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Applications Of Fuzzy Number Mathematics

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ABSTRACT

Fuzzy sets have been introduced by Lotfi.A.Zadeh(1965)[16] and Dieter Klaua(1965)[7]. Fuzzy set theory permits the gradual assessment of the membership of elements in a set which is described in the interval [0, 1]. It can be used in a wide range of domains where information is incomplete and imprecise. Interval arithmetic was first suggested by Dwyer[7] in 1951,by means of Zadeh's extension principle[15,16], the usual Arithmetic operations on real numbers can be extended to the ones defined on Fuzzy numbers. D.Dubois and H.Prade[3] in 1978 has defined any of the fuzzy numbers as a fuzzy subset of the real line[4,5,6,8]. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single-valued numbers. Among the various shapes of fuzzy numbers, Triangular fuzzy number and Trapezoidal fuzzy number are the most commonly used membership function(Dubois and Prade[3],1980,Zimmermann[17], 1996) In this paper a new operation of Decagonal fuzzy numbers has been introduced with its basic membership function followed by the properties of its arithmetic operations of fuzzy numbers[1,2,3,9,13]. In few cases Triangular or Trapezoidal is not applicable to solve the problem if it has ten different points; hence we make use of this new operation of Decagonal fuzzy number to solve in such cases.

Key words : Fuzzy number, Addition, Subtraction, Multiplication and division

HEXAGONAL FUZZY NUMBERS

A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by $\tilde{A}_H$ $(a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given below.

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right) & \text{for } a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right) & \text{for } a_2 \le x \le a_3 \\ 1 & \text{for } a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x - a_4}{a_5 - a_4}\right) & \text{for } a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6 - x}{a_6 - a_5}\right) & \text{for } a_5 \le x \le a_6 \\ 0 & \text{for } x > a_6 \end{cases}$$

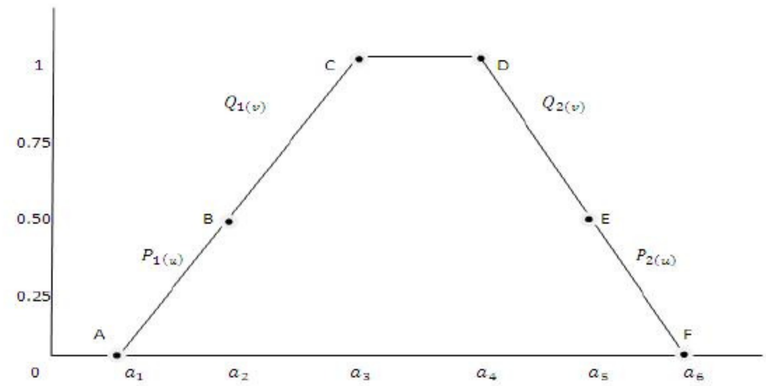

Figure 1 Graphical representation of a normal hexagonal fuzzy number for $x \in [0, 1]$

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Decagonal fuzzy number :

A fuzzy number $\tilde{A}_D$ is a decagonal fuzzy number denoted by $\tilde{A}_D$ $(a_1, a_2, a_3, a_4, a_5\ a_6, a_7, a_8, a_9, a_{10})$ where $a_1, a_2, a_3, a_4, a_5\ a_6, a_7, a_8, a_9, a_{10}$ are real numbers and its membership function $\mu_{\tilde{A}_D}(x)$ given below.

$$\mu_{\tilde{A}_D}(x) = \begin{cases} \frac{1}{4}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ \frac{1}{4}+\frac{1}{4}\left(\frac{x-a_2}{a_3-a_2}\right) & a_2 \le x \le a_3 \\ \frac{1}{2}+\frac{1}{4}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ \frac{3}{4}+\frac{1}{4}\left(\frac{x-a_4}{a_5-a_4}\right) & a_4 \le x \le a_5 \\ 1 & a_5 \le x \le a_6 \\ 1-\frac{1}{4}\left(\frac{x-a_6}{a_7-a_6}\right) & a_6 \le x \le a_7 \\ \frac{3}{4}-\frac{1}{4}\left(\frac{x-a_7}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ \frac{1}{2}-\frac{1}{4}\left(\frac{x-a_8}{a_9-a_8}\right) & a_8 \le x \le a_9 \\ \frac{1}{4}\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & a_9 \le x \le a_{10} \\ 0 & \text{Otherwise} \end{cases}$$

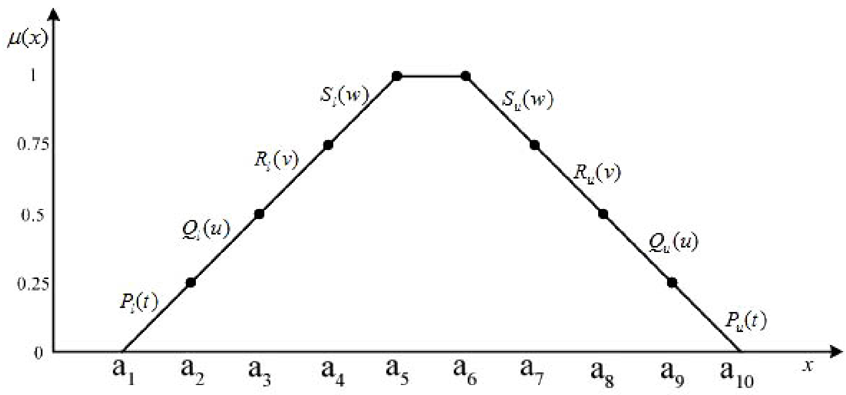

Fig – 2 Graphical representation of a normal decagonal fuzzy number for $x \in [0, 1]$

ALPHA CUT :

The classical set $\tilde{A}_\alpha$ called alpha cut set is the set of elements whose degree of membership is the set of elements whose degree of membership in $\tilde{A}_D = (a_1, a_2, a_3, a_4, a_5\ a_6, a_7, a_8, a_9, a_{10})$ is no less than α it is defined as,

$$A_\alpha = \{ x \in X / \mu_{\tilde{A}_D}(x) \geq \alpha \}$$

$$\tilde{D}_\alpha = \begin{cases} P_1(\alpha), & P_2(\alpha) \quad \text{for} \quad \alpha \in [0, 0.25) \\ Q_1(\alpha), & Q_2(\alpha) \quad \text{for} \quad \alpha \in [0.25, 0.5) \\ R_1(\alpha), & R_2(\alpha) \quad \text{for} \quad \alpha \in [0.5, 0.75) \\ S_1(\alpha), & S_2(\alpha) \quad \text{for} \quad \alpha \in [0.75, 1) \end{cases}$$

α Cut Operations

If the crisp interval by α cut operations interval A_α shall be obtained as follows for all $\alpha \in [0, 1]$.

