ISSN 2348 – 7968

Further Results on Square Divisor Cordial Labeling of Graphs

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Abstract

In this paper, the square divisor cordial labeling of switching of a pendent vertex in path P_n, $K_{1,n,n} \cup K_{1,m,m}$, $S(B_{n,n})$, $B_{n,n} \cup K_{1,m,m}$, the graph G obtained by identifying all the apex vertices of m wheels $W_{n_1}, W_{n_2}, \ldots, W_{n_m}$, the graph G obtained by identifying all the apex vertices of m wheels $W_{n_1}, W_{n_2}, \ldots, W_{n_m}$ and an apex vertex of a fan Fan_n and $< K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)} >$ are presented.

Keywords: *Divisor Cordial labeling, Divisor Cordial Graphs, Square Divisor Cordial labeling, Square Divisor Cordial Graphs.*

1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [4]. For standard terminology and notations related to number theory we refer to Burton [2] and graph labeling, we refer to Gallian [3]. In [1], Cahit introduce the concept of cordial labeling of graph. In [7], Varatharajan et al. introduce the concept of divisor cordial labeling of graph. In [5], Murugesan et al introduce a new special type of cordial labeling called square divisor cordial labeling of graph. The square divisor cordial labeling of various types of graph is presented in [6]. The brief summaries of definition which are necessary for the present investigation are provided below.

2. Definitions

Definition :2.1

A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph.

Definition :2.2

A mapping $f : V(G) \rightarrow \{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f. If for an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = | f(u) - f(v) |$. Then $v_f(i)$ = number of vertices of having label i under f and $e_f(i)$ = number of edges of having label i under f^*.

Definition :2.3

A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

Definition :2.4

Let a and b be two integers. If a divides b means that there is a positive integer k such that b = ka. It is denoted by a | b. If a does not divide b, then we denote $a \nmid b$.

Definition :2.5

Let G = (V(G), E(G)) be a simple graph and $f : V(G) \rightarrow \{1,2,...,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if f(u) | f(v) or f(v) | f(u) and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph with a divisor cordial labeling is called a divisor cordial graph.

Definition :2.6

Let G = (V(G), E(G)) be a simple graph and $f : V \rightarrow \{1,2,\ldots,|V|\}$ be a bijection. For each edge uv, assign the label 1 if either $[f(u)]^2 | f(v)$ or $[f(v)]^2 | f(u)$ and the label 0 otherwise. f is called a square divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph with a square divisor cordial labeling is called a square divisor cordial graph.

Definition :2.7

The join $G = G_1 + G_2$ of graphs G_1 and G_2 with disjoint vertex sets V_1 and V_2, and edge sets E_1 and E_2, is the graph union $G_1 \cup G_2$ together with all the edges joining V_1 and V_2. A wheel graph W_n is defined as $K_1 + C_n$, where C_n denotes the cycle with n vertices. The fan graph Fan_n is defined as $K_1 + P_n$, where P_n is the path on n vertices.

Definition :2.8

Bistar $B_{n,n}$ is the graph obtained by joining the center (apex) vertices of two copies of $K_{1,n}$ by an edge.

Definition :2.9

A subdivision of the edge e = uv of a graph G is the replacement of the edge e by a new vertex w and two new edges uw and wv. This operation is also called an elementary subdivision of G.

A graph H obtained by a sequence of elementary subdivisions from a graph G is said to be a subdivision graph of G.

Definition :2.10

A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing all the edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition :2.11

Consider t copies of stars namely $K_{1,n}^{(1)}, K_{1,n}^{(2)}, \ldots, K_{1,n}^{(t)}$. Then $G = < K_{1,n}^{(1)}, K_{1,n}^{(2)}, \ldots, K_{1,n}^{(t)} >$ is the graph obtained by joining apex vertices of each $K_{1,n}^{(m-1)}$ and $K_{1,n}^{(m)}$ to a new vertex x_{m-1} where $2 \le m \le t$ and G has $t(n+2)-1$ vertices and $t(n+2)-2$ edges.

