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Grey Relational Analysis Method for Multiple Attribute Decision Making in Rough Intuitionistic Fuzzy Setting

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Abstract:

The focus of this paper is to investigate the multiple attribute decision making problems using rough intuitionistic fuzzy information. Based on evidence theory and grey relational analysis (GRA) technique, we design a new method for decision making problems. We verify the developed approach and to demonstrate its practicality and its effectiveness by an example.

1. Introduction

Business sector is one of the most challenging and emerging field of today's scenario. This sector flourishes well depending on the effectiveness of the decision makers need on effective tool to decide between the given set of alternatives. In this context the MADM has come as an aid to the decision makers which has drawn a great deal of attention from researchers of various disciplines.

Grey relational analysis (GRA) method was originally developed by Deng [6] and has been widely used to solve the uncertainty problems under the discrete data and incomplete information [5-7, 15, 29, 33]. In addition, GRA method is one of the very popular method to analyze various relationships among the discrete data sets and make decisions in multiple attribute situations. The major advantages of the GRA method are that the results are based on the original data, the calculations are simple and straightforward, it is one of the best methods to make decisions under business environment.

Atanassov[1] introduced the concept of intuitionistic fuzzy set which is a generalization concept of fuzzy set. Rough sets [17-18]was originally proposed by Pawlak[17] and is an extension of crisp set theory for the study of intelligent system characterized by inexact, uncertain or insufficient information. It is a useful tool for dealing with uncertainty or imprecision information. Rough sets have successfully found its application indifferent fields such as artificial intelligence[10], pattern recognition[31], medical diagnosis[25-27], data mining[13-14, 23], conflict analysis[19], decision support system[21-22], intelligent control [20]etc. These are combined with various other sets such as fuzzy sets[32], vague sets[11], intuitionistic fuzzy set[1] neutrosophic are developed as rough fuzzy sets, fuzzy rough set[8], rough intuitionistic fuzzy set, intuitionistic fuzzy rough set, rough intuitionistic fuzzy set[24], rough neutrosophic set etc to explore many concepts.

As decision makers tends to express their preferences in rough intuitionistic fuzzy form, in this paper we extend the concept of GRA to develop a methodology for solving MADM problems with rough intuitionistic fuzzy information.

2. Preliminaries

Definition 2.1[1]: Let X be a nonempty fixed set. An *intuitionistic fuzzy set* (IFS, for short) A is an object having the form A= $\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the function $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element x$\in$X to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each x $\in$ X.

Obviously, every fuzzy set A on a nonempty set X is an IFS having the form A= $\{\langle x, \mu_A(x), 1-\mu_A(x)\rangle : x \in X\}$.

Definition 2.2[12]: For each intuitionistic fuzzy set A in X, if $\pi_A(X) = 1-\mu_A(x)-\nu_A(x),\ \forall x \in X$. Then $\pi_A(X)$ is called the degree of indeterminacy of x to A.

Definition 2.3[12]: Let $\tilde{a}_1 = (\mu_1, \nu_1)$ and $\tilde{a}_2 = (\mu_2, \nu_2)$ be two intuitionistic fuzzy numbers, then the Hamming distance between $\tilde{a}_1 = (\mu_1, \nu_1)$ and $\tilde{a}_2 = (\mu_2, \nu_2)$ is defined as follows:

$$p(\tilde{a}_1, \tilde{a}_2) = \frac{1}{2}(|\mu_1 - \mu_2| + |\nu_1 - \nu_2|).$$

Definition 2.4[24]: Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ be an intuitionistic fuzzy set of X. Then the rough intuitionistic fuzzy set of A (RIFS) denoted as $Apr = (\underline{Apr}, \overline{Apr})$ and defined as $\underline{Apr} = \{\langle x, \underline{\mu}_A(x), \underline{\nu}_A(x)\rangle\}$ where $\underline{\mu}_A(x) = \wedge_{x' \in [x]_\theta} \mu_A(x')$ and $\underline{\nu}_A(x) = \vee_{x' \in [x]_\theta} \nu_A(x')$

$\overline{Apr} = \{\langle x, \overline{\mu}_A(x), \overline{\nu}_A(x)\rangle\}$,where $\overline{\mu}_A(x) = \vee_{x' \in [x]_\theta} \mu_A(x')$ and $\overline{\nu}_A(x) = \wedge_{x' \in [x]_\theta} \nu_A(x')$, where θ is an equivalence relation on X and $[x]_\theta$ is an equivalence class containing x.

