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Quantum Schrödinger String Theory for Frictional Medium & Collision

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Abstract
Maxwell's equation for decaying wave due to friction, beside a classical and quantum expression for oscillating string energy are used to derive a useful expression for particle energy in frictional resistive media. This expression is used to derive Schrödinger Equation for oscillating string in a resistive media. This new equation reduces to the ordinary. Schrödinger equation in the absence of friction it also gives collision probability similar to that obtained by transport equation. This new equation is used to derive an expression for energy lost by friction by the vibrating string. This energy is shown to be quantized.

1. Introduction:

Atoms are building blocks of matter. Any material consists of a large number of a toms. Each atom consists of electrons, protons and neutrons .Atom and subatomic particles are described by the laws of quantum mechanics. For slow moving particles one utilizes Schrödinger equation .Fast particles can be described by relativistic are quantum mechanics[1] spin less particles are described by Klein-Gordon equation , while those having spin are described by Dirac equation [2,3,4].

The laws of quantum mechanics were found to be successful in describing single particles and atoms. However, when particles and atoms accumulate them self to from a bulk matter, quantum laws suffers from some long standing problems [5,6].One of them is known as many body problem[7].The behavior of superconductors(Sc) at high temperature, till know, have no well defined simple full quantum theory to describe them [8,9]. Theory that can describe the early universe and unify gravity with other forces [10, 11].This may be attributed to the fact that, when particles ender a medium, quantum equations accounts for the effect of potential only, does not account for the effect of friction anther particles. Some attempts were made by the first authore do consider the friction effect. This work is concerned with alternative derivations based on Maxwell's and classical oscillator equations. The derivations were done in sections 2,3 and 4.The new equation is applied to oscillators in sec 5. Discussion and conclusion. The new equation is applied to oscillators in sections 6 and 7.

2. Maxwell's Equations for time Decaying Wave in Resistive Medium

Consider an electromagnetic wave enters a medium of conductivity σ , and electric polarization P. Maxwell's equations for this medium are given by:

$$-\nabla^2 E - \mu\sigma \,\partial E/\partial t + \mu\varepsilon\, \partial^2 E/\partial t^2 = -\mu \frac{\partial^2 P}{\partial t^2} \quad (2.1)$$

The electric field intensity decays in this case and can be described by the relation:

$$E = E_o\, e^{-\alpha t}\, e^{i(k\ x-\omega t)} \quad (2.2)$$

The corresponding displacement is given by:

$$x = x_o\, e^{-\alpha t}\, e^{i(k\ x-\omega t)} = \frac{x_o}{E_o} E \quad (2.3)$$

The electric polarization terms is defined to be:

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$$P = e\,n\,x \;= e\,n\,\frac{x_o}{E_o}\,E \qquad (2.4)$$

With the aid of equations (2.2) and (2.4) of equations (2.1) becomes:

$$K^2 + \frac{\alpha^2}{c^2} - \frac{\omega^2}{c^2} - \mu\,\sigma\,\alpha + \frac{2i\,\alpha\,\omega}{c^2} - \mu\,\sigma\,\omega i = \frac{e\,n\,\omega^2 \mu_o x_o}{E_o} \quad (2.5)$$

Equation (2.5) can be simplified by using the relation:

$$K = \frac{2\,\pi}{\lambda} = \frac{2\,\pi f}{\lambda f} = \frac{\omega}{c} \qquad (2.6)$$

and by assuming:

$$\alpha << c \qquad (2.7)$$

Where c is the speed of light in vacuum which is large, Thus equation (2.5) becomes:

$$-\mu\,\sigma\,\alpha + \frac{2i\,\alpha\,\omega}{c^2} - \mu\,\sigma\,\omega i = + e\,n\,\omega^2 \mu\,\frac{x_o}{E_o} \quad (2.8)$$

Comparing real parts on both sides of equation (2.8) yields:

$$\sigma\,\alpha = -\;e\,n\,\omega^2\,\frac{x_o}{E_o} \qquad (2.9)$$

3. Friction Coefficient and Relaxation time:

It is quite natural to relate frictional coefficient α to the relaxation time τ .

