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# Hamiltonian Decomposition Of Complete Fuzzy Graphs And Some Results Using Fuzzy Matrices

**Vanadhi Kamalakannan, Madurai, Tamil Nadu, India.**

Abarna Parthiban, Christ the king matriculation Higher Secondary School,
Karamadai, Coimbatore, Tamil Nadu, India.

Mahalakshmi Senthilkumar, Islamiyah matriculation Higher Secondary School,
Karumbukkadai, Coimbatore, Tamil Nadu, India.

## 1. Abstract

In this paper we show that the Hamiltonian decomposition of complete fuzzy graphs with 2n vertices can be decomposed into the integer value of (2n – 1)/2 Hamiltonian fuzzy cycles and the rest of the edges is n which forms the 1- factorization. And also we discuss about some results of complete fuzzy graphs and regular fuzzy graphs using fuzzy matrices.

**Keywords** Fuzzy graph, Complete fuzzy graph, Hamiltonian fuzzy cycles, Regular fuzzy graph, Fuzzy matrices.

## 2. Introduction

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and the relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a fuzzy graph model.

Fuzzy graph is also a symmetric binary fuzzy relation on a fuzzy subset. The concept of fuzzy sets and fuzzy relations was introduced by L.A. Zadeh in 1965[1] and further studied in [2]. In 1975 Rosenfeld [3] introduced the notion of fuzzy graph and several analogs of graph theoretic concepts sucsh as path, cycles and connectedness. The concept of decomposition of regular graphs was introduced by Klas Markstrom [4]. Decomposition of complete graphs into Hamiltonian cycles discussed in [5] and

Decomposition of fuzzy graphs introduced by Dr.G.Nirmala and M.Vijaya [6]. Here we discuss about the decomposition of complete fuzzy graphs with 2n vertices and some results of complete fuzzy graph and regular fuzzy graphs using fuzzy matrices.

This paper is organized as follows: The section 1 focuses the abstract of this paper. In section 2, introduction of fuzzy graph is given. Basic definitions included in section 3. Section 4 explains decomposition of complete fuzzy graphs with 2n vertices into Hamiltonian fuzzy cycles. Some results using fuzzy matrices are discussed in section 5. Conclusion is included in section 6.

## 3. Basic definitions

**Definition 3.1** In a fuzzy graph G a fuzzy cycle C covers all the vertices of G exactly once except the end vertices then the cycle is called Hamiltonian fuzzy cycle.

**Definition 3.2** A fuzzy graph G: (σ, μ) is said to be complete if $\mu(x,y) = \sigma(x)\Lambda\sigma(y)$ for all x and y.

**Definition 3.3** Let X and Y be two fuzzy matrices we define the addition of fuzzy matrices as follows, X+Y= max{X,Y} or min {X,Y}.

## 4. Decomposition of Complete fuzzy graphs with 2n vertices into Hamiltonian fuzzy Cycles

### Theorem

The complete fuzzy graph of 2n vertices can be decomposed into the integer value of (2n-1)/2 Hamilton fuzzy cycles and the number of rest of the edges is n which forms the 1-factorisation and cannot form the Hamiltonian fuzzy cycles.

### Proof

First we have to proof the complete fuzzy graph of 2n vertices decomposed into the integer value of (2n-1)/2 Hamiltonian fuzzy cycles.

From [6] we have for any n≥1, $K_{2n+1}$ is decomposable into n Hamiltonian fuzzy cycles $C_{2n+1}$. By the wonderful walecki construction, each vertex of $K_{2n+1}$ have exactly 2n neighbors.

But in the definition of Hamilton fuzzy cycle, no vertex is placed next to the same vertex on either side more than once. Therefore the complete fuzzy graph $K_{2n+1}$ can be decomposed into 2n/2 Hamilton fuzzy cycles.

i.e., the complete fuzzy graph $K_{2n+1}$ can be decomposed into n Hamilton fuzzy cycles.

Using the same manner, we have to prove for $K_{2n}$. Here each vertex of $K_{2n}$ has exactly 2n-1 neighbors. But in the definition of Hamilton fuzzy cycle, the complete fuzzy graph $K_{2n}$ can be decomposed into (2n-1)/2 Hamilton fuzzy cycles.

