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Generalized Special Relativistic Quantum Equation &String Mass Quantization

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Abstract
A new quantum mechanical equation based on generalized special relativity energy expression is derived. This new equation reduces to both Klein – Gordon equation and Schrodinger equation. If one considers elementary particles as vibrating harmonic oscillator strings, the solution shows energy and mass quantization. It is very interesting to note that one can adjust the frequency of strings and the quantum number to make the masses of elementary particles fit the predicted values. .

Keywords: *generalized special relativity, string, mass quantization, Klein – Gordon equation*

1. Introduction
The real history of quantum mechanics starts with the Bohr atomic theory and quantum Plank protests. Plank assumed that light which was thought at that time to be a wave, sometimes behaves as stream of particles [1, 2]. Later on De Broglie proposed that particles can also behave as wave. This proposal was verified experimentally [3]. This dual nature of atomic and subatomic particles was for mutated mathematically by Schrodinger and Heisenberg in to two famous quantum equations [4, 5]. One of them is knows as Schrodinger equation, in which space – time evolution of the quantum system is described by the wave function. In contrary, the space – time evolution of the system in Heisenberg equation is described by operators [6, 7]. The two representations succeeded in describing a wide variety of atomic phenomena including atomic spectra and interaction of radiation with matter [8, 9]. However, the Schrodinger equation is not suitable for very fast relativistic particles. This led Klein, Gordon and Dirac to for new relativistic equations [10]. This equation succeeded to describe the behavior of the fast particles but unfortunately they suffer from noticeable setbacks. For example, the quantum equations does not account for the rest mass term and potential energy at the same time. Thus it is not sensitive to string harmonic vibration in string theories. Thus there is a need to account for the two energy types. This can be

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done within the frame work of generalized special relativity (GSR). The success of GSR in describing many phenomena beside its ability to be reduced to SR motivates to construct new quantum equation based on GSR. This is done in section 2. Section 3 is concerned with reducibility of the new quantum equation to Schrodinger equation; section 4 is concerned with time dependent potential form of the new equation. The solution for oscillator is made in section 5. Section 6 and 7 are concerned with discussion and conclusion.

2. Generalized Special Relativistic Quantum Equation

According to GSRE the energy given by [10]

$$g_{oo}\,E^2 = C^2P^2 + g_{oo}m_o^2\,C^4 \qquad (2.1)$$

The feeling of this equation by the wave function Ψ can be mad multiplying both sides (2.1) by Ψ to get

$$g_{oo}\,E^2 = C^2P^2\,\Psi + g_{oo}m_o^2\,C^4\Psi \qquad (2.2)$$

The GSRE for quantum system can be obtained by taking into account the dual nature of atomic particles which are assumed to be in the form of a wave bracket

$$\Psi = Ae^{\frac{i}{\hbar}}\,(P_x - E_t) \qquad (2.3)$$

Differentiating both sides with respect to time and space twice yields.

$$\frac{\partial\Psi}{\partial t} = \frac{i}{\hbar}E\Psi$$

$$\frac{\partial\psi}{\partial t} = \frac{i}{h}\,E\psi$$

$$\frac{\partial^2\psi}{\partial t^2} = i^2\,\frac{E^2}{\hbar}\psi = -\frac{E}{\hbar^2}\psi$$

$$-\hbar^2\,\frac{\partial^2\psi}{\partial t^2} = E^2\psi \qquad (2.4)$$

$$\frac{\partial\psi}{\partial x} = \frac{i}{h}\,\rho\psi$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{i}{\hbar^2}\,\rho^2\psi = -\,\frac{\rho^2}{\hbar^2}\psi$$

$$-\hbar^2\,\frac{\partial^2\psi}{\partial x^2} = -\rho^2\psi$$

In 3 dimension

$$-\hbar^2\nabla^2 = \rho^2\psi \qquad (2.5)$$

Substituting (2.4) and (2.5) in (2.2)

$$-\hbar^2 g_{oo}\frac{\partial^2\psi}{\partial t^2} = C^2\hbar^2\nabla^2\psi + g_{oo}m_0^2C^4\gamma \qquad (2.6)$$

Where

$$g_{oo} = 1 + \frac{2\emptyset}{C^2} = 1 + \frac{2m_0\emptyset}{m_0C^2} = 1 + \frac{2V}{m_0C^2} \quad (2.7)$$

Where the relativistic mass assumed to be equal to the rest mass. This approximating is true for slow particles in a week field, i.e., where

$$\frac{\emptyset}{C^2} < 1 \qquad \frac{v^2}{C^2} < 1 \qquad\qquad m \approx m_0 \qquad\qquad (2.8)$$

Thus the quantum generalized special relativistic equation becomes

$$-\hbar^2\left(1 + \frac{2V}{m_0C^2}\right)\frac{\partial^2\psi}{\partial t^2} = C^2\hbar^2\nabla^2\psi + \left(1 + \frac{2V}{m_0C^2}\right)m_0^2C^4\psi \;(2.9)$$