3. Main Results

Theorem 3.1

Switching of a pendent vertex in path P_n is square divisor cordial graph.

Proof.

Let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n.

The graph G is obtained by switching of a pendent vertex in path P_n. v_1 and v_n are pendent vertex of path P_n.

Without loss of generality, let the switched vertex be v_1.

Then $|V(G)| = n$ and $|E(G)| = 2n - 4$.

Define $f : V(G) \rightarrow \{1, 2, 3, \ldots, n\}$ by

$f(v_i) = i, \quad 1 \le i \le n.$

Then $e_f(0) = e_f(1) = n - 2$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.

Hence G is square divisor cordial graph.

Example 3.1

The square divisor cordial labeling of switching of a pendent vertex in path P_6 is given in figure 3.1.

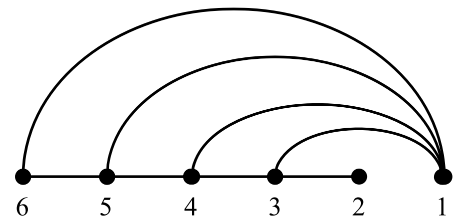

Figure 3.1

Theorem 3.2

The graph $S(B_{n,n})$ is square divisor cordial graph, where $n \ge 2$.

Proof .

Let $B_{n,n}$ be a graph and $v, u, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n$ be the vertices of $B_{n,n}$. Let G be the graph $S(B_{n,n})$. The vertex set $V(G) = \{u, w, v, u_i, v_i, u'_i, v'_i, : 1 \le i \le n\}$ and the edge set $E(G) = \{uw, wv, uu'_i, u'_iu_i, vv'_i, v'_iv_i, : 1 \le i \le n\}$.

Then $|V(G)| = 4n+3$ and $|E(G)| = 4n+2$.

Define vertex labeling $f : V(G) \rightarrow \{1, 2, \ldots, 4n+3\}$ as follows

$f(u) = 1,$

$f(v) = 2,$

$f(w) = 4n+3,$

$f(v_i) = 3+4(i-1), \quad 1 \le i \le n.$

$f(v'_i) = 4+4(i-1), \quad 1 \le i \le n.$

$f(u_i) = 6+4(i-1), \quad 1 \le i \le n.$

$f(u'_i) = 5+4(i-1), \quad 1 \le i \le n.$

Then $e_f(0) = e_f(1) = 2n+1$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.

Hence G is square divisor cordial graph.

Example 3.2

The square divisor cordial labeling of $S(B_{4,4})$ is given in figure 3.2.

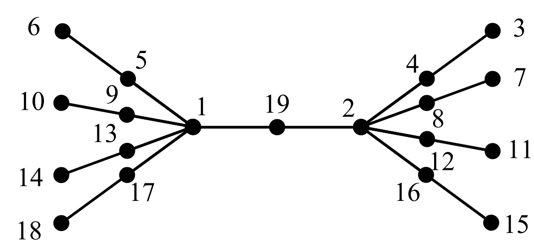

Figure 3.2

Theorem 3.3

The graph $K_{1,n,n} \cup K_{1,m,m}$ is square divisor cordial graph, where $n,m \geq 2$.

Proof .

Let G be a graph $K_{1,n,n} \cup K_{1,m,m}$, $n,m \geq 2$.

Let $v, v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}, \ldots, v_{2n}$ and $u, u_1, u_2, \ldots, u_m, u_{m+1}, u_{m+2}, \ldots, u_{2m}$ be the vertices of $K_{1,n,n}$ and $K_{1,m,m}$ respectively.

Then $|V(G)| = 2n+2m+2$ and $|E(G)| = 2n+2m$.

Consider $n \leq m$. Since $K_{1,n,n} \cup K_{1,m,m}$ is isomorphic to $K_{1,m,m} \cup K_{1,n,n}$.