Consider $P_1(x) = \alpha$,

(i.e) $\frac{1}{4}\left(\frac{x - a_1}{a_2 - a_1}\right) = \alpha$

$\left(\frac{x - a_1}{a_2 - a_1}\right) = 4\alpha$

$x - a_1 = 4\alpha \ (a_2 - a_1)$

$x = 4\ \alpha\ (a_2 - a_1) + a_1$

(i.e) $P_1(\alpha) = 4\ \alpha\ (a_2 - a_1) + a_1$

Similarly from $P_2(x) = \alpha$,

(i.e) $\frac{1}{4}\left(\frac{a_{10} - x}{a_{10} - a_9}\right) = \alpha,$

$\left(\frac{a_{10} - x}{a_{10} - a_9}\right) = 4\alpha$

$a_{10} - x = 4\alpha\ (a_{10} - a_9)$

$-x = 4\alpha\ (a_{10} - a_9) - a_{10}$

$x = -\ 4\alpha\ (a_{10} - a_9) + a_{10}$

(i.e) $P_2(\alpha) = -\ 4\ \alpha\ (a_{10} - a_9) + a_{10}$

This implies

$[P_1(\alpha), P_2(\alpha)] = [\ 4\ \alpha\ (a_2 - a_1) + a_1, -\ 4\ \alpha\ (a_{10} - a_9) + a_{10}]$

Consider $Q_1(x) = \alpha$,

(i.e)

$\frac{1}{4} + \frac{1}{4}\left(\frac{x - a_2}{a_3 - a_2}\right) = \alpha,$

$\frac{1}{4}\left(\frac{x - a_2}{a_3 - a_2}\right) = \left(\alpha - \frac{1}{4}\right)$

$\left(\frac{x - a_2}{a_3 - a_2}\right) = \left(\alpha - \frac{1}{4}\right) 4$

$$\left(\frac{x-a_2}{a_3-a_2}\right) = (4\alpha - 1)$$

$$(x - a_2) = (4\alpha - 1)(a_3 - a_2)$$

$$(x - a_2) = 4\alpha(a_3) - 4\alpha(a_2) - a_3 + a_2$$

$$= 4\alpha(a_3 - a_2) - a_3 + a_2$$

$$x = 4\alpha(a_3 - a_2) - a_3 + a_2 + a_2$$

$$x = 4\alpha(a_3 - a_2) - a_3 + 2a_2$$

$$x = 4\alpha(a_3 - a_2) - 2a_2 - a_3$$

$$Q_1(\alpha) = 4\alpha\ (a_3 - a_2) + 2a_2 - a_3$$

Similarly from

$$Q_2(x) = \alpha$$

(i.e) $$\frac{1}{2} - \frac{1}{4}\left(\frac{x-a_8}{a_9-a_8}\right) = \alpha$$

$$-\frac{1}{4}\left(\frac{x-a_8}{a_9-a_8}\right) = \alpha - \frac{1}{2}$$

$$-\left(\frac{x-a_8}{a_9-a_8}\right) = \left(\alpha - \frac{1}{2}\right) 4$$

$$-(x - a_8) = (4\alpha - 2)(a_9 - a_8)$$

$$-x + a_8 = (4\alpha - 2)(a_9 - a_8)$$

$$-x = 4\alpha(a_9) - 4\alpha(a_8) - 2a_9 + 2a_8 - a_8$$

$$-x = 4\alpha(a_9 - a_8) - 2a_9 + a_8$$

$$x = -4\alpha(a_9 - a_8) + 2a_9 - a_8$$

(i.e) $$Q_2(\alpha) = -4\alpha(a_9 - a_8) + 2a_9 - a_8$$

This implies

$$[Q_1(\alpha),\ Q_2(\alpha)] = [4\alpha\ (a_3 - a_2) - a_3 + 2a_2,\ -4\alpha\ (a_9 - a_8) + 2a_9 - a_8]$$

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Consider $R_1(x) = \alpha$

(i.e) $\frac{1}{2} + \frac{1}{4}\left(\frac{x - a_3}{a_4 - a_3}\right) = \alpha$

$\frac{1}{4}\left(\frac{x - a_3}{a_4 - a_3}\right) = \left(\alpha - \frac{1}{2}\right)$

$\left(\frac{x - a_3}{a_4 - a_3}\right) = \left(\alpha - \frac{1}{2}\right)4$

$\left(\frac{x - a_3}{a_4 - a_3}\right) = (4\alpha - 2)$

$(x - a_3) = (4\alpha - 2)(a_4 - a_3)$

$(x - a_3) = 4\alpha(a_4) - 4\alpha(a_3) - 2a_4 + 2a_3$

$x = 4\alpha(a_4 - a_3) - 2a_4 + 2a_3 + a_3$

$x = 4\alpha(a_4 - a_3) - 2a_4 + 3a_3$

$x = 4\alpha(a_4 - a_3) + 3a_3 - 2a_4.$

(i.e) $R_1(\alpha) = 4\alpha(a_4 - a_3) + 3a_3 - 2a_4$

Similarly from,

$R_2(x) = \alpha$

(i.e) $\frac{3}{4} - \frac{1}{4}\left(\frac{x - a_7}{a_8 - a_7}\right) = \alpha$

$-\frac{1}{4}\left(\frac{x - a_7}{a_8 - a_7}\right) = \left(\alpha - \frac{3}{4}\right)$

$-\left(\frac{x - a_7}{a_8 - a_7}\right) = \left(\alpha - \frac{3}{4}\right)4$

$-\left(\frac{x - a_7}{a_8 - a_7}\right) = (4\alpha - 3)$

$-(x - a_7) = (4\alpha - 3)(a_8 - a_7)$

$-(x - a_7) = 4\alpha(a_8) - 4\alpha(a_7) - 3a_8 + 3a_7$

$-x + a_7 = 4\alpha(a_8 - a_7) - 3a_8 + 3a_7$

$$- x = 4\alpha (a_8 - a_7) - 3a_8 + 3a_7 - a_7$$

$$x = - 4\alpha (a_8 - a_7) + 3a_8 - 2a_7$$

$$x = - 4\alpha (a_8 - a_7) - 2a_7 + 3a_8$$

(i.e) $R_2 (\alpha) = - 4\alpha (a_8 - a_7) - 2a_7 + 3a_8$

This implies

$$[R_1, (\alpha), R_2 (\alpha)] = [4\alpha (a_4 - a_3) + 3a_3 - 2a_4, - 4\alpha (a_8 - a_7) - 2a_7 + 3a_8]$$