3. Rough intuitionistic fuzzy multi-attribute decision making based on grey relational analysis

Grey relational analysis method was originally developed by Deng (1989) and has been successfully applied in solving a variety of MADM problems. The main procedure of GRA is firstly to translate the performance of all alternatives into a comparability sequence. This step is called grey relational generating. The gray relational coefficient between all comparability sequences and ideal sequence is calculated. If comparability sequence translated from an alternative has the highest gray relational degree between the ideal target sequence and itself, then that alternative will be the best choice. We apply GRA method to solve rough intuitionistic fuzzy MADM with known weight information. The method involves the following steps.

A multi-attribute decision making problem with m alternatives and n attributes is considered. Let $A_1, A_2, ...A_m$ and $C_1, C_2, ...C_n$ represent the discrete set of alternatives and attributes respectively.

The ranking reflects the performance of the alternative A_1 against the attribute C_j. For MADM weight vector $W = \{w_1, w_2, ...w_n\}^T$ is assigned to the attributes. The weight w_j (j=1, 2……..n) reflects the relative importance of the attribute C_j (j=1,2…..m) to the decision making process where $w_j \in [0,1], \sum_{j=1}^{n} w_j = 1$. The weights of the attributes are usually determined on subject basis.

Grey system theory deals with primarily on multi-input, incomplete, or uncertain information. GRA is suitable for solving problems with complicated relationships between multiple factors and variables. The values associated with the alternatives for MADM problems are presented in the Table1

	C_1	C_2	………	C_n
A_1	$\langle \underline{d}_{11}, \overline{d}_{11} \rangle$	$\langle \underline{d}_{12}, \overline{d}_{12} \rangle$	………	$\langle \underline{d}_{1n}, \overline{d}_{1n} \rangle$
A_2	$\langle \underline{d}_{21}, \overline{d}_{21} \rangle$	$\langle \underline{d}_{22}, \overline{d}_{22} \rangle$	………	$\langle \underline{d}_{2n}, \overline{d}_{2n} \rangle$
. . .	……… ……… ………	……… ……… ………	……… ……… ………	……… ……… ………
A_m	$\langle \underline{d}_{m1}, \overline{d}_{m1} \rangle$	$\langle \underline{d}_{m2}, \overline{d}_{m2} \rangle$	………	$\langle \underline{d}_{mn}, \overline{d}_{mn} \rangle$

Where $\langle \underline{d}_{ij}, \overline{d}_{ij} \rangle$ is rough intuitionistic number according to the *i*-th alternative and the *j*-th attribute.

Step 1: Determine the most important criteria.
The most important thing is to select a proper criteria for decision making which is based on experts opinion.

Step 2: Consider a MADM problem with m alternatives and n attributes. Here we use the traditional GRA method to deal with this problem

Step 3: Construct the decision matrix.
Here the rating of an alternative with respect to an attribute is a rough intuitionistic fuzzy set represented in the following form

$$A_i = [\frac{C_1}{(\langle \underline{\mu}_{i1}, \underline{\nu}_{i1} \rangle, \langle \overline{\mu}_{i1}, \overline{\nu}_{i1} \rangle)}, \frac{C_2}{(\langle \underline{\mu}_{i2}, \underline{\nu}_{i2} \rangle, \langle \overline{\mu}_{i2}, \overline{\nu}_{i2} \rangle)}, ..., \frac{C_n}{(\langle \underline{\mu}_{in}, \underline{\nu}_{in} \rangle, \langle \overline{\mu}_{in}, \overline{\nu}_{in} \rangle)}] = \frac{C_j}{\langle \underline{\mu}_{ij}, \underline{\nu}_{ij} \rangle, \langle \overline{\mu}_{ij}, \overline{\nu}_{ij} \rangle},$$

j=1,2..n…………………(1)

Here $\langle \underline{\mu}_{ij}, \underline{\nu}_{ij} \rangle$ and $\langle \overline{\mu}_{ij}, \overline{\nu}_{ij} \rangle$ are the degree of membership and non-membership of an attribute A_i satisfying the attribute C_j respectively.

