IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 2 Issue 11, November 2015.
www.ijiset.com
ISSN 2348 – 7968

Special Class Of Extended Mean Cordial Graphs

Dr. A. Nellai Murugan and J. Shiny Priyanka
Department of Mathematics
V.O.Chidambaram College
Tuticorin 628 008

Abstract – Let G = (V,E) be a graph with p vertices and q edges. A Mean Extended Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1,2} such that each edge uv is assigned the label $(\lceil f(u)+f(v))\rceil/2$where ⌈ x ⌉ is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a Extended Mean Cordial Labeling is called Extended Mean Cordial Graph. In this paper, we proved that the graphs H-graph, $c_3@s_n$, Book $K_{1,2n}$ X K_4 ,Tp_n ,are Extended Mean Cordial Graphs.

Keywords – Extended Mean Cordial Graph, Extended Mean Cordial Labeling.

2000 Mathematics Subject classification 05C78.

I.INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {u,v} of vertices in E is called edges or a line of G. In this paper, we proved that the graphs H-graph, $c_3@s_n$, Book $K_{1,2n}$ X K_4 ,Tp_n ,are Extended Mean Cordial Graphs. For graph theory terminology, we follow [2].

II.PRELIMINARIES

Let G = (V,E) be a graph with p vertices and q edges. A Mean Square Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1,2} such that each edge uv is assigned the label $(\lceil f(u)+f(v))\rceil/2$ where ⌈ x ⌉ is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

The graph that admits a Extneded Mean Cordial Labeling is called Extended Mean Cordial Graph. In this paper, we proved that the graphs H-graph, $c_3@s_n$, Book $K_{1,2n}$ X K_4 ,Tp_n ,are Extended Mean Cordial Graphs.

Definition: 2.1

H-graph H_n is a graph obtained from two copies of path P_n with vertices $(u_1,u_2,........,u_n)$ and $(v_1,v_2,........,v_n)$ by joining the vertices $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ if n is even.

Definition: 2.2

The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 points) and P_1 copies of G_2 and joining the i^{th} point of G_1 to every point in the i^{th} copy of G. $C_n \Theta K_1$ is called a Crown.

Definition :2.3

The product G_1 x G_2 of two graphs G_1 and G_2 is defined to be the graph whose vertex set is V_1 x V_2 and two vertices u= (u_1, u_2) and v= (v_1, v_2) in $V=V_1 x V_2$ are adjacent in G_1 x G_2 if either $u_1=v_1$ and u_2 is adjacent to v_2 or $u_2=v_2$ and u_1 is adjacent to v_1. $K_{1,2n}$ X K_2 is called a Book.

Definition: 2.4

It is a graph obtained from a triangle C_3, by having each side as P_n and every internal vertex of each side joined with a single vertex of other sides. So that it divides in to sub triangles giving in Triangular numbers. Tp_n.

III. MAIN RESULTS

Theorem :3.1

H-graph is a Extended Mean Cordial Graph.

Proof:

Let G =(V,E)

Case(i)

Let V(H) ={ u_i , v_i : $1 \le i \le n$}

Let E(H) ={$[(u_i u_{i+1}) \cup (v_i v_{i+1}) : 1 \le i \le n-1] \cup [(u_{n+2/2} v_{n+1/2})]$}

Define f:V(H)→{0,1,2} by

$$f(u_i) = \begin{cases} 0 \ if \ i \equiv 0 \ mod \ 2 \\ 2 \ if \ i \equiv 1 \ mod \ 2 \end{cases}, \ 1 \le i \le n$$

$$f(v_i) = \begin{cases} 0 \ if \ i \equiv 0 \ mod \ 2 \\ 1 \ if \ i \equiv 1 \ mod \ 2 \end{cases}, \ 1 \le i \le n$$

The induced edge labeling are

$f^*(u_i u_{i+1}) = 1$, $1 \le i \le n-1$

$f^*(v_i\, v_{i+1}) = 0,\ 1 \le i \le n-1$

$f^*(u_{\frac{n+1}{2}}\ v_{\frac{n+1}{2}}) = \begin{cases} 0 \text{ if } n \equiv 3 \bmod 4 \\ 1 \text{ if } n \equiv 1 \bmod 4 \end{cases},\ 1 \le i \le n$

Here the graph satisfies the condition,

ef(1) =ef(0) +1

Hence H is Satisfies the condition| ef(0) – ef(1) | ≤ 1

Therefore , H is a Extended Mean Cordial Graph.

