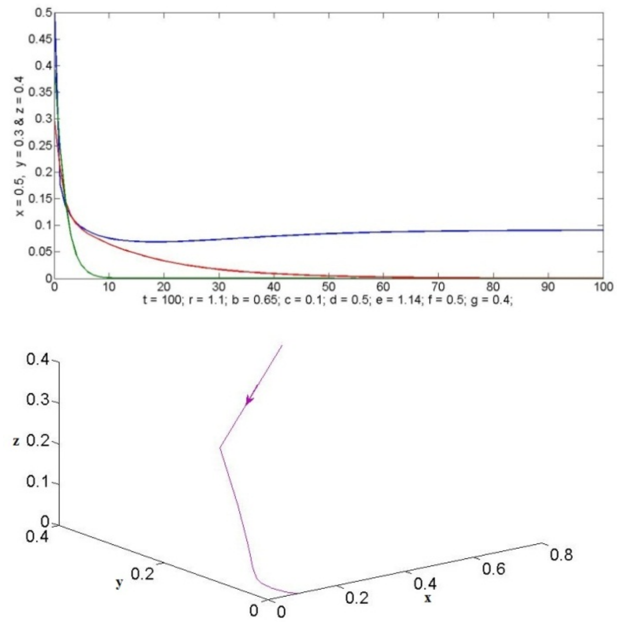


Figure 2: Time Series Plot and Phase Portrait at E_1.

Example3: We shall consider the parameter values $r = 1.5, b = 0.65, c = 0.1, d = 0.5, e = 1.14,$ $f = 0.5$ and $g = 0.4$. The equilibrium point $E_2 = (0.2, 0.308, 0)$ and the eigenvalues are $\lambda_1 = 0.623,$ $\lambda_2 = 0.9$ and $\lambda_3 = 0.8$ so that $|\lambda_{1,2,3}| < 1$. Thus system (1) is stable (see Figure-3).

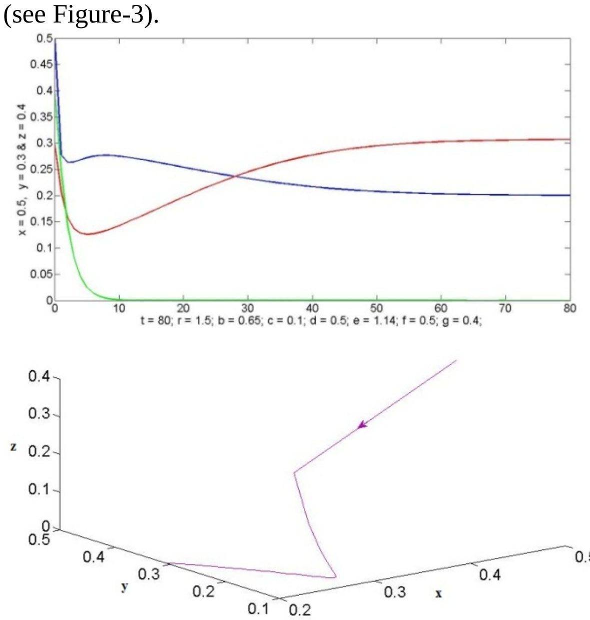


Figure 3: Time Series Plot and Phase Portrait at E_2.

Example4: When $r = 2.65$, $b = 0.45$, $c = 0.31$, $d = 0.8$, $e = 0.5$, $f = 0.5$ and $g = 1.1$. The eigenvalues are $\lambda_1 = -0.3828$ and $\lambda_{2,3} = 0.9686 \pm i\,0.2549$ so that $|\lambda_1| < 1$ and $|\lambda_{2,3}| = 1.0016 > 1$. The system (1) is unstable (see Figure-4).