Define $f : V(G) \rightarrow \{1, 2, 3, \ldots, 2n+2m+2\}$ by

$f(v) = 2$,

$f(u) = 1$.

Case (i) : n = m

$f(v_i) = 4+4(i-1)$, $1 \leq i \leq n$.

$f(v_{n+i}) = 3+4(i-1)$, $1 \leq i \leq n$.

$f(u_i) = 5+4(i-1)$, $1 \leq i \leq n$.

$f(u_{n+i}) = 6+4(i-1)$, $1 \leq i \leq n$.

Then $e_f(0) = e_f(1) = 2n$.

Case (ii) : n < m

$f(v_i) = 4+4(i-1)$, $1 \leq i \leq n$.

$f(v_{n+i}) = 3+4(i-1)$, $1 \leq i \leq n$.

$f(u_i) = 5+4(i-1)$, $1 \leq i \leq n$.

$f(u_{n+i}) = 4n+1+2i$, $1 \leq i \leq m-n$.

$f(u_{m+i}) = 6+4(i-1)$, $1 \leq i \leq n$.

$f(u_{m+n+i}) = 4n+2+2i$, $1 \leq i \leq m-n$.

Then $e_f(0) = e_f(1) = n+m$.

Thus from the above cases, $|e_f(0) - e_f(1)| \leq 1$.

Hence G is square divisor cordial graph.

Example 3.3

The square divisor cordial labeling of $K_{1,4,4} \cup K_{1,6,6}$ is given in figure 3.3.

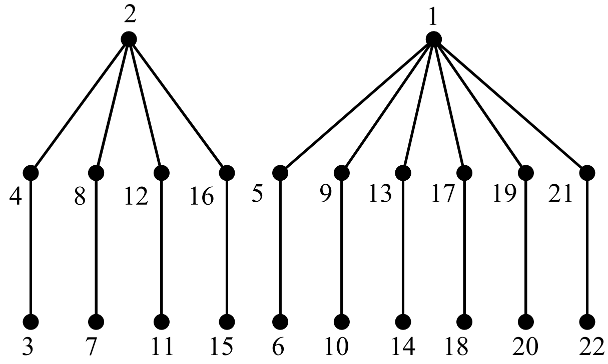

Figure 3.3

Theorem 3.4

The graph $B_{n,n} \cup K_{1,m,m}$ is square divisor cordial graph, where $n,m \geq 2$.

Proof .

Let G be a graph $B_{n,n} \cup K_{1,m,m}$, where $n,m \geq 2$.

Let $u, v, u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n$ and $w, w_1, w_2, \ldots, w_{2m}$ be the vertices of $B_{n,n}$ and $K_{1,m,m}$ respectively.

Then $|V(G)| = 2n+2m+3$ and $|E(G)| = 2n+2m+1$.

Let p be the highest prime number such that $p < 2n+2m+3$.

Define vertex labeling $f : V(G) \rightarrow \{1, 2, ..., 2n+2m+3\}$ as follows

Case (i) : n = m

$f(u) = 4n+3$,

$f(v) = 2$,

$f(u_i) = 6+4(i-1)$, $1 \leq i \leq n$.

$f(v_i) = 4+4(i-1)$, $1 \leq i \leq n$.

$f(w) = 1$,

$f(w_i) = 3+4(i-1)$, $1 \leq i \leq n$.

$f(w_{n+i}) = 5+4(i-1)$, $1 \leq i \leq n$.

Then $e_f(0) = 2n+1$ and $e_f(1) = 2n$.

Case (ii) : n < m

$f(u) = 4n+3$,

$f(v) = 2$,

$f(u_i) = 6+4(i-1)$, $1 \leq i \leq n$.

$f(v_i) = 4+4(i-1)$, $1 \leq i \leq n$.

$f(w) = 1$,

$f(w_i) = 3+4(i-1)$, $1 \leq i \leq n$.

$f(w_{n+i}) = 5+4(i-1)$, $1 \leq i \leq n$.