For example, H_4 is a Extended Mean Cordial Graph as shown in the figure 3.2

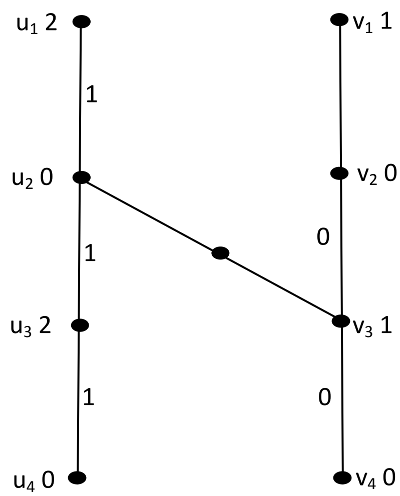

Figure 3.2

Case(ii) : n-even

$V(H)=\{[u_i,v_i:\ 1 \le i \le n]\}$

$E(H)=\{[(u_iu_{i+1}) \cup (v_iv_{i+1}) : 1 \le i \le n-1] \cup [(u_{n/2}\, v_{n+1/2})]\}$

$f(u_i) = \begin{cases} 0 \text{ if } i \equiv 0 \bmod 2 \\ 2 \text{ if } i \equiv 1 \bmod 2 \end{cases},\ 1 \le i \le n$

$f(v_i) = \begin{cases} 0 \text{ if } i \equiv 0 \bmod 2 \\ 1 \text{ if } i \equiv 1 \bmod 2 \end{cases},\ 1 \le i \le n$

$f^*(u_i\, u_{i+1}) = 1,\ 1 \le i \le n-1$

$f^*(v_i\, v_{i+1}) = 0,\ 1 \le i \le n-1$

$f^*(u_{\frac{n}{2}} v_{\frac{n+1}{2}}) = \begin{cases} 0 \; if \; n \equiv 0 \; mod \; 4 \\ 1 \; if \; n \equiv 2 \; mod \; 4 \end{cases}$, $1 \le i \le n$

Here the graph satisfies the condition,

ef(1) =ef(0) +1

Hence H is Satisfies the condition| ef(0) – ef(1) | ≤ 1

Therefore , H is a Extended Mean Cordial Graph.

For example, H_5 is a Extended Mean Cordial Graph as shown in the figure 3.3

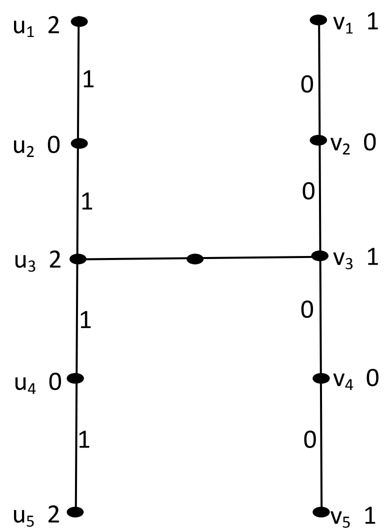

Figure 3.3

Theorem :3.4

Let $c_3@s_n$ be a Extended Mean Cordial Graph

Proof:

Let G(V,E)

Let V ($c_3@s_n$) ={[(u_i,u_2) : 1 ≤ i ≤ n]U[u_{i1} : 1 ≤ i ≤ n]}

Let E($c_3@s_n$) = {[(u_1u_2) : 1 ≤ i ≤ n]U[(u_2u_3) : 1 ≤ i ≤ n]U(u_1u_3) : 1 ≤ i ≤ n]