Consider,

$$S_1 (x) = \alpha$$

(i.e) $\frac{3}{4} + \frac{1}{4}\left(\frac{x - a_4}{a_5 - a_4}\right) = \alpha$

$$\frac{1}{4}\left(\frac{x - a_4}{a_5 - a_4}\right) = \left(\alpha - \frac{3}{4}\right)$$

$$\left(\frac{x - a_4}{a_5 - a_4}\right) = \left(\alpha - \frac{3}{4}\right) 4$$

$$\left(\frac{x - a_4}{a_5 - a_4}\right) = (4\alpha - 3)$$

$$x - a_4 = (4\alpha - 3) (a_5 - a_4)$$

$$x - a_4 = 4\alpha (a_5) - 4\alpha (a_4) - 3a_5 + 3a_4$$

$$x - a_4 = 4\alpha (a_5 - a_4) - 3a_5 + 3a_4$$

$$x = 4\alpha (a_5 - a_4) - 3a_5 + 3a_4 + a_4$$

$$x = 4\alpha (a_5 - a_4) - 3a_5 + 4a_4$$

(i.e) $S_1 (\alpha) = 4 \alpha (a_5 - a_4) + 4a_4 - 3a_5$

Similarly from,

$$S_2 (x) = \alpha$$

(i.e) $1 - \frac{1}{4}\left(\frac{x - a_6}{a_7 - a_6}\right) = \alpha$

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$$-\frac{1}{4}\left(\frac{x-a_6}{a_7-a_6}\right) = \alpha - 1$$

$$-\left(\frac{x-a_6}{a_7-a_6}\right) = (\alpha - 1)\,4$$

$$\begin{aligned}
-(x - a_6) &= (4\alpha - 4)(a_7 - a_6) \\
-(x - a_6) &= (4\alpha)(a_7) - 4\alpha(a_6) - 4a_7 + 4a_6 \\
-x + a_6 &= 4\alpha(a_7 - a_6) - 4a_7 + 4a_6 \\
-x &= 4\alpha(a_7 - a_6) - 4a_7 + 4a_6 - a_6 \\
-x &= 4\alpha(a_7 - a_6) - 4a_7 + 3a_6 \\
x &= -4\alpha(a_7 - a_6) + 4a_7 - 3a_6 \\
\text{(i.e) } S_2(\alpha) &= -4\alpha(a_7 - a_6) + 4a_7 - 3a_6
\end{aligned}$$

This implies

$$[S_1(\alpha), S_2(\alpha)] = [4\alpha(a_5 - a_4) + 4a_4 - 3a_5,\ -4\alpha(a_7 - a_6) + 4a_7 - 3a_6]$$

Hence

$$A_\alpha = \begin{cases} [4\alpha(a_2 - a_1) + a_1,\ -4\alpha(a_{10} - a_9) + a_{10}] & \text{for } \alpha \in [0, 0.25) \\ [4\alpha(a_3 - a_2) + 2a_2 - a_3,\ -4\alpha(a_9 - a_8) + 2a_9 - a_8] & \text{for } \alpha \in [0.25, 0.5) \\ [4\alpha(a_4 - a_3) + 3a_3 - 2a_4,\ -4\alpha(a_8 - a_7) + 3a_8 - 2a_7] & \text{for } \alpha \in [0.5, 0.75) \\ [4\alpha(a_5 - a_4) + 4a_4 - 3a_5,\ -4\alpha(a_7 - a_6) + 4a_7 - 3a_6] & \text{for } \alpha \in [0.75, 1] \end{cases}$$

Operations of decagonal fuzzy numbers : [1, 2, 3, 4]

Following are the three operations that can be performed on decagonal fuzzy numbers. Suppose

Let $\tilde{A}_D$ = $(a_1, a_2, a_3, a_4, a_5\ a_6, a_7, a_8, a_9, a_{10})$

and $\tilde{B}_D$ = $(b_1, b_2, b_3, b_4, b_5\ b_6, b_7, b_8, b_9, b_{10})$

Addition :

$\tilde{A}_D (+) \tilde{B}_D$ = $(a_1 + b_1, a_2 + b_2 , a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7,$
$a_8 + b_8, a_9 + b_9, a_{10} + b_{10})$.

Subtraction :

$\tilde{A}_D (-) \tilde{B}_D$ = $(a_1 - b_1, a_2 - b_2 , a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6, a_7 - b_7,$
$a_8 - b_8, a_9 - b_9, a_{10} - b_{10})$.

Multiplication :

$\tilde{A}_D (*) \tilde{B}_D$ = $(a_1 * b_1, a_2 * b_2 , a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6, a_7 * b_7,$
$a_8 * b_8, a_9 * b_9, a_{10} * b_{10})$.

Division :

$\tilde{A}_D \div \tilde{B}_D$ = $(a_1 \div b_1, a_2 \div b_2 , a_3 \div b_3, a_4 \div b_4, a_5 \div b_5, a_6 \div b_6, a_7 \div b_7,$
$a_8 \div b_8, a_9 \div b_9, a_{10} \div b_{10})$.

Example : 1

Let $\tilde{A}_D$ = (3.3, 3.6, 3.9, 4.2, 4.5, 4.8, 5.1, 5.4, 5.7, 6.0)

and $\tilde{B}_D$ = (4.8, 5.1, 5.4, 5.7, 6.0, 6.3, 6.6, 6.9, 7.2, 7.5)

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Then

$$\tilde{A}_D \,(+)\, \tilde{B}_D \;=\; (8.1, 8.7, 9.3, 9.9, 10.5, 11.1, 11.7, 12.3, 12.9, 13.5)$$

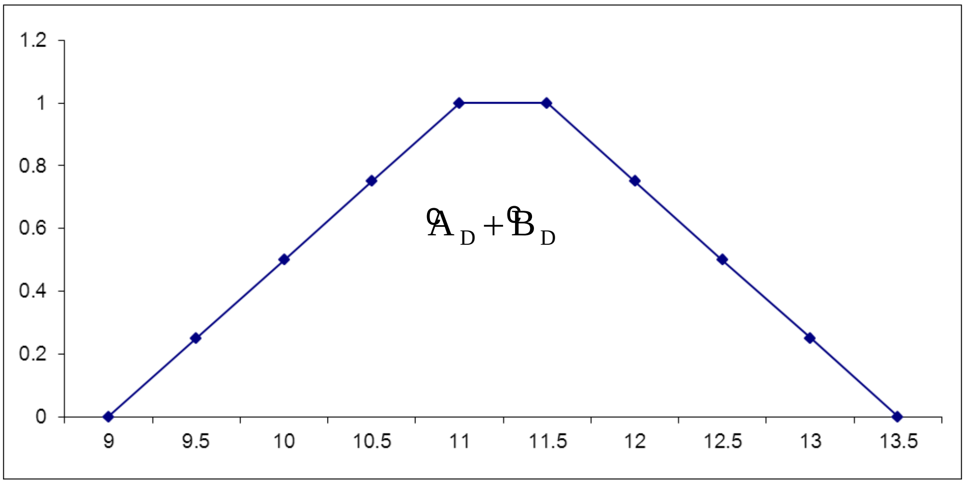

Figure -1

A New Operation for addition, subtraction, Multiplication and division on Decagonal fuzzy number.