Clearly (2n-1)/2 gives the decimal value. But we cannot form Hamilton fuzzy cycles using decimal value. Therefore without loss of generality we can consider only the integer value of (2n-1)/2.

Now we have to prove the number of the rest of the edges is n.

From [6] we have the complete fuzzy graph with 2n vertices can be decomposed into Hamiltonian fuzzy cycles with 1-factorisation. Here we prove only the number of rest of the edges is n.

If a fuzzy graph is complete with n vertices then it has n (n-1)/2 edges. This is true for all odd and even number of n. For this theorem we consider only even number of vertices which is denoted by 2n.

Therefore the number of total edges in a complete fuzzy graph of 2n vertices is 2n(2n-1)/2. But every Hamiltonian fuzzy cycle contains equal number of vertices and edges.

By our first part, the number of edges used in the Hamiltonian fuzzy cycles of 2n vertices is (n-1)2n. Subtracting edges in Hamiltonian fuzzy cycles from the total edges we get n edges which are less than 2n.

Therefore the number of remaining edges is n and which is less than 2n. Hence it cannot forms the Hamiltonian fuzzy cycles.

Thus, the complete fuzzy graph $K_{2n}$ can be decomposed into the integer value of (2n-1)/2 Hamilton fuzzy cycles and the number of rest of the edges is n which forms the 1-factorisation and cannot form the Hamiltonian fuzzy cycles.

By using this theorem we can decompose the $K_4$ fuzzy graph into one Hamiltonian fuzzy cycle. The edges $v_1$ $v_3$ and $v_2$ $v_4$ cannot form the fuzzy cycles which is equal to the value of n.

Similarly we can decompose the $K_6$ fuzzy graph into two Hamiltonian fuzzy cycles and the edges $v_1$ $v_4$, $v_1$ $v_6$, $v_3$ $v_4$ cannot form the circuit.

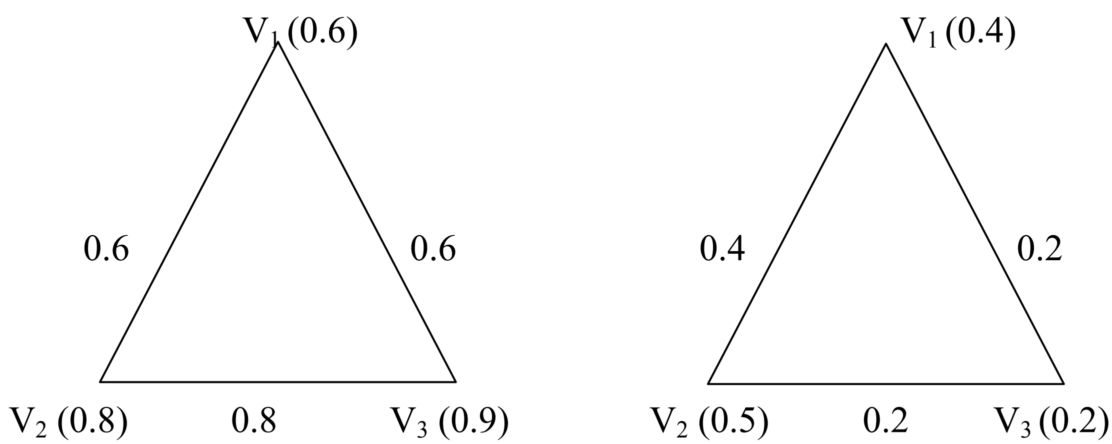
Proceeding like this we can find $K_8$ is decomposable into 3 Hamiltonian fuzzy cycles, $K_{10}$ is decomposable into 4 Hamiltonian fuzzy cycles and so on.

## 5. Some Results Using Fuzzy Matrices

### 5.1 Fuzzy matrix Addition of complete fuzzy graphs

If we add the matrices of edge membership values of any two complete fuzzy graphs then the solutions are same as the given complete fuzzy graph.

Consider the two complete fuzzy graphs of 3 vertices.




Using the above diagram we can form the following matrices.

$$\begin{pmatrix} 0 & 0.8 & 0.6 \\ 0.8 & 0 & 0.6 \\ 0.6 & 0.6 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 0.2 & 0.4 \\ 0.2 & 0 & 0.2 \\ 0.4 & 0.2 & 0 \end{pmatrix}$$

Now we use the fuzzy matrix addition, we get the solutions are same as the above matrices. (ie) We get the given complete fuzzy graphs are our solution.