U(u_1u_{i1}) : 1 ≤ i ≤ n]}

Define f: V($c_3@s_n$)→ {0,1,2}

The vertex labeling are,

f(u_i) =1 , i = 1,3 , 1 ≤ i ≤ n

ISSN 2348 – 7968

$f(u_2)=0$, $1 \leq i \leq n$

$f(u_{i1})= \begin{cases} 1 & i \equiv 1 \bmod 2 \\ 0 & i \equiv 0 \bmod 2 \end{cases}, 1 \leq i \leq n$

The edge labeling are,

$f^*(u_1u_2) = 0$, $1 \leq i \leq n$

$f^*(u_2u_3) = 0$, $1 \leq i \leq n$

$f^*(u_1u_3) = 1$, $1 \leq i \leq n$

$f^*(u_1u_{i1})= \begin{cases} 1 & i \equiv 1 \bmod 2 \\ 0 & i \equiv 0 \bmod 2 \end{cases}, 1 \leq i \leq n$

Here , $ef(1) = ef(0)+1$

Hence $c_3@s_n$ is Satisfies the condition$|$ $ef(0) – ef(1)$ $| \leq 1$

Therefore , $c_3@s_n$ is a Extended Mean Cordial Graph.

For example, $c_3@s_4$ is a Extended Mean Cordial Graph as shown in the figure 3.5

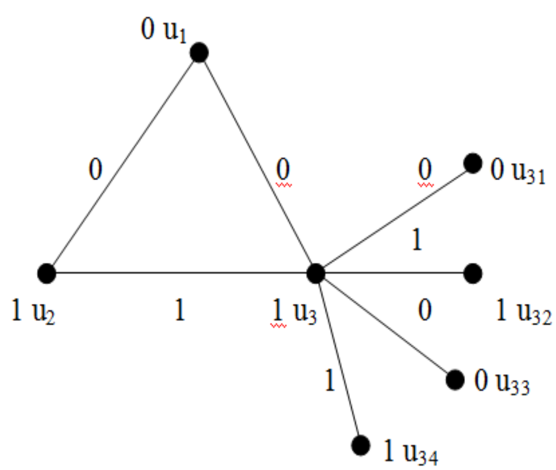

Figure 3.5

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www.ijiset.com
ISSN 2348 – 7968

Theorem 3.6

Let Book $K_{1,2n}$ X K_4 be the Extended Mean Cordial Graph

Proof:

Let V($K_{1,2n}$ X K_4) = {u_i, v_i, v_1 $1 \leq i \leq n$}

Let E($K_{1,2n}$ X K_4) ={[(u_1v_1)U(u_iv_i)U(u_iu_{i+1})U(v_1v_{i+1}) : $1 \leq i \leq n-1$]}

Define f :v($K_{1,2n}$ X K_4)→{0,1,2}by

The vertex labeling are,

$$f(u_i) = \begin{cases} 1 & i \equiv 0 \mod 2 \\ 0 & i \equiv 1 \mod 2 \end{cases}, 1 \leq i \leq n$$

$f(v_1) = 2$, $1 \leq i \leq n$

$f(v_i) = 0$, $i = 2,3$

$$f(v_i) = \begin{cases} 1 & i \equiv 0 \mod 2 \\ 0 & i \equiv 1 \mod 2 \end{cases}, 4 \leq i \leq n$$

The edge labeling are,

$f^*(u_1 v_1) = 1$, $1 \leq i \leq n$

$f^*(u_i v_i) = 0$, $2 \leq i \leq n$

$f^*(u_1 v_{i+1}) = 0$, $1 \leq i \leq n-1$

$f^*(v_1 v_{i+1}) = 1$, $1 \leq i \leq n-1$

Here, ef(1) =ef(0)+1

ef(1) = ef(0)

Hence Book $K_{1,2n}$ X K_4 is Satisfies the condition| ef(0) – ef(1) | ≤ 1

Therefore , Book $K_{1,2n}$ X K_4 is a Extended Mean Cordial Graph.