α cut of a normal decagonal fuzzy number. The α cut of a normal decagonal fuzzy number $\tilde{A}_D = (a_1, a_2, a_3, a_4, a_5\ a_6, a_7, a_8, a_9, a_{10})$ given by the definition

(i.e) W = 1 for all $\alpha \in [0, 1]$ is

$$A_\alpha = \begin{cases} [4\alpha\,(a_2 - a_1) + a_1,\ -4\alpha\,(a_{10} - a_9) + a_{10}] & \text{for } \alpha \in [0, 0.25) \\ [4\alpha(a_3 - a_2) + 2a_2 - a_3,\ -4\,\alpha(a_9 - a_8) + 2a_9 - a_8] & \text{for } \alpha \in [0.25, 0.5) \\ [4\alpha(a_4 - a_3) + 3a_3 - 2a_4,\ -4\alpha(a_8 - a_7) + 3a_8 - 2a_7] & \text{for } \alpha \in [0.5, 0.75) \\ [4\alpha(a_5 - a_4) + 4a_4 - 3a_5,\ -4\alpha(a_7 - a_6) + 4a_7 - 3a_6] & \text{for } \alpha \in [0.75, 1] \end{cases}$$

Addition of two decagonal fuzzy numbers :

Let $\tilde{A}_D = (a_1, a_2, a_3, a_4, a_5\ a_6, a_7, a_8, a_9, a_{10})$

and $\tilde{B}_D = (b_1, b_2, b_3, b_4, b_5\ b_6, b_7, b_8, b_9, b_{10})$ be two decagonal fuzzy numbers for all $\alpha \in [0, 1]$.

Let us add the alpha cuts A_α and B_α of $\tilde{A}_D$ and $\tilde{B}_D$ using interval arithmetic.

$$A_\alpha + B_\alpha = \begin{cases} [4\alpha (a_2 - a_1) + a_1, - 4\alpha (a_{10} - a_9) + a_{10}] + \\ [4\alpha (b_2 - b_1) + b_1, - 4\alpha (b_{10} - b_9) + b_{10}] \text{ for } \alpha \in [0, 0.25) \\ [4\alpha(a_3 - a_2) + 2a_2 - a_3, - 4\alpha (a_9 - a_8) + 2a_9 - a_8] + \\ [4\alpha (b_3 - b_2) + 2b_2 - b_3, - 4\alpha (b_9 - b_8) +2b_9 - b_8] \\ \qquad \text{for } \alpha \in [0.25, 0.5) \\ [4\alpha (a_4 - a_3) + 3a_3 - 2a_4 , - 4\alpha (a_8 - a_7) + 3a_8 - 2a_7] + \\ [4\alpha (b_4 - b_3) + 3b_3 - 2b_4, - 4\alpha (b_8 - b_7) + 3b_8 - 2b_7] \\ \qquad \text{for } \alpha \in [0.5, 0.75) \\ [4\alpha (a_5 - a_4) + 4a_4 - 3a_5, - 4\alpha (a_7 - a_6) + 4a_7 - 3a_6] \\ + [4\alpha (b_5 - b_4) + 4b_4 - 3b_5, - 4\alpha (b_7 - b_6) + 4b_7 - 3b_6] \\ \qquad \text{for } \alpha \in [0.75, 1] \end{cases}$$

Addition Operation :

1. $A_\alpha = [4\alpha (a_2 - a_1) + a_1, - 4\alpha (a_{10} - a_9) + a_{10}]$

$B_\alpha = [4\alpha (b_2 - b_1) + b_1, - 4\alpha (b_{10} - b_9) + b_{10}]$

$a_1 = 3.3,\ a_2 = 3.6,\ a_3 = 3.9,\ a_4 = 4.2,\ a_5 = 4.5$

$a_6 = 4.8,\ a_7 = 5.1,\ a_8 = 5.4,\ a_9 = 5.7,\ a_{10} = 6.0$

$b_1 = 4.8,\ b_2 = 5.1,\ b_3 = 5.4,\ b_4 = 5.7,\ b_5 = 6.0$

$b_6 = 6.3,\ b_7 = 6.6,\ b_8 = 6.9,\ b_9 = 7.2,\ b_{10} = 7.5$

$A_\alpha = [4\alpha (3.6 - 3.3) + 3.3, - 4\ \alpha (6.0 - 5.7) + 6.0]$

$= \quad [4\alpha\,(0.3) + 3.3,\; -4\alpha\,(0.3) + 6.0]$

$A_\alpha = [1.2\,\alpha + 3.3,\; -1.2\,\alpha + 6.0]$

$B_\alpha = [4\alpha\,(5.1 - 4.8) + 4.8,\; -4\alpha\,(7.5 - 7.2) + 7.5]$

$= \quad [4\alpha\,(0.3)+4.8,\; -4\alpha\,(0.3) + 7.5]$

$B_\alpha = [1.2\alpha + 4.8,\; -1.2\alpha+ 6.0]$

For $\alpha \in [0, 0.25)$

$A_\alpha = [1.2\alpha + 3.3, -1.2\alpha + 6.0]$

$B_\alpha = [1.2\alpha + 4.8, -1.2\alpha + 7.5]$

$A_\alpha + B_\alpha = [2.4\,\alpha + 8.1, -2.4\,\alpha + 13.5]$

2. $A_\alpha = [4\alpha\,(a_3 - a_2) + 2a_2,\; -a_3, -4\alpha\,(a_9 - a_8) + 2a_9 - a_8]$

$B_\alpha = [4\alpha\,(b_3 - b_2) + 2b_2 - b_3 - 4\alpha\,(b_9 - b_8) + 2b_9 - b_8]$

$A_\alpha = [4\alpha\,(3.9 - 3.6) + 2\,(3.6) - 3.9,$

$-4\alpha\,(5.7 - 5.4) + 2\,(5.7) - 5.4]$

$= \quad [4\alpha\,(0.3) + 7.2 - 3.9, -4\alpha\,(0.3) + 11.4 - 5.4]$

$A_\alpha = [1.2\,\alpha+ 3.3,\; -1.2\,\alpha + 6.0]$

$B_\alpha = [4\alpha(5.4 - 5.1)+2\,(5.1) - 5.4, -4\alpha(7.2 - 6.9)+2\,(7.2) - 6.9]$

$= \quad [4\alpha\,(0.3) + 4.8, -4\alpha\,(0.3) + 7.5]$

$B_\alpha = [1.2\,\alpha + 4.8,\; -1.2\,\alpha + 7.5]$

For $\alpha \in [0.25, 0.5)$

$A_\alpha = [1.2\,\alpha + 3.3,\; -1.2\,\alpha + 6.0]$

$B_\alpha = [1.2\,\alpha + 4.8, -1.2\,\alpha + 7.5]$

$A_\alpha + B_\alpha = [2.4\,\alpha+ 8.1, -2.4\,\alpha + 13.5]$

3. $A_\alpha = [4\alpha\,(a_4 - a_3) + 3a_3 - 2a_4, -4\alpha\,(a_8 - a_7) + 3a_8 - 2a_7]$

$B_\alpha = [4\alpha\,(b_4 - b_3) + 3b_3 - 2b_4, -4\alpha\,(b_8 - b_7) + 3b_8 - 2b_7]$

A_α = [4α (4.2 – 3.9) + 3 (3.9) – 2 (4.2),

- 4α (5.4 - 5.1) + 3 (5.4) – 2(5.1)].