This is due to the fact that by physical intuition are one can deduce that that shorter the relaxation time , the bigger frictional coefficient .This can show also mathematics, by using the expression of energy dissipated by frication, which for oscillation particle is given by:

$$E_f = \frac{m}{\tau} = v_e$$

$$E_f = \frac{m\;\omega A^2}{2\tau} = \frac{m\;\omega^2 A^2}{2\tau\,\omega} = \frac{E}{\tau\;\omega} \qquad (3.1)$$

Where the effective displacement x_e and velocity v_e are related to the maximum displacement A by:

$$x_e = \frac{A}{\sqrt{2}} \qquad v_e = \omega\,x_e = \frac{\omega A}{\sqrt{2}}$$

Therefore

Where the classical energy of the oscillator is:

$$E = \frac{1}{2}\;m\;\omega^2 A^2 \qquad (3.2)$$

The oscillator frequency for classical and quantum system is the same. Thus one can write the quantum oscillator energy as:

$$E = \hbar\,\omega \qquad (3.3)$$

Thus inserting (3.3) in (3.1) yields:

$$E_f = \frac{\hbar\,E}{\tau\,\hbar\,\omega} = \frac{\hbar}{\tau} = \frac{E}{E} = \frac{\hbar}{\tau} \qquad (3.4)$$

The same result can be obtained by using equation (2.9) where:

$$\alpha = \frac{e\,n\,\omega^2}{\sigma}\left(\frac{x_o}{E_o}\right) \qquad (3.5)$$

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This equation can be simplified the using equation of motion of the electron in a frictional medium, in the presence of electric field. The equation is given by:

$$m\ddot{x} = +eE - \frac{mv}{\tau} \qquad (3.6)$$

Consider the solutions:

$$x = x_o e^{i\omega t}$$

$$E = E_o e^{i\omega t}$$

Therefore

$$\ddot{X} = -\omega^2 x$$

$$V = \dot{x} = i\omega x \qquad (3.7)$$

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Inserting equation (3.7) in equation (3.6) yields:

$$+m\left[i\frac{\omega}{\tau} - \omega^2\right] x_o e^{i\omega t} = eE_o e^{i\omega t}$$

$$\frac{x_o}{E_o} = \frac{e}{m\omega\left[\frac{i}{\tau} - \omega\right]\tau} \qquad (3.8)$$

For higher frequency and relativity large τ ,i. e when:

$$\omega >> \frac{1}{\tau} \qquad (3.9)$$

One gets:

$$\frac{x_o}{E_o} = \frac{e}{m\omega^2} \qquad (3.10)$$

Considering the conductivity to be:

$$\sigma = \frac{ne^2\tau}{m}$$

Then equation (3.5) reduces to:

$$\alpha = \frac{\omega^2 ne}{\sigma}\left(\frac{x_o}{E_o}\right) = +\frac{enm}{ne^2\tau}\left(\frac{+e}{m}\right)\frac{\omega^2}{\omega^2} = \frac{1}{\tau} \qquad (3.11)$$

Thus the electric field is decaying a according to equation (2.2) to be:

$$E = E_o e^{-t/\tau} e^{i(kx - \omega t)} \qquad (3.12)$$

Another approach to relate a to τ can also be obtained by using the equation of motion of the electron under the action of oscillating electric field of the form:

$$E = E_o e^{i\omega t} \qquad (3.13)$$

In this case the equation of motion of the electron is given by:

$$m\frac{dv}{dt} = eE - \frac{mv}{\tau} \qquad (3.14)$$

Consider the solution of the form: $V = v_o e^{i\omega t}$

Therefore:

$$m\left[i\,\omega+\frac{1}{\tau}\right]v = e\;E \qquad v = \frac{e\;E}{m\left[i\,\omega+\frac{1}{\tau}\right]} \qquad (3.15)$$

The conductivity σ can be found from the expression of the current density J , where:

$$J = n\,e\,v = \frac{n e^2\left[\frac{1}{\tau} - i\,\omega\right]\tau}{m\left[\left(\frac{1}{\tau}\right)^2 + \omega^2\right]} = \sigma\;E = \left[\sigma_1 + i\,\sigma_2\right]E \quad (3.16)$$