Clearly this equation reduced to Klein Gordon equation when $V = 0$, where

$$-\,\hbar\,\frac{\partial^2\psi}{\partial t^2} = -C^2\hbar^2\nabla^2\psi + m_0^2C^4\psi \qquad (2.10)$$

3. Time Independent Quantum Equation for time Independent Potential

Most of electrons atoms are affected by nuclear time independent potential resulting from the electrostatic the form.

$$V = v(r, q, \phi) = v)(x, y, z) \qquad (3.1)$$

According to equation (2.7)

$$g_{oo}1+\frac{2v}{m_0C^2}=g_{oo}(x,y,z) \qquad (3.2)$$

Dividing both sides of (2.6) yields

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2}=-\frac{C^2\hbar^2}{g_{oo}}\nabla^2\psi+m_0^2C^4\psi \qquad (3.3)$$

$$\psi=e^{iwt}\,u=e^{iwt}\,u\,(x,y,z) \qquad (3.4)$$

Where $E=\hbar\omega$ (3.5)

$$\frac{\partial\psi}{\partial t}=i\omega\psi$$

$$\frac{\partial^2\psi}{\partial t^2}=i^2\omega^2\psi=-\omega^2\psi$$

$$\hbar^2\omega^2ue^{iwt}=\left[-\frac{C^2\hbar^2}{g_{oo}}\nabla^2u+m_0^2C^4u\right]e^{iwt}$$

$$E^2u^2=-C^2\hbar^2\nabla^2=g_{oo}\,E^2u-m_0^2C^4u \qquad (3.6)$$

Equation is the time independent quantum GER equation. It is very interesting to note this equation reduced to the time independent Klein – Gordon equation when

$$V=0 \qquad g_{oo}=1 \qquad (3.7)$$

Where

$$-C^2\hbar^2\nabla^2u=E^2u^2m_0^2C^4u \qquad (3.8)$$

Consider particles with energy E

$$W=mc^2$$

$$g_{oo}=1+\frac{2\emptyset}{c^2}=1+\frac{2m\emptyset}{mc^2}=1+\frac{2v}{E} \qquad (3.9)$$

Divide both sides of (3.6) by $2mc^2$ to get

$$-\frac{\hbar^2}{2m}\nabla^2 u = \left(\frac{1}{2mc^2}+\frac{2v}{2mc^2}\right)\,E^2-\frac{m_0^2C^4u}{2mc^2}$$

$$-\frac{\hbar^2}{2m}\nabla^2 u = \left(\frac{1}{2E}+\frac{2V}{E^2}\right)\,Eu^2-\frac{1}{2}\,\frac{m_0^2cu}{mc^2}$$

$$\frac{\hbar^2}{2m}\nabla^2 u = vu\frac{1}{2}E-\frac{1}{2}\frac{m_0^2c}{E} \qquad (3.10)$$

The Newtonian energy is given by

$$E_N = \frac{1}{2}\,mc^2 = \frac{1}{2}E \qquad (3.11)$$

Neglecting mass term for very small particles where

$$M_0c^2 \ll 1 \qquad (3.12)$$

And substituting (3.11) and (3.12) and replacing v by –v yields

$$-\frac{\hbar^2}{2m}\nabla^2\mu + v\mu = E_Nu \quad (3.13)$$

This is the ordinary time independent Schrodinger equation.

4. Time Independent Quantum Equation for time Dependent Potential

Consider now time dependent and spatial in dependent potential of the form

$$v = v(t) \qquad (4.1)$$

In this ease, equation (2.7) becomes

$$g_{oo} = 1+\frac{2v(t)}{m_0c^2} = g_{oo}(t) \qquad (4.2)$$

Thus equation (2.6) becomes

$$\hbar^2c^2\nabla^2\psi = g_{oo}\hbar^2\frac{\partial^2\psi}{\partial t^2} + g_{oo}m_0c^4\psi \qquad (4.3)$$

Consider the solution of the form

$$\psi = e^{ikr} = f(t) \qquad (4.4)$$

Thus

$$\nabla^2\psi = -k^2\psi = -K^2\, e^{i\,k-r} f \qquad (4.5)$$

Inserting (4.5) and (4.4) in (4.3) yields

$$-c^2k^2fe = g_{oo}\hbar^2 e^{0\,k-v}\,\partial f^2 + g_{oo}m_0^2\,c^4 e^{i\,k-v} f$$

$$-c^2\hbar^2k^2fe == g_{oo}\hbar^2\frac{\partial f^2}{\partial\tau^2} + g_{oo}\,m_0c^4 f \qquad (4.6)$$

But

$$\hbar k = P \qquad (4.7)$$

Thus

$$g_{oo}\hbar^2\frac{\partial f^2}{\partial\tau^2} + g_{oo}m_0^2\,c^4\,f = c^2P^2f \qquad (4.8)$$