= [4α (0.3) + 3 (3.9) – 2 (4.2), - 4α (0.3) + 3 (5.4) – 2 (5.1)]

A_α = 1.2 α + 3.3, - 1.2 α + 6.0]

B_α = [4α (6.0 – 5.7) + 3 (5.4) – 2 (5.7),

- 4α (6.9 – 6.6) + 3 (6.9) – 2 (6.6)]

= [4α (0.3) + 3 (5.4) – 2(5.7) , - 4α (0.3) + 3 (6.9) – 2 (6.6)]

B_α = [1.2 α + 4.8, - 1.2α + 7.5]

For α ∈ [0.5, 0.75)

A_α = [1.2 α + 3.3, - 1.2 α + 6.0]

B_α = [1.2 α + 4.8 – 1.2 α + 7.5]

$A_\alpha + B_\alpha$ = [2.4 α + 8.1, - 2.4 α + 13.5]

4. A_α = $[4\alpha (a_5 – a_4) + 4a_4 – 3a_5, - 4\alpha (a_7 – a_6) + 4a_7 - 3a_6]$

B_α = $[4\alpha (b_5 – b_4) + 4b_4 – 3b_5, - 4\alpha (b_7 – b_6) + 4b_7 – 3b_6]$

A_α = [4α (4.5 – 4.2) + 4 (4.2) – 3 (4.5) ,

- 4 α (5.1 – 4.8) + 4 (5.1) – 3 (4.8)]

= [4α (0.3) + 4 (4.2) – 3 (4.5), - 4α (0.3) + 4 (5.1) – 3 (4.8)]

A_α = 1.2 α + 3.3, - 1.2 α + 6.0

B_α = [4α (6.0 – 5.7) + 4 (5.7) – 3 (6.0)]

- 4α (6.6 – 6.3) + 4 (6.6) – 3 (6.3)]

= [4 α (0.3)+ 4 (5.7) – 3 (6.0), - 4α (0.3) + 4 (6.6) – 3 (6.3)]

B_α = [1.2 α + 4.8, - 1.2 α + 7.5]

For α ∈ [0.75, 1]

A_α = [1.2 α + 3.3, - 1.2 α + 6.0]

B_α = [1.2 α + 4.8, - 1.2 α + 7.5]

$A_\alpha + B_\alpha = [2.4\,\alpha + 8.1, -2.4\,\alpha + 13.5]$

To verify this new addition operation with ordinary addition operation :

$\tilde{A}_D = (3.3, 3.6, 3.9, 4.2, 4.5, 4.8, 5.1, 5.4, 5.7, 6.0)$

$\tilde{B}_D = (4.8, 5.1, 5.4, 5.7, 6.0, 6.3, 6.6, 6.9, 7.2, 7.5)$

For $\alpha \in [0, 0.25)$

$A_\alpha = [1.2\,\alpha + 3.3, -1.2\,\alpha + 6.0]$

$B_\alpha = [1.2\,\alpha + 4.8, -1.2\,\alpha + 7.5]$

For $\alpha \in [0.25, 0.5)$

$A_\alpha = [1.2\,\alpha + 3.3, -1.2\,\alpha + 6.0]$

$B_\alpha = [1.2\,\alpha + 4.8, -1.2\,\alpha + 7.5]$

$A_\alpha + B_\alpha = [2.4\,\alpha + 8.1, -2.4\,\alpha + 13.5]$

For $\alpha \in [0.5 , 0.75)$

$A_\alpha = [1.2\,\alpha + 3.3, -1.2\,\alpha + 6.0]$

$B_\alpha = [1.2\,\alpha + 4.8, -1.2\,\alpha + 7.5]$

$A_\alpha + B_\alpha = [2.4\,\alpha + 8.1, -2.4\,\alpha + 13.5]$

For $\alpha \in [0.75, 1]$

$A_\alpha = [1.2\,\alpha + 3.3, -1.2\,\alpha + 6.0]$

$B_\alpha = [1.2\,\alpha + 4.8, -1.2\,\alpha + 7.5]$

$A_\alpha + B_\alpha = [2.4\,\alpha + 8.1, -2.4\,\alpha + 13.5]$

As for $\alpha \in [0, 0.25)$, $\alpha \in [0.25, 0.5)$

$\alpha \in [0.5, 0.75)$, $\alpha \in [0.75, 1]$

Arithmetic intervals are same.

$A_\alpha + B_\alpha = [2.4\,\alpha + 8.1, -2.4\,\alpha + 13.5]$ for $\alpha \in [0, 1]$

When $\alpha = 0$,

$$\Rightarrow A_0 + B_0 = [2.4\,(0) + 8.1,\ - 2.4\,(0) + 13.5]$$

$$A_0 + B_0 = [8.1, 13.5]$$

$\alpha = 0.25$,

$$\Rightarrow A_{0.25} + B_{0.25} = [2.4\,(0.25) + 8.1,\ - 2.4\,(0.25) + 13.5]$$

$$A_{0.25} + B_{0.25} = [8.7, 12.9]$$

$\alpha = 0.5$,

$$\Rightarrow A_{0.5} + B_{0.5} = [2.4\,(0.5) + 8.1,\ - 2.4\,(0.5) + 13.5]$$

$$A_{0.5} + B_{0.5} = [9.3, 12.3]$$

$\alpha = 0.75$,

$$\Rightarrow A_{0.75} + B_{0.75} = [2.4\,(0.75) + 8.1,\ - 2.4\,(0.75) + 13.5]$$

$$A_{0.75} + B_{0.75} = [9.9, 11.7]$$

$\alpha = 1$,

$$\Rightarrow A_1 + B_1 = [2.4\,(1) + 8.1,\ - 2.4\,(1) + 13.5]$$

$$A_1 + B_1 = [10.5, 11.1]$$

Hence $A_\alpha + B_\alpha = [8.1, 8.7, 9.3, 9.9, 10.5, 11.1, 11.7, 12.3, 12.9, 13.5]$

Hence all the points coincides with the sum of the two decagonal fuzzy number.

Therefore addition of two α - cuts lies within the interval.

Subtraction of two decagonal fuzzy numbers :

Let $\tilde{A}_D = (a_1, a_2, a_3, a_4, a_5\ a_6, a_7, a_8, a_9, a_{10})$

and $\tilde{B}_D = (b_1, b_2, b_3, b_4, b_5\ b_6, b_7, b_8, b_9, b_{10})$ be two decagonal fuzzy numbers for all $\alpha \in [0, 1]$.