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Similarly we can add the complete fuzzy graphs $K_4$, $K_5$, …… it yields the same result.

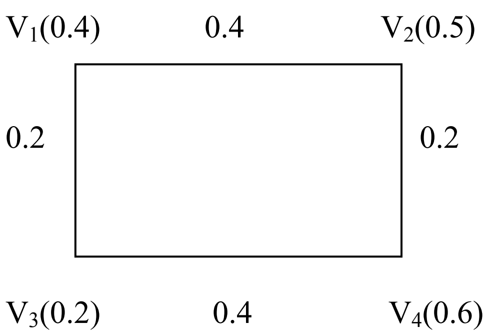
Therefore fuzzy matrix addition of any two complete fuzzy graphs is same as the given complete fuzzy graphs.

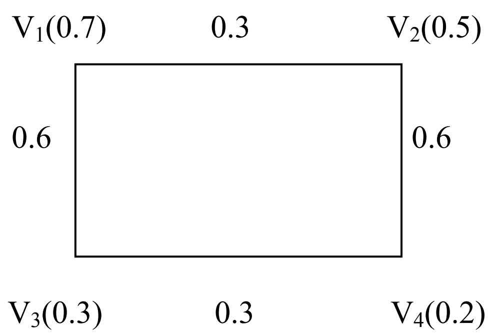
## 5.2 Fuzzy Matrix Addition of regular fuzzy graphs of odd and even Cycles

If we add the matrices of edge membership values of any two regular fuzzy graphs with odd cycles then the solutions are same as the given regular fuzzy graph.

If we add the matrices of edge membership values of any two regular fuzzy graphs with even cycles then the solutions are the regular fuzzy graphs of one pair of edge membership values interchanged between the given fuzzy regular graphs.

Consider the two regular fuzzy graphs of 4 vertices.

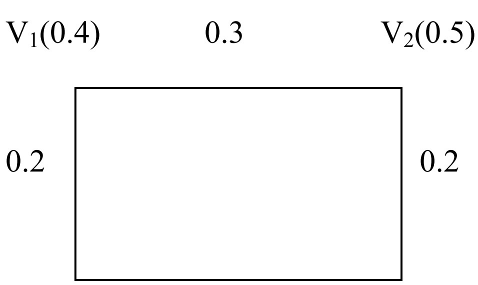


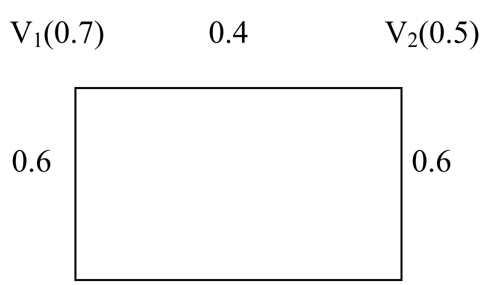




Using the above diagram we can form the following matrices.

$$\begin{pmatrix} 0 & 0.4 & 0 & 0.2 \\ 0.4 & 0 & 0.2 & 0 \\ 0 & 0.2 & 0 & 0.4 \\ 0.2 & 0 & 0.4 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 0.3 & 0 & 0.6 \\ 0.3 & 0 & 0.6 & 0 \\ 0 & 0.6 & 0 & 0.3 \\ 0.6 & 0 & 0.3 & 0 \end{pmatrix}$$

Now we use the fuzzy matrix addition, we get the solution regular fuzzy graphs are



$V_3(0.2)$ 0.3 $V_4(0.6)$ $V_3(0.3)$ 0.4 $V_4(0.2)$

Therefore the solutions of fuzzy matrix addition of two regular fuzzy graphs are the regular fuzzy graphs of one pair of edge membership values interchanged between the given fuzzy regular graphs.

Similarly we can add the regular fuzzy graphs with even cycles of 6, 8, …. vertices also yields the same result.

## 6. Conclusion

The complete fuzzy graph of even number of vertices can be decomposed into the integer value of (2n-1)/2 Hamiltonian fuzzy cycles and the rest of the edges is n which forms the 1-factorisation. The remaining edges cannot form the Hamiltonian fuzzy cycle. Some results of complete fuzzy graphs and regular fuzzy graphs using fuzzy matrix addition can be found in this paper.

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