For example, Book $K_{1,2n}$ X K_4 is a Extended Mean Cordial Graph as shown in the figure 3.7

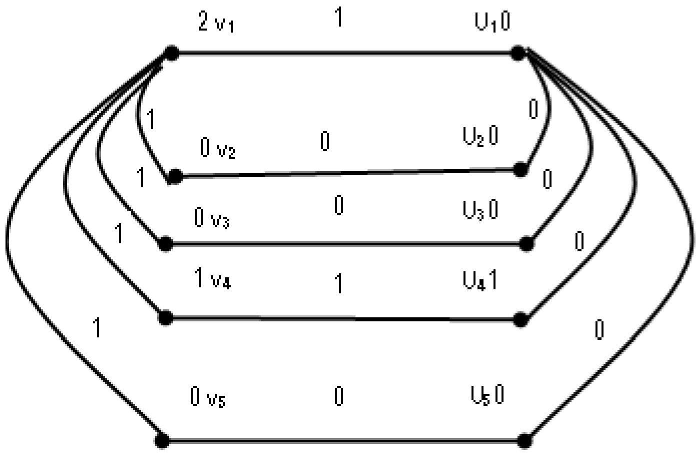

Figure 3.7

Theorem 3.8

Graph TP_n is a Extended Mean Cordial Graph.

Proof:

Let $V(TP_n) = \{u_{ij} : 1 \leq i \leq n , 1 \leq j \leq i\}$

Let $E(TP_n) = \{[(u_{ij}\ u_{j+1}) : 2 \leq i \leq n ;\ 1 \leq j \leq i-1] U [(u_{ij}\ u_{i+1j}) : 1 \leq j \leq n-1 ;\ j \leq i \leq n-1] U$

$[(u_{ij}\ u_{i+1,j+1}) : 1 \leq j \leq n-1 ;\ j \leq i \leq n-1]\}$

The vertex labeling are,

$f(u_{11}) = 0 \quad 1 \leq i \leq n , 1 \leq j \leq i$

$f(u_{21}) = 0 \quad 1 \leq i \leq n , 1 \leq j \leq i$

$f(u_{22}) = 0 \quad 1 \leq i \leq n , 1 \leq j \leq i$

for $3 \leq i \leq n \qquad i \equiv 1\ mod\ 2$

$$f(u_{ij}) = \begin{cases} 1 , & 1 \leq j \leq \frac{i}{2} \\ 0 & , \frac{i+1}{2} \leq j \leq i \end{cases}$$

The edge labeling are

$f^*(u_{21}\ u_{22}) = 1$

for $3 \leq i \leq n$ $\quad i \equiv 1 \bmod 2$

$$f^*(u_{ij}u_{ij+1}) = \begin{cases} 1 & 1 \leq j \leq \frac{i-1}{2} \\ 0 & \frac{i+1}{2} \leq j \leq i-1 \end{cases} \quad , \quad i \equiv 0 \bmod 2$$

$$f^*(u_{ij}u_{ij+1}) = \begin{cases} 1 & 1 \leq j \leq \frac{i}{2} - 1 \\ 0 & \frac{i}{2} \leq j \leq i-1 \end{cases} \quad , \quad i \equiv 0 \bmod 2$$