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Let us subtract the alpha cuts A_α and B_α of $\tilde{A}_D$ and $\tilde{B}_D$ using interval arithmetic.

$$A_\alpha - B_\alpha = \begin{cases} [4\alpha (a_2 - a_1) + a_1, - 4\alpha (a_{10} - a_9) + a_{10}] - \\ [4\alpha (b_2 - b_1) + b_1, - 4\alpha (b_{10} - b_9) + b_{10}] \text{ for } \alpha \in [0, 0.25) \\ [4\alpha(a_3 - a_2) + 2a_2 - a_3, - 4\alpha (a_9 - a_8) + 2a_9 - a_8] - \\ [4\alpha (b_3 - b_2) + 2b_2 - b_3, - 4\alpha (b_9 - b_8) + 2b_9 - b_8] \\ \qquad \text{for } \alpha \in [0.25, 0.5) \\ [4\alpha (a_4 - a_3) + 3a_3 - 2a_4 , - 4\alpha (a_8 - a_7) + 3a_8 - 2a_7] - \\ [4\alpha (b_4 - b_3) + 3b_3 - 2b_4, - 4\alpha (b_8 - b_7) + 3b_8 - 2b_7] \\ \qquad \text{for } \alpha \in [0.5, 0.75) \\ [4\alpha (a_5 - a_4) + 4a_4 - 3a_5, - 4\alpha (a_7 - a_6) + 4a_7 - 3a_6] - \\ [4\alpha (b_5 - b_4) + 4b_4 - 3b_5, - 4\alpha (b_7 - b_6) + 4 b_7 - 3b_6] \\ \qquad \text{for } \alpha \in [0.75, 1] \end{cases}$$

To verify this new Subtraction operation with ordinary subtraction operation :

$$\tilde{A}_D = (3.3, 3.6, 3.9, 4.2, 4.5, 4.8, 5.1, 5.4, 5.7, 6.0)$$

$$\tilde{B}_D = (4.8, 5.1, 5.4, 5.7, 6.0, 6.3, 6.6, 6.9, 7.2, 7.5)$$

For $\alpha \in [0, 0.25)$

$$A_\alpha = [4\alpha (a_2 - a_1) + a_1, - 4 \alpha (a_{10} - a_9) + a_{10}]$$

$$B_\alpha = [4\alpha (b_2 - b_1) + b_1, - 4\alpha (b_{10} - b_9) + b_{10}]$$

$$A_\alpha = [4\alpha (3.6 - 3.3) + 3.3, - 4\alpha (6.1 - 5.7) + 6.0]$$

$$A_\alpha = [4\alpha (0.3) + 3.3, - 1.2 \alpha - 6.0]$$

$$A_\alpha = [1.2\alpha + 3.3, - 4\alpha (0.3) + 6.0]$$

$$B_\alpha = [4\alpha (5.1 - 4.8) + 4.8, - 4\alpha (7.5 - 7.2) + 7.5]$$

$$B_\alpha = [4\alpha (0.3) + 4.8, - 4\alpha (0.3)+ 7.5]$$

$$B_\alpha = [1.2\alpha + 4.8, - 1.2\alpha + 7.5]$$

For $\alpha \in [0, 0.25)$

A_α = [1.2 α + 3.3, - 1.2 α + 6.0]

B_α = [1.2 α + 4.8, - 1.2 α + 7.5]

A_α - B_α = [- 1.5, - 1.5]

For α ∈ [0.25, 0.5)

A_α = [1.2 α + 3.3, - 1.2 α + 6.0]

B_α = [1.2 α + 4.8, - 1.2 α + 7.5]

A_α - B_α = [- 1.5, - 1.5]

For α ∈ [0.5, 0.75)

A_α = [1.2 α + 3.3, - 1.2 α + 6.0]

B_α = [1.2 α + 4.8, - 1.2 α + 7.5]

A_α - B_α = [- 1.5, - 1.5]

For α ∈ [0.75, 1]

A_α = [1.2 α + 3.3, - 1.2 α + 6.0]

B_α = [1.2 α + 4.8, - 1.2 α + 7.5]

A_α - B_α = [- 1.5, - 1.5]

As For α ∈ [0, 0.25), α ∈ [0.25, 0.5), α ∈ [0.5, 0.75)
α ∈ [0.75, 1] arithmetic intervals are
Same

Therefore A_α - B_α = [2.4 α + 8.1, - 2.4 α + 13.5] for α ∈ [0, 1]

when α = 0, $A_0 – B_0$ = [- 1.5, - 1.5]

when α = 0.25, $A_{0.25} – B_{0.25}$ = [- 1.5, - 1.5]

when α = 0.75, $A_{0.75} – B_{0.75}$ = [- 1.5, - 1.5]

when α = 1, $A_1 – B_1$ = [- 1.5, - 1.5]

Hence A_α - B_α = [- 1.5, - 1.5, - 1.5, - 1.5, - 1.5, - 1.5, - 1.5, - 1.5, - 1.5, - 1.5,] Hence all the points coincides with the difference of the two decagonal fuzzy number.

Therefore subtraction of two α - cuts lies within the intervals.

Multiplication of two decagonal fuzzy number :

Let $\tilde{A}_D = (a_1, a_2, a_3, a_4, a_5\ a_6, a_7, a_8, a_9, a_{10})$

and $\tilde{B}_D = (b_1, b_2, b_3, b_4, b_5\ b_6, b_7, b_8, b_9, b_{10})$ be two decagonal fuzzy numbers for all $\alpha \in [0, 1]$.

Let us multiply the alpha cuts A_α and B_α of $\tilde{A}_D$ and $\tilde{B}_D$ using interval arithmetic.

$$A_\alpha * B_\alpha = \begin{cases} [4\alpha (a_2 - a_1) + a_1, - 4\alpha (a_{10} - a_9) + a_{10}] * \\ [4\alpha (b_2 - b_1) + b_1, - 4\alpha (b_{10} - b_9) + b_{10}] \text{ for } \alpha \in [0, 0.25) \\ [4\alpha(a_3 - a_2) + 2a_2 - a_3, - 4\alpha (a_9 - a_8) + 2a_9 - a_8] * \\ [4\alpha (b_3 - b_2) + 2b_2 - b_3, - 4\alpha (b_9 - b_8) +2b_9 - b_8] \\ \qquad \text{for } \alpha \in [0.25, 0.5) \\ [4\alpha (a_4 - a_3) + 3a_3 - 2a_4 , - 4\alpha (a_8 - a_7) + 3a_8 - 2a_7] * \\ [4\alpha (b_4 - b_3) + 3b_3 - 2b_4, - 4\alpha (b_8 - b_7) + 3b_8 - 2b_7] \\ \qquad \text{for } \alpha \in [0.5, 0.75) \\ [4\alpha (a_5 - a_4) + 4a_4 - 3a_5, - 4\alpha (a_7 - a_6) + 4a_7 - 3a_6] * \\ [4\alpha (b_5 - b_4) + 4b_5 - 3b_5, - 4\alpha (b_7 - b_6) + 4b_7 - 3b_6] \\ \qquad \text{for } \alpha \in [0.75, 1] \end{cases}$$

To verify the new multiplication operation with ordinary multiplication operation :