Rough intuitionistic fuzzy decision matrix

Table 2

$d = (\langle \underline{\mu}_{ij}, \underline{\nu}_{ij} \rangle, \langle \overline{\mu}_{ij}, \overline{\nu}_{ij} \rangle)_{m \times n}$ =

	C_1	C_2	………	C_n
A_1	$(\langle \underline{\mu}_{11}, \underline{\nu}_{11} \rangle, \langle \overline{\mu}_{11}, \overline{\nu}_{11} \rangle)$	$(\langle \underline{\mu}_{12}, \underline{\nu}_{12} \rangle, \langle \overline{\mu}_{12}, \overline{\nu}_{12} \rangle)$	………	$(\langle \underline{\mu}_{1n}, \underline{\nu}_{1n} \rangle, \langle \overline{\mu}_{1n}, \overline{\nu}_{1n} \rangle)$
A_2	$(\langle \underline{\mu}_{21}, \underline{\nu}_{21} \rangle, \langle \overline{\mu}_{21}, \overline{\nu}_{21} \rangle)$	$(\langle \underline{\mu}_{22}, \underline{\nu}_{22} \rangle, \langle \overline{\mu}_{22}, \overline{\nu}_{22} \rangle)$	………	$(\langle \underline{\mu}_{2n}, \underline{\nu}_{2n} \rangle, \langle \overline{\mu}_{2n}, \overline{\nu}_{2n} \rangle)$
. . .	……… ……… ………	……… ……… ………	……… ……… ………	……… ……… ………
A_n	$(\langle \underline{\mu}_{n1}, \underline{\nu}_{n1} \rangle, \langle \overline{\mu}_{n1}, \overline{\nu}_{n1} \rangle)$	$(\langle \underline{\mu}_{n2}, \underline{\nu}_{n2} \rangle, \langle \overline{\mu}_{n2}, \overline{\nu}_{n2} \rangle)$	………	$(\langle \underline{\mu}_{mn}, \underline{\nu}_{mn} \rangle, \langle \overline{\mu}_{mn}, \overline{\nu}_{mn} \rangle)$

Where $(\langle \underline{\mu}_{ij}, \underline{\nu}_{ij} \rangle, \langle \overline{\mu}_{ij}, \overline{\nu}_{ij} \rangle)$ are the lower and upper approximation of the rough intuitionistic fuzzy set.

Step 4: Determine the accumulation geometric operator.
Transform the rough intuitionistic fuzzy set to intuitionistic fuzzy set by the following operator .
The accumulated geometric operator is defined in the following way

$$\langle \mu_{ij}, \nu_{ij} \rangle = \langle (\underline{\mu}_{ij}, \overline{\mu}_{ij})^{1/2}, (\underline{\nu}_{ij}, \overline{\nu}_{ij})^{1/2} \rangle . \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The decision matrix is transformed in the form of intuitionistic fuzzy set as follows
Table 3

	C_1	C_2	………	C_n
A_1	$\langle \mu_{11}, \nu_{11} \rangle$	$\langle \mu_{12}, \nu_{12} \rangle$	………	$\langle \mu_{1n}, \nu_{1n} \rangle$
A_2	$\langle \mu_{12}, \nu_{21} \rangle$	$\langle \mu_{22}, \nu_{22} \rangle$	………	$\langle \mu_{2n}, \nu_{2n} \rangle$
. . .	……… ……… ………	……… ……… ………	……… ……… ………	……… ……… ………
A_n	$\langle \mu_{mn}, \nu_{mn} \rangle$	$\langle \mu_{mn}, \nu_{mn} \rangle$	………	$\langle \mu_{mn}, \nu_{mn} \rangle$

Let us denote the decision matrix by $D = (d_{ij})_{m\times n} = \langle \mu_{ij}, \nu_{ij} \rangle_{m\times n}$ .And let $w = (w_1, w_2, \ldots . w_n)^T$ be the weighting vector.

Step 5: Determine the positive ideal solution based on intuitionistic fuzzy numbers

$$d^+ = (\langle \mu_1^+, \nu_1^+ \rangle, \langle \mu_2^+, \nu_2^+ \rangle, \ldots\ldots\langle \mu_n^+, \nu_n^+ \rangle) \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Where

$$d_j^+ = \langle \mu_j^+, \nu_j^+ \rangle = \langle \max \mu_{ij}, \min \nu_{ij} \rangle \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Step 6: Calculate the grey relational coefficient of each alterative by using the following equation

$$\Lambda_{ij}^+ = \frac{\min\limits_{1\le i\le m} \min\limits_{1\le j\le m} p(d_{ij}, d_j^+) + \rho \max\limits_{1\le i\le m} \max\limits_{1\le j\le m} p(d_{ij}, d_j^+)}{p(d_{ij}, d_j^+) + \rho \max\limits_{1\le i\le m} \max\limits_{1\le j\le m} p(d_{ij}, d_j^+)} \qquad i = 1,2\ldots\ldots m,\ j = 1,2\ldots\ldots. n \quad \ldots\ldots\ldots.(5)$$

Where the identification coefficient $\rho = 0.5$.