With the aid of equation (4.2), (4.8) becomes

$$\left(1+\frac{2v[t]}{m_0c^4}\right)\left[\hbar^2\frac{\partial^2 f}{\partial t^2} + m_0c^4\right] = -c^2p^2\,f \qquad (4.9)$$

$$\hbar^2 g_{oo}\frac{\partial^2 f}{\partial t^2} - g_{oo}{}^2m_0^2\,c^4\,f = c_0 f \qquad (4.10)$$

5. Harmonic Oscillator Wave Function and Energy

For particle like pendulum the displacement should be small as to execute simple harmonic motion i.e.

$$x \ll 1 \qquad (5.1)$$

Thus one expects the potential v to be a small, i.e.

$$V = \frac{1}{2}\quad kx^2 \ll 1 \qquad (5.2)$$

Since the displacement is in the form

$$x = x_0\sin wt \qquad (5.3)$$

From equation (5.6) by dividing both sides by

$$\hbar^2 g_{oo}\frac{\partial^2 f}{\partial t^2} - g_{oo}{}^2 m_0^2\, c^4\, f = -c_0 g_{oo}{}^{-1} f \quad (5.4)$$

Since v small. Thus

$$g_{oo}{}^{-1} = \left(1+\frac{2\phi}{c^2}\right) = \left(1+\frac{2m_0\phi}{m_0c^2}\right)^{-1} = \left(1+\frac{2V}{m_0c^2}\right)^{-1}$$

$$g_{oo}{}^{-1} = (1+c_1V^{-1}) = 1-c_1V \qquad (5.5)$$

Where $c_1 = \frac{2}{m_0c^2}$

A direct substitution of (5.4) in (5.5) yields

$$\hbar^2 f^{11} + (1+c_1V)m_0c^2 f = -c_0(1-c_1V)f \qquad (5.6)$$

Let the solution be

$$f = A\sin at \qquad f^{11} = a^2 f \quad (5.7)$$

$$-a^2\hbar^2 f + m_0^2c^4\, f + c_1m_0^2c^4Vf = -c_0 f + c_0c_1Vf \qquad (5.8)$$

Equating the coefficients of f and vf on both sides gives

$$-a^2\hbar^2 + m_0^2c^4 = -c_0$$

$$c_1m_0^2c^4 = c_0c_1$$

$$c_0 = m_0^2c^4$$

$$a^2\hbar^2 = m_0^2c^4 + c_0 = 2\, m_0^2c^4$$

$$a^2 = \frac{2\, m_0^2c^4}{\hbar^2} \qquad (5.9)$$

$$f = A\sin at \qquad (5.10)$$

The periodicity condition requires

$$f(t) = f(t+T)$$

$$A\sin at = A\sin(a\tau + aT) \qquad (5.11)$$

This requires

$$\cos aT = 1 \qquad sing\ aT - 0$$

Hence

$$aT = 2\pi n$$

$$a = \frac{2n\pi}{T} = n\,(\pi f) = nw \qquad (5.12)$$

$$m_0 c^2 = \frac{1}{\sqrt{2}}\,\hbar\alpha = \frac{1}{\sqrt{2}} n\hbar\omega \qquad (5.13)$$

In view of equations (5.4), (5.5)

$$c_0 = E^2 = m_0^c\, c^4$$

$$E = \pm m_0 c^2 = \pm\frac{1}{\sqrt{2}} n\hbar\omega \qquad (5.14)$$

$$E = m_0 c^2 = \frac{1}{\sqrt{2}} n\hbar\omega$$

But the approximation used in equation

$$V = m\emptyset \quad \approx m_0\emptyset$$

$$E = mc^2 \approx m_0 c^2 = \frac{1}{\sqrt{2}}\, n\hbar\omega$$

$$m_0 = \frac{n\hbar\omega}{c^2\sqrt{2}} \qquad (5.15)$$

6. Discussion

The new relativistic quantum equation (2.9) is obtained by using the GSR energy expression (2.1), beside the particle wave equation (2.3). In view of equation (2.9), it is clear that it's more general than Schrodinger, Klein, Gordon and Dirac equation. This is since it incorporates the rest mass energy and potential energy at the same time. Schrodinger equation is not sensitive to rest mass energy. this means that particles having different rest mass energies have the same behavicur, which is not physically acceptable, Klein - Gordon and Dirac equation are not sensitive to field potential, except electromagnetic potential. Thus for particles of certain rest mass energy, some of them in free space and some of them in a certain medium of specific potential, relativistic equations. Cannot differentiate between them but this new equation can do this. It is also interesting to note thank equation (5.13) predict that the masses of elementary particles are quantized if one treats particles as vibrating strings. By adjusting frequency ω and quantum number n one can determine the max of any elementary particle

7. Conclusion

The new quantum equation based on GSR can describe particles having rest mass energy and moving in afield. It can also predict masses of any elementary particle which is quantized.

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