$\tilde{A}_D = (3.3, 3.6, 3.9, 4.2, 4.5, 4.8, 5.1, 5.4, 5.7, 6.0)$

$\tilde{B}_D = (4.8, 5.1, 5.4, 5.7, 6.0, 6.3, 6.6, 6.9, 7.2, 7.5)$

For $\alpha \in [0, 0.25)$

$A_\alpha = [4\alpha (a_2 - a_1) + a_1, - 4 \alpha (a_{10} - a_9) + a_{10}]$

$B_\alpha = [4\alpha (b_2 - b_1) + b_1, - 4\alpha (b_{10} - b_9) + b_{10}]$

$A_\alpha = [4\alpha (3.6 - 3.3) + 3.3, - 4\alpha (6.1 - 5.7) + 6.0]$

$A_\alpha = [4\alpha (0.3) + 3.3, - 4 \alpha (0.3) + 6.0]$

$A_\alpha = [1.2\alpha + 3.3, - 1.2\alpha + 6.0]$

$B_\alpha = [4\alpha (5.1 - 4.8) + 4.8, - 4\alpha (7.5 - 7.2) + 7.5]$

$B_\alpha = [4\alpha (0.3) + 4.8, - 4\alpha (0.3)+ 7.5]$

$B_\alpha = [1.2\alpha + 4.8, - 1.2\alpha + 7.5]$

For $\alpha \in [0, 0.25)$

$A_\alpha = [1.2 \alpha + 3.3, - 1.2 \alpha + 6.0]$

$B_\alpha = [1.2 \alpha + 4.8, - 1.2 \alpha + 7.5]$

$A_\alpha * B_\alpha = [1.44 \alpha + 15.84, 1.44 \alpha + 45]$

For $\alpha \in [0.25, 0.5)$

$A_\alpha = [1.2 \alpha + 3.3, - 1.2 \alpha + 6.0]$

$B_\alpha = [1.2 \alpha + 4.8, - 1.2 \alpha + 7.5]$

$A_\alpha * B_\alpha = [1.44 \alpha + 15.84, 1.44 \alpha + 45]$

For $\alpha \in [0.5, 0.75)$

$A_\alpha = [1.2 \alpha + 3.3, - 1.2 \alpha + 6.0]$

$B_\alpha = [1.2 \alpha + 4.8, - 1.2 \alpha + 7.5]$

$A_\alpha * B_\alpha = [1.44 \alpha + 15.84, 1.44 \alpha + 45]$

For $\alpha \in [0.75, 1]$

$A_\alpha = [1.2 \alpha + 3.3, - 1.2 \alpha + 6.0]$

$B_\alpha = [1.2 \alpha + 4.8, - 1.2 \alpha + 7.5]$

$A_\alpha * B_\alpha = [1.44 \alpha + 15.84, 1.44 \alpha + 45]$

As For $\alpha \in [0, 0.25)$, $\alpha \in [0.25, 0.5)$, $\alpha \in [0.5, 0.75)$

and $\alpha \in [0.75, 1]$ arithmetic intervals are same

Therefore $A_\alpha * B_\alpha = [1.44\,\alpha + 15.84, 1.44\,\alpha + 45]$ for $\alpha \in [0, 1]$

when $\alpha = 0 \Rightarrow$

$A_0 * B_0 = [1.44\,(0) + 15.84, 1.44\,(0) + 45]$

$A_0 * B_0 = [15.84, 45]$

when $\alpha = 0.25 \Rightarrow$

$A_{0.25} * B_{0.25} = [1.44\,(0.25) + 15.84, 1.44\,(0.25) + 45]$

$A_{0.25} * B_{0.25} = [18.36, 41.04]$

when $\alpha = 0.5 \Rightarrow$

$A_{0.5} * B_{0.5} = [1.44\,(0.5) + 15.84, 1.44\,(0.5) + 45]$

$A_{0.5} * B_{0.5} = [21.06, 37.26]$

when $\alpha = 0.75 \Rightarrow$

$A_{0.75} * B_{0.75} = [1.44\,(0.75) + 15.84, 1.44\,(0.75) + 45]$

$A_{0.75} * B_{0.75} = [23.94, 33.66]$

when $\alpha = 1 \Rightarrow$

$A_1 * B_1 = [1.44\,(1) + 15.84, 1.44\,(1) + 45]$

$A_1 * B_1 = [27, 30.43]$

Hence $A_\alpha * B_\alpha = [15.84, 18.36, 21.06, 23.94, 27, 30.43, 33.66, 37.26, 41.04, 45]$

Hence all the points coincides with the multiply of two decagonal fuzzy number.

Therefore multiplication of two α - cuts lies within the intervals.

Symmetric image :

If $\tilde{A}_D = (a_1, a_2, a_3, a_4, a_5\ a_6, a_7, a_8, a_9, a_{10})$ is the decagonal fuzzy number then $-\tilde{A}_D = (-a_{10}, -a_9, -a_8, -a_7, -a_6, -a_5, -a_4, -a_3, -a_2, -a_1)$ which in the symmetric image of $\tilde{A}_D$ is also an decagonal fuzzy number.

Example :

If $\tilde{A}_D = (3.3, 3.6, 3.9, 4.2, 4.5, 4.8, 5.1, 5.4, 5.7, 6.0)$

Then

$-\tilde{A}_D = (-6.0, -5.7, -5.4, -5.1, -4.8, -4.5, -4.2, -3.9, -3.6, -3.3)$

which is again an decagonal fuzzy number.

Dividing of two decagonal fuzzy number :

Let $\tilde{A}_D = (a_1, a_2, a_3, a_4, a_5\ a_6, a_7, a_8, a_9, a_{10})$

and $\tilde{B}_D = (b_1, b_2, b_3, b_4, b_5\ b_6, b_7, b_8, b_9, b_{10})$ be two decagonal fuzzy numbers for all $\alpha \in [0, 1]$.

Let us divided the alpha cuts A_α and B_α of $\tilde{A}_D$ and $\tilde{B}_D$ using interval arithmetic.

$$A_\alpha \div B_\alpha = \begin{cases} [4\alpha\,(a_2 - a_1) + a_1,\ - 4\alpha\,(a_{10} - a_9) + a_{10}] \div \\ [4\alpha\,(b_2 - b_1) + b_1,\ - 4\alpha\,(b_{10} - b_9) + b_{10}] \text{ for } \alpha \in [0, 0.25) \\ [4\alpha(a_3 - a_2) + 2a_2 - a_3,\ - 4\alpha\,(a_9 - a_8) + 2a_9 - a_8] \div \\ [4\alpha\,(b_3 - b_2) + 2b_2 - b_3,\ - 4\alpha\,(b_9 - b_8) + 2b_9 - b_8] \\ \qquad \text{for } \alpha \in [0.25, 0.5) \\ [4\alpha\,(a_4 - a_3) + 3a_3 - 2a_4,\ - 4\alpha\,(a_8 - a_7) + 3a_8 - 2a_7] \div \\ [4\alpha\,(b_4 - b_3) + 3b_3 - 2b_4,\ - 4\alpha\,(b_8 - b_7) + 3b_8 - 2b_7] \\ \qquad \text{for } \alpha \in [0.5, 0.75) \\ [4\alpha\,(a_5 - a_4) + 4a_4 - 3a_5,\ - 4\alpha\,(a_7 - a_6) + 4a_7 - 3a_6] \div \\ [4\alpha\,(b_5 - b_4) + 4b_4 - 3b_5,\ - 4\alpha\,(b_7 - b_6) + 4b_7 - 3b_6] \\ \qquad \text{for } \alpha \in [0.75, 1] \end{cases}$$