Hence:

$$\sigma_1 = \frac{n\,e^2/\tau}{m\left[1/\tau^2 + \omega^2\right]} \qquad \sigma_2 = \frac{-n\,e^2\omega}{m\left[1/\tau^2 + \omega^2\right]} \qquad (3.17)$$

The a absorption coefficient equation for non polarization: system can be given by equation to be:

$$-\frac{2i\,\alpha\,\omega}{c^2} - \mu\left(\sigma_1 + i\,\sigma_2\right)\alpha - \mu\left(\sigma_1 + i\,\sigma_2\right)\omega i = 0 \quad (3.18)$$

Equation imaginary parts in this equation yields:

$$-2\,\alpha\,\frac{\omega}{c^2} - \sigma_2\alpha - \mu\,\sigma_1\omega = 0 \qquad (3.19)$$

Since C^2 is extremely large compared to ordinary radio frequencies or current frequencies ie.

$$\frac{\omega}{c^2} << 1$$

Therefore from(3.170 and(3.19):

$$\alpha = -\frac{\sigma_1}{\sigma_2}\omega = \frac{1}{\tau} \qquad (3.20)$$

In view of equation (2.2) one gets:

$$E = E_o = e^{-t/\tau} e^{i\left(k\,x - \omega t\right)} \qquad (3.21)$$

4. Derivation of frictional Schrödinger Equation on the basis of frictional energy equation:

Ordinary Schrödinger is based on the postulates. The first postulate is related to the wave nature of micro particles. In this case the wave function takes the from:

$$\psi = A\;e^{i\left(k\,x - \omega t\right)} \qquad (4.1)$$

Using the fact that:

$$P = \hbar\,k \qquad E_o = \hbar\,\omega \qquad (4.2)$$

Therefore:

$$\psi = A\;e^{\frac{i}{\hbar}\left(P\,x - E\,t\right)} \qquad (4.3)$$

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The second postulate is based on the classical expression of energy:

$$E = \frac{P^2}{2m} + V \qquad (4.4)$$

Using these two postulate one can derive Schrödinger equation, where:

$$E_o \psi = \frac{P^2}{2m}\psi + V\psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = E_o \psi \qquad -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = \frac{P^2}{2m}\psi$$

Thus in three dimensions:

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m}\nabla^2 \psi + V\psi \qquad (4.5)$$

The expression of energy (4.4) in the presence of fiction is given, with the aid of equation (2.4) to be:

$$E = E_o + E_f$$

$$= \frac{P^2}{2m} + V - \frac{i\hbar}{\tau} \qquad (4.6)$$

Multiply both sides by ψ , one gets:

$$E\psi = \frac{P^2}{2m}\psi + V\psi + \frac{i\hbar}{\tau}\psi \qquad (4.7)$$

But:

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} E\psi \qquad i\hbar \frac{\partial \psi}{\partial t} = E\psi$$

$$\frac{\partial \psi}{\partial x} = \frac{i}{\hbar} P.\psi \qquad \frac{\partial^2 \psi}{\partial x^2} = -\frac{P^2}{\hbar^2}\psi$$

In three dimensions:

$$-\hbar^2 \nabla^2 \psi = P^2 \psi \qquad (4.8)$$

Inserting equation (4.8) in equation (4.7), one gets:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \psi + V\psi - i\frac{\hbar}{\tau}\psi \qquad (4.9)$$

Which is the Schrödinger equation for resistive medium

5. Harmonic oscillator string Theory:

To solve (4.9) consider the wave function to be:

$$\psi = f(t)\, u(r)$$

Inserting is (4.9) yields:

$$\frac{i\hbar}{f}\frac{\partial f}{\partial t} = -\frac{\hbar^2}{2mu}\nabla^2 u + V - \frac{i\hbar}{\tau} = E_o \qquad (5.1)$$

Hence:

$$i\hbar \frac{\partial f}{\partial t} = E_o f \qquad (5.2)$$

For harmonic oscillator string vibrating in one dimension the potential is given by:

$$V = \frac{1}{2} k x^2$$

Thus equation (5.1) reads:

$$-\frac{\hbar^2}{2m}\nabla^2 u + \frac{1}{2} k x^2 u = \left(E_o + \frac{i\hbar}{\tau}\right) u = E u \qquad (5.3)$$