$f(w_{n+i}) = 4n+2+2i$, $1 \leq i \leq m-n$.

$f(w_{m+n+i}) = 4n+3+2i$, $1 \leq i \leq m-n$.

Then $e_f(0) = n+m+1$ and $e_f(1) = n+m$.

Case (iii) : n > m

$f(u) = 2n+2m+3$,

$f(v) = 1$,

$f(u_i) = 3+4(i-1)$, $1 \leq i \leq m$.

$f(v_i) = 6+4(i-1)$, $1 \leq i \leq m$.

$f(u_{m+i}) = 4m+1+2i$, $1 \leq i \leq n-m$.

$f(v_{m+i}) = 4m+2+2i$, $1 \leq i \leq n-m$.

$f(w) = 2$,

$f(w_i) = 4+4(i-1)$, $1 \leq i \leq m$.

$f(w_{m+i}) = 5+4(i-1)$, $1 \leq i \leq m$.

Then $e_f(0) = n+m+1$ and $e_f(1) = n+m$.

Thus from the above cases, $|e_f(0) - e_f(1)| \leq 1$.

Hence G is square divisor cordial graph.

Example 3.4

The square divisor cordial labeling of $B_{4,4} \cup K_{1,6,6}$ is given in figure 3.4.

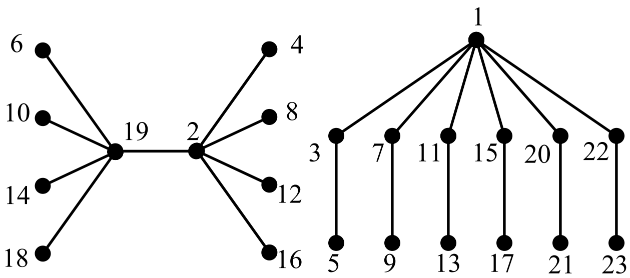

Figure 3.4

Theorem 3.5

The graph G obtained by identifying all the apex vertices of m wheels $W_{n_1}, W_{n_2}, \ldots, W_{n_m}$ is square divisor cordial graph.

Proof.

The graph G obtained by identifying all the apex vertices of m wheels $W_{n_1}, W_{n_2}, \ldots, W_{n_m}$.

Let the common apex vertex of G be u_0 and the consecutive rim vertices of each of the wheels W_{n_i} be $u_{i,1}, u_{i,2}, \ldots, u_{i,n_i}$ for $1 \le i \le m$.

$n_1, n_2, \ldots, n_m$ is the number of rim vertices of $W_{n_1}, W_{n_2}, \ldots, W_{n_m}$ respectively.

Then $|V(G)| = 1+n_1+n_2+\ldots+n_m$ and $|E(G)| = 2(n_1+n_2+\ldots+n_m)$.

Let p_1 be the highest prime number, p_2 be the second highest prime number, … , p_m be the m^{th} highest prime number such that $p_m < \ldots < p_2 < p_1 \le 1+n_1+n_2+\ldots+n_m$.

Define vertex labeling

$f : V(G) \rightarrow \{1,2,3,\ldots, (1+n_1+n_2+\ldots+n_m)\}$ as follows.

$f(u) = 1$,

$f(u_{i,n_i}) = p_{m+1-i} \qquad 1 \le i \le m$.

Then assign the remaining labels to the remaining vertices $u_{i,1}, u_{i,2}, \ldots, u_{i,n_i-1}$ for $1 \le i \le m$.

Since 1 divides any integer, the edges incident to u_0 contribute $n_1+n_2+\ldots+n_m$ to $e_f(1)$.

The label of every pair of adjacent rim vertices in G is either consecutive number or relatively prime to each other.

Therefore, each edges formed by the rim vertices contribute $n_1+n_2+\ldots+n_m$ to $e_f(0)$.

Then $e_f(0) = e_f(1) = n_1+n_2+\ldots+n_m$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.

Hence G is square divisor cordial graph.

Example 3.5

The square divisor cordial labeling of graph G obtained by identifying all the apex vertices of 3 wheels W_4, W_5 and W_4 is given in figure 3.5.

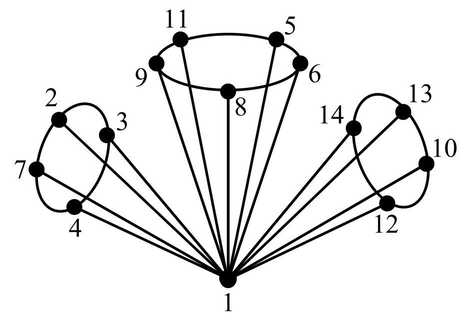

Figure 3.5

Theorem 3.6

The graph G obtained by identifying all the apex vertices of m wheels $W_{n_1}, W_{n_2}, \ldots, W_{n_m}$ and an apex vertex of a fan Fan_n is square divisor cordial graph.

Proof.

The graph G obtained by identifying all the apex vertices of m wheels $W_{n_1}, W_{n_2}, \ldots, W_{n_m}$ and an apex vertex of a fan Fan_n.

Let the common apex vertex of G be u_0 and the consecutive rim vertices of each of the wheels W_{n_i} be $u_{i,1}, u_{i,2}, \ldots, u_{i,n_i}$ for $1 \le i \le m$ and $v_1, v_2, \ldots, v_n$ be the path vertices of the fan Fan_n.

$n_1, n_2, \ldots, n_m$ is the number of rim vertices of $W_{n_1}, W_{n_2}, \ldots, W_{n_m}$ respectively and n is the number path vertices of the fan Fan_n.

Then $|V(G)| = 1+n_1+n_2+\ldots+n_m+n$ and $|E(G)| = 2(n_1+n_2+\ldots+n_m+n) - 1$.

Let p_1 be the highest prime number, p_2 be the second highest prime number, … , p_m be the m^{th} highest prime number such that $p_m < \ldots < p_2 < p_1 \le 1+n_1+n_2+\ldots+n_m+n$.

Define vertex labeling

$f : V(G) \rightarrow \{1,2,3,\ldots, (1+n_1+n_2+\ldots+n_m+n)\}$ as follows.

$f(u) = 1$,

$f(u_{i,n_i}) = p_{m+1-i} \qquad 1 \le i \le m$.

Then assign the remaining labels to the remaining vertices $u_{i,1}, u_{i,2}, \ldots, u_{i,n_i-1}$ for $1 \le i \le m$ and v_i for $1 \le i \le n$.

Since 1 divides any integer, the edges incident to u_0 contribute $n_1+n_2+\ldots+n_m+ n$ to $e_f(1)$.

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The label of every pair of adjacent rim vertices in G is either consecutive number or relatively prime to each other.

Therefore, each edges formed by the rim vertices contribute $n_1+n_2+\ldots+n_m$ to $e_f(0)$ and each edges formed by the path vertices contribute n – 1 to $e_f(0)$.

Then $e_f(0) = n_1+n_2+\ldots+n_m+n-1$ and
$e_f(1) = n_1+n_2+\ldots+n_m+n$.

Therefore, $|e_f(0) - e_f(1)| \leq 1$.
Hence G is square divisor cordial graph.

Example 3.5

The square divisor cordial labeling of graph G obtained by identifying all the apex vertices of 3 wheels W_4, W_5 and W_4 and a fan Fan_5 is given in figure 3.6.

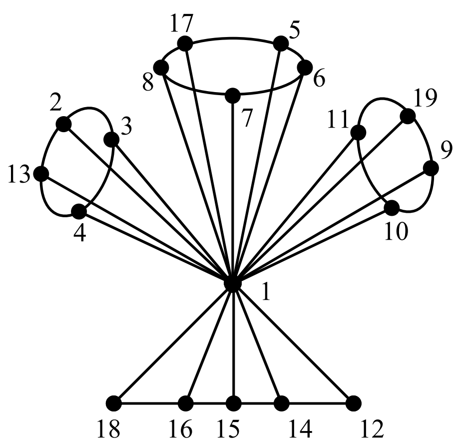

Figure 3.6

Theorem 3.7

The graph $G = <K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)}>$ is square divisor cordial.

Proof.

Let $G = <K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)}>$ be a graph with 3n + 5 vertices and 3n + 4 edges.

Let $v_1^{(i)}, v_2^{(i)}, \ldots, v_n^{(i)}$ be the pendant vertices of $K_{1,n}^{(i)}$ and let c_i be the apex vertex of $K_{1,n}^{(i)}$ for i = 1, 2, 3 .

Here, c_1 and c_2 are adjacent to x_1 and c_2 and c_3 are adjacent to x_2.

Define vertex labeling as follows. : $f : V(G) \rightarrow \{1,2,3,\ldots, 3n+5\}$ as follows.

Assign the label 2 to c_1, 1 to c_2 , 3 to x_1 , p to c_3, where p is the largest prime number such that $p \leq 3n+5$ and the even labels from 4,6,…, 2n+2 to the vertices $v_1^{(1)}, v_2^{(1)}, \ldots, v_n^{(1)}$ respectively.

Then assign the remaining labels to the remaining vertices $v_1^{(2)}, v_2^{(2)}, \ldots, v_n^{(2)}, v_1^{(3)}, v_2^{(3)}, \ldots, v_n^{(3)}$ and x_2 .

Since 1 divides any integer, the edges incident to c_2 contribute n + 2 to $e_f(1)$.

Since p^2 does not divide any labels of the vertices which are adjacent to c_3 and square of the label of adjacent vertices of c_3 does not divide p, the edges incident to c_3 also contribute n + 1 to $e_f(0)$.

Since 4 divides the even labels from 4,6,…, 2n+2 which are multiple of 4 and does not divide the even labels from 4,6,…, 2n+2 which are not multiple of 4, 4 does not divide 3 and 9 does not divide 2, the edges incident to c_1 contribute $e_f(1) = \frac{n}{2}$ and $e_f(0) = \frac{n+2}{2}$, if n is even and $e_f(1) = \frac{n+1}{2}$ and $e_f(1) = \frac{n+1}{2}$, if n is odd.

Thus, $e_f(0) = e_f(1) = \frac{3n+4}{2}$, if n is even.

$e_f(0) = \frac{3n+3}{2}$ and $e_f(1) = \frac{3n+5}{2}$, if n is odd.

Therefore $|e_f(0) - e_f(1)| \leq 1$.

Hence, G is square divisor cordial.

Example 3.7

The square divisor cordial labeling of $G = <K_{1,5}^{(1)}, K_{1,5}^{(1)}, K_{1,5}^{(1)}>$ is given in figure 3.7.

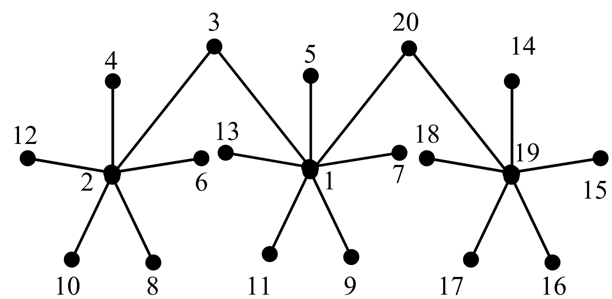

Figure 3.7

4. Conclusions

In this paper, we prove the square divisor cordial labeling of switching of a pendent vertex in path P_n, $S(B_{n,n})$, $K_{1,n,n} \cup K_{1,m,m}$, $B_{n,n} \cup K_{1,m,m}$, the graph G obtained by identifying all the apex vertices of m wheels $W_{n_1}, W_{n_2}, \ldots, W_{n_m}$, the graph G obtained by identifying

ISSN 2348 – 7968

all the apex vertices of m wheels