$f^*(u_{11}\, u_{21}) = 0$

$f^*(u_{ij}u_{i+1j}) = 1: \quad 2 \leq i \leq n-1, \quad j=1,2$

when $\quad , \quad n \equiv 0 \bmod 2 \quad 3 \leq j \leq \frac{n}{2}$

$$f^*(u_{ij}u_{ij+1}) = \begin{cases} 0 & j \leq i \leq 2j-2 \\ 1 & 2j-1 \leq i \leq n-1 \end{cases}$$

$$\frac{n+1}{2} \leq j \leq n-1$$

$f^*(u_{ij}u_{i+1j}) = 0: \quad j \leq i \leq n-1,$

when , $\quad n \equiv 0 \bmod 2 \quad 3 \leq j \leq \frac{n-1}{2}$

$$f^*(u_{ij}u_{ij+1}) = \begin{cases} 0 & j \leq i \leq 2j-2 \\ 1 & 2j-1 \leq i \leq n-1 \end{cases}$$

$$\frac{n+1}{2} \leq j \leq n-1$$

$f^*(u_{ij}u_{i+1j}) = 0: \quad j \leq i \leq n-1,$

when $\quad n \equiv 0 \bmod 2 \quad 1 \leq j \leq \frac{n-1}{2}$

$$f^*(u_{ij}u_{ij+1}) = \begin{cases} 0 & j \leq i \leq 2j-1 \\ 1 & 2j \leq i \leq n-1 \end{cases}$$

$$\frac{n}{2} \leq j \leq n-1$$

$f^*(u_{ij}u_{i+1j}) = 0: \quad j \leq i \leq n-1,$

when $\quad n \equiv 1 \bmod 2 \quad 1 \leq j \leq \frac{n-1}{2}$

$f^*(u_{ij}u_{ij+1}) = \begin{cases} 0 & j \le i \le 2j-1 \\ 1 & 2j \le i \le n-1 \end{cases}$

$$\frac{n+1}{2} \le j \le n-1$$

$f^*(u_{ij}u_{i+1j}) = 0$: $j \le i \le n-1$,

Here ef(0) = ef(1)+1

Hence Tp_n is Satisfies the condition$|ef(0) - ef(1)| \le 1$

Therefore , Tp_n is a Extended Mean Cordial Graph.

For example, Tp_4 is a Extended Mean Cordial Graph as shown in the figure 3.9

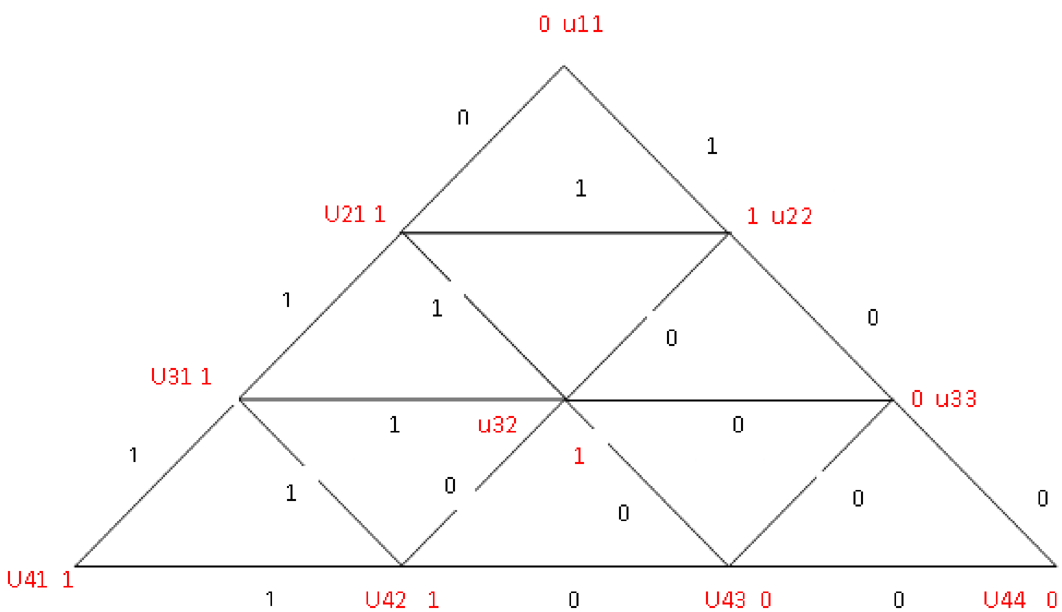

Figure 3.9

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