This is the ordinary harmonic oscillator equation, with have quantized energy:

$$E = \left(n_1 + \frac{1}{2}\right)\hbar\omega \qquad (5.4)$$

For equation (5.1) one can suggest the solution:

$$f = A e^{-i \alpha_o t} \qquad (5.5)$$

$$\hbar \alpha_o = E_o$$

The periodicity condition require

$$f(t+T) = f(t) \qquad (5.6)$$

Hence from (5.4):

$$A e^{-i \alpha_o (t+T)} = A e^{-i \alpha_o t}$$

$$e^{-i \alpha_o T} = e^{-i\theta} = \cos\theta - i \sin\theta = 1$$

$$\theta = \alpha_o T = 2 n_2 \pi$$

$$\alpha_o = \frac{2}{T}\pi \quad \frac{2\pi n_2}{T} \qquad n_2 = n_2 \omega$$

From (5.5):

$$E_o = n_2 \hbar \omega \qquad (5.7)$$

In view of equation (5.3):

$$E = E_o - \frac{i\hbar}{\tau} \qquad (5.8)$$

Using equation (5.4) , (5.7) and (5.8) yields:

$$\left(n_1 + \frac{1}{2}\right)\hbar\omega = n_2 \hbar \omega + \frac{i\hbar}{\tau}$$

$$\frac{i}{\tau} = \left[\left(n_2 - n_1\right) - \frac{1}{2}\right]\omega = \left(-n - \frac{1}{2}\right)\omega$$

$$(\tau)^{-1} = -\frac{\left(n + \frac{1}{2}\right)}{i}\omega = \left(\frac{1}{2} + n\right)\omega i \qquad (5.9)$$

The physical meaning of complex relaxation can be known from equation (4.6):

$$E = E_o + \frac{i\hbar}{\tau} = E_o + E_f$$

Thus the energy lost due to friction is given by:

$$E_f = \frac{-i\hbar}{\tau} = +\left(n+\frac{1}{2}\right)\hbar\omega \qquad (5.10)$$

The minus sign indicates that the energy is lost by the particle.

6. Discussion:

It is very interesting to note that the force due to friction is found to be related to plank constant which reflects the quantum behavior. The friction force is also inversely proportional to the relaxation time. Solving Schrödinger equation (4.4) for free particles, the wave function is given by:

$$\psi = A\, e^{\frac{-t}{\tau}}\, e^{i(k\,x-\omega t)\frac{-t}{\tau}}$$

Thus the probability of suffering from friction (collision) is given by:

$$n = |\psi|^2 = A^2\, e^{\frac{-2t}{\tau}}$$

It is very string to note that this resembles collision probability P found from transport equation where:

$$P = p_o\, e^{\frac{-t}{\tau}}$$

The doubling of τ may be related to the fact that the probability of collision of single particles:

$$n_1 \sim e^{\frac{-t}{\tau}} \qquad n_2 \sim e^{\frac{-t}{\tau}}$$

$$n = n_1 n_2 \sim e^{\frac{-t}{\tau}} \quad e^{\frac{-2t}{\tau}}$$

The expression (3.4) for E_f is an exact relaxation obtained from classical expression for frictional force of oscillating particle, beside classical and quantum expression of energy for oscillating particle. The same express E_f is obtained by using Maxwell's equations and expressions for conductivity for oscillating particles by considering uniform and non uniform motion. For uniform motion, one considers approximation where the frequency is large compared to τ^{-1} see equations (3.9) and(3.11) for non uniform motions one considered low frequency compared to the speed of light ,this such. Equation (4.4) is applied tom the see equations (3.20) this new harmonic oscillating in section (5).The energy in (5.4) resembles that of ordinary harmonic. Oscillating. The frictional energy in (5.10) is quantized 7.

Conclusion:

The new Schrödinger equation that accounts for friction give collocation typical to that obtained from transport equations. It reduces to ordinary such. Equation in the basemen officiates.

6. Conclusion:

The quantum equation derived from Max Wells equation for decaying wave in a resistive medium is more advanced than ordinary schord Eqn. This is since it reduces to sch. eqn. It also gives expression for collision probability similar to that obtained by transport Eqn. It also predicts quantized frictional energy.

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