Step 7: Calculate the degree of grey relational coefficient of each alternative using the following equation

$$\Lambda_i^+ = \sum_{j=1}^{n} w_j \Lambda_{ij}^+ \quad \ldots\ldots\ldots\ldots\ldots(6)$$

Step 8: Rank all the alternatives A_i (i=1, 2,……m). The ranking order of all the alternatives can be determined according to Λ_i^+ (i=1, 2,……m). The ranking order of all the alternatives can be determined according to decreasing order of the rough relational degree. The alternative with the highest value is the best alternative.

4. Application of the method

We consider a rough intuitionistic fuzzy MADM to demonstrate the application and the effectiveness of the proposed approach.
Suppose a person wants to buy a car out of the three brands 1) Maruthi Suzuki A_1 2)Honda A_2 3) Ford A_3, he should decide to choose a car based on the four criteria 1) safety C_1 2) Cost C_2 3) mileage C_3 4) comfort C_4.We obtain the rough intuitionistic fuzzy decision matrix based on the experts assessment:

Table 4

Decision matrix with rough intuitionistic fuzzy number

	C_1	C_2	C_3	C_4
A_1	$(\langle .6,.4\rangle,\langle .7,.3\rangle)$	$(\langle .7,.2\rangle,\langle .8,.2\rangle)$	$(\langle .7,.3\rangle,\langle .8,.2\rangle)$	$(\langle .6,.3\rangle,\langle .7,.2\rangle)$
A_2	$(\langle .8,.1\rangle,\langle .9,.1\rangle)$	$(\langle .5,.4\rangle,\langle .5,.3\rangle)$	$(\langle .6,.3\rangle,\langle .8,.1\rangle)$	$(\langle .5,.4\rangle,\langle .6,.3\rangle)$
A_3	$(\langle .6,.2\rangle,\langle .8,.1\rangle)$	$(\langle .6,.4\rangle,\langle .7,.2\rangle)$	$(\langle .7,.3\rangle,\langle .7,.2\rangle)$	$(\langle .8,.2\rangle,\langle .9,.1\rangle)$

Determination of the decision matrix in form of intuitionistic fuzzy set: By using the (AGO) accumulation geometric operator from equation (2) we have the decision matrix in intuitionistic fuzzy set form is presented as

Table 5

	C_1	C_2	C_3	C_4
A_1	$(0.6481, 0.3464)$	$(0.7483, 0.2)$	$(0.7483, 0.2449)$	$(0.6481, 0.2449)$
A_2	$(0.8485, 0.1)$	$(0.5, 0.3464)$	$(0.6928, 0.1732)$	$(0.5477, 0.3464)$
A_3	$(0.6928, 0.1414)$	$(0.6481, 0.2828)$	$(0.7, 0.2449)$	$(0.8485, 0.1414)$

The information of the attribute weights is known as follows:

$w = (0.40, 0.20, 0.25, 0.15)^T$

We determine the positive ideal solution by using equation (3)

$d^+ = (\langle 0.8485, 0.1\rangle, \langle 0.7483, 0.2\rangle, \langle 0.7483, 0.1732\rangle, \langle 0.8485, 0.1414\rangle)$

Calculate the grey relational coefficient of each alternative from PIS

	C_1	C_2	C_3	C_4

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Table 6

A_1	0.361441	1	0.7791	0.4542
A_2	1	0.3905	0.82004	0.3333
A_3	0.562	0.58017	0.6782	1

$\Lambda^+ = (\Lambda_{ij}{}^+)_{4\times 5} =$

Thus the rough intuitionistic fuzzy relative degree of each alternative can be obtained with the help of equation (6) as

$\Lambda_1{}^+ = 0.6075 \quad \Lambda_2{}^+ = 0.73312 \quad \Lambda_3{}^+ = 0.6604.$

According to the relative relational degree, the ranking order according to the value of rough intuitionistic fuzzy relational degree is $\Lambda_2{}^+ > \Lambda_3{}^+ > \Lambda_1$. It is seen that the highest value of rough intuitionistic relational degree is $\Lambda_2{}^+$ therefore A_2 is the most desirable alternative.

Conclusion

In this paper we have introduced rough intuitionistic fuzzy multi attribute decision making problem based on traditional GRA method. We have used accumulation geometric operator to transform rough intuitionistic fuzzy set to intuitionistic fuzzy set. Rough intuitionistic fuzzy grey relation coefficient was proposed for solving multi attribute decision-making problem. Finally the applicability of the proposed approach is useful with a suitable illustration. In future we shall continue working in the application of rough intuitionistic fuzzy multi attribute decision-making to other domains such as decision making, pattern recognition, medical diagnosis and clustering analysis.

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