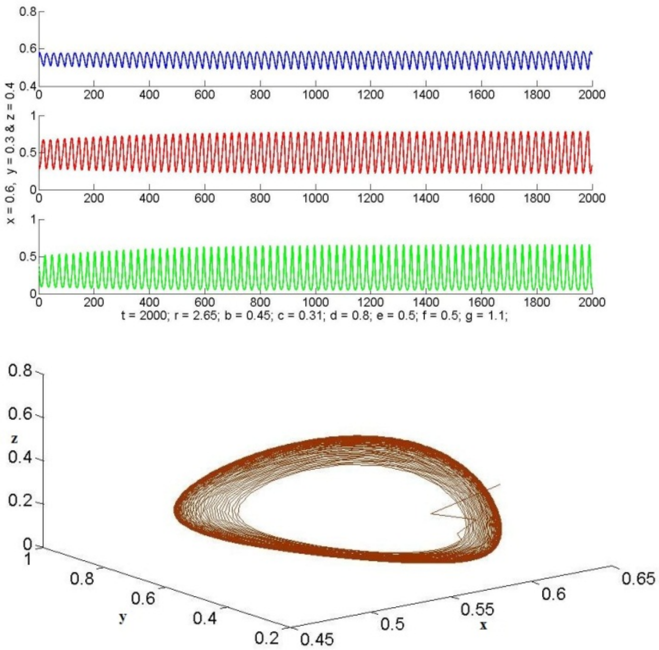
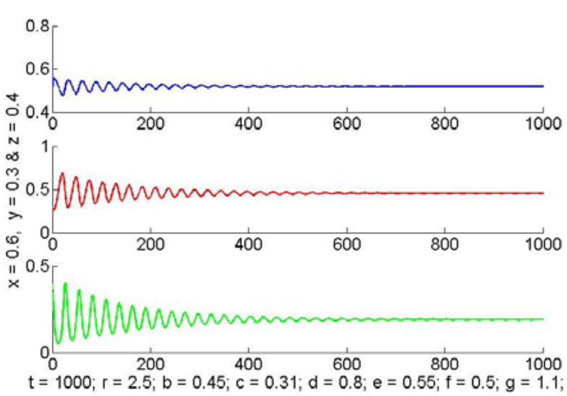


Figure 4: Time Series Plot and Phase Portrait at E_3.

While with $r = 2.5, e = 0.55$ and keeping all other parameters same, we obtain $E_3 = (0.518, 0.454, 0.191)$ and the eigenvalues are $\lambda_1 = -0.2288$ and $\lambda_{2,3} = 0.9667 \pm i\,0.2329$ so that $|\lambda_1| < 1$ and $|\lambda_{2,3}| = 0.9943 < 1$. We observe the system (1) is stable (see Figure-5).



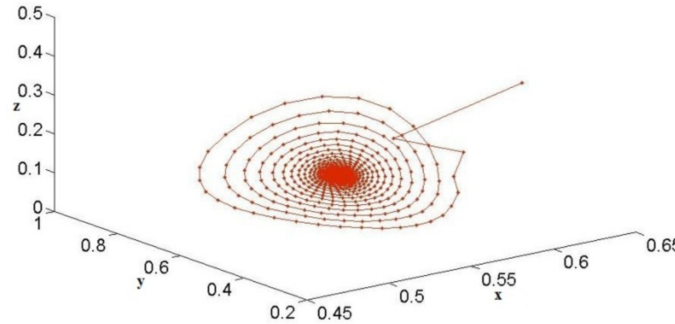


Figure 5: Time Series Plot and Phase Portrait at E_3.

7. BIFURCATION ANALYSIS VIA NUMERICAL SIMULATIONS

Bifurcation is a change of the dynamical behaviors of the system as its parameters pass through a *bifurcation* (*critical*) *value*. Bifurcation usually occurs when the stability of an equilibrium changes. In this section, we focus on exploring the possibility of chaotic behavior for the prey and the mid-predator and the top predator respectively.

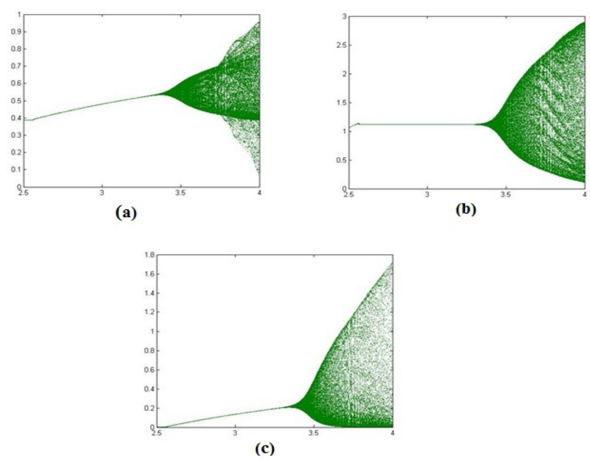


Figure 6: Bifurcation diagrams for (a) prey population, (b) mid-predator population and (c) top predator.

Figure (6) presents the bifurcation diagram for prey and the mid-predator and the top predator densities of the system (1) with initial conditions $x = 0.6, y = 0.3$ and $z = 0.4$ as above and we consider the parameters values $b = 0.5, c = 0.31$, $d = 0.8, e = 0.55$, $f = 0.95$, $g = 0.85$ and $r = 2.5{:}0.001{:}4$. The bifurcation diagrams imply the existence of chaos. Also these results reveal far richer dynamics of the discrete-time models.

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