To verify this new division operation with ordinary division operation :

$\tilde{A}_D$ = (3.3, 3.6, 3.9, 4.2, 4.5, 4.8, 5.1, 5.4, 5.7, 6.0)

$\tilde{B}_D$ = (4.8, 5.1, 5.4, 5.7, 6.0, 6.3, 6.6, 6.9, 7.2, 7.5)

For $\alpha \in [0, 0.25)$

$A_\alpha = [4\alpha\,(a_2 - a_1) + a_1,\ - 4\,\alpha\,(a_{10} - a_9) + a_{10}]$

$B_\alpha = [4\alpha\,(b_2 - b_1) + b_1,\ - 4\alpha\,(b_{10} - b_9) + b_{10}]$

$A_\alpha = [4\alpha\,(3.6 - 3.3) + 3.3,\ - 4\alpha\,(6.0 - 5.7) + 6.0]$

$A_\alpha = [4\alpha\,(0.3) + 3.3,\ - 4\,\alpha\,(0.3) + 6.0]$

$A_\alpha = [1.2\alpha + 3.3,\ - 1.2\alpha + 6.0]$

$B_\alpha = [4\alpha\,(5.1 - 4.8) + 4.8,\ - 4\alpha\,(7.5 - 7.2) + 7.5]$

$B_\alpha = [4\alpha\,(0.3) + 4.8,\ - 4\alpha\,(0.3) + 7.5]$

$B_\alpha = [1.2\alpha + 4.8,\ - 1.2\alpha + 7.5]$

For $\alpha \in [0, 0.25)$

$A_\alpha = [1.2\,\alpha + 3.3,\ -1.2\,\alpha + 6.0]$

$B_\alpha = [1.2\,\alpha + 4.8,\ -1.2\,\alpha + 7.5]$

$A_\alpha \div B_\alpha = \left[\frac{1.2\alpha + 3.3}{1.2\alpha + 4.8},\ \frac{-1.2\alpha + 6.0}{-1.2\alpha + 7.5}\right]$

For $\alpha \in [0.25, 0.5)$

$A_\alpha = [1.2\,\alpha + 3.3,\ -1.2\,\alpha + 6.0]$

$B_\alpha = [1.2\,\alpha + 4.8,\ -1.2\,\alpha + 7.5]$

$A_\alpha \div B_\alpha = \left[\frac{1.2\alpha + 3.3}{1.2\alpha + 4.8},\ \frac{-1.2\alpha + 6.0}{-1.2\alpha + 7.5}\right]$

For $\alpha \in [0.5, 0.75)$

$A_\alpha = [1.2\,\alpha + 3.3,\ -1.2\,\alpha + 6.0]$

$B_\alpha = [1.2\,\alpha + 4.8,\ -1.2\,\alpha + 7.5]$

$A_\alpha \div B_\alpha = \left[\frac{1.2\alpha + 3.3}{1.2\alpha + 4.8},\ \frac{-1.2\alpha + 6.0}{-1.2\alpha + 7.5}\right]$

For $\alpha \in [0.75, 1]$

$A_\alpha = [1.2\,\alpha + 3.3,\ -1.2\,\alpha + 6.0]$

$B_\alpha = [1.2\,\alpha + 4.8,\ -1.2\,\alpha + 7.5]$

$A_\alpha \div B_\alpha = \left[\frac{1.2\alpha + 3.3}{1.2\alpha + 4.8},\ \frac{-1.2\alpha + 6.0}{-1.2\alpha + 7.5}\right]$

As for $\alpha \in [0, 0.25)$, $\alpha \in [0.25, 0.5)$, $\alpha \in [0.5, 0.75)$ and $\alpha \in [0.75, 1]$ arithmetic intervals are same

$A_\alpha \div B_\alpha = \left[\frac{1.2\alpha + 3.3}{1.2\alpha + 4.8},\ \frac{-1.2\alpha + 6.0}{-1.2\alpha + 7.5}\right]$

When $\alpha = 0$

$\Rightarrow A_0 \div B_0 = \left[\frac{1.2\,(0) + 3.3}{1.2\,(0) + 4.8},\ \frac{-1.2\,(0) + 6.0}{-1.2\,(0) + 7.5}\right]$

$$= \left[\frac{3.3}{4.8}, \frac{6.0}{7.5}\right]$$

$$A_0 \div B_0 = 0.6875,\ 0.8$$

When $\alpha = 0.25$

$$\Rightarrow \quad A_{0.25} \div B_{0.25} = \left[\frac{1.2(0.25)+3.3}{1.2(0.25)+4.8}, \frac{-1.2(0.25)+6.0}{-1.2(0.25)+7.5}\right]$$

$$= \left[\frac{3.6}{5.1}, \frac{5.7}{7.2}\right]$$

$$A_{0.25} \div B_{0.25} = [0.705,\ 0.79]$$

When $\alpha = 0.5$

$$\Rightarrow \quad A_{0.5} \div B_{0.5} = \left[\frac{1.2(0.5)+3.3}{1.2(0.5)+4.8}, \frac{-1.2(0.5)+6.0}{-1.2(0.5)+7.5}\right]$$

$$= \left[\frac{3.9}{5.4}, \frac{5.4}{6.9}\right]$$

$$A_{0.5} \div B_{0.5} = [0.722,\ 0.782]$$

When $\alpha = 0.75$

$$\Rightarrow \quad A_{0.75} \div B_{0.75} = \left[\frac{1.2(0.75)+3.3}{1.2(0.75)+4.8}, \frac{-1.2(0.75)+6.0}{-1.2(0.75)+7.5}\right]$$

$$= \left[\frac{4.2}{5.7}, \frac{5.1}{6.6}\right]$$

$$A_{0.75} \div B_{0.75} = [0.736,\ 0.772]$$

When $\alpha = 1$

$$\Rightarrow \quad A_1 \div B_1 = \left[\frac{1.2(1)+3.3}{1.2(1)+4.8}, \frac{-1.2(1)+6.0}{-1.2(1)+7.5}\right]$$

$$= \left[\frac{4.5}{6.0}, \frac{4.8}{6.3}\right]$$

$A_1 \div B_1 \quad = \quad [0.75, 0.761]$

Hence

$A_1 \div B_1 = [0.6875, 0.705, 0.722, 0.736, 0.75, 0.761, 0.772, 0.782, 0.79, 0.8]$

Hence all the points coincides with the divide of the two decagonal fuzzy number. Therefore division of two α - cuts lies within the interval.

CONCLUSIONS

In this paper decagonal Fuzzy number has been newly introduced and the alpha cut operations of arithmetic function principles using addition, subtraction multiplication and division has been fully modified with some conditions and has been explained with numerical examples. In a particular case of the growth rate in bacteria which consists of ten points is difficult to solve using trapezoidal or triangular fuzzy numbers, therefore decagonal fuzzy numbers plays a vital role in solving the problem. It also helps us to solve many optimization problems in future which has ten parameters as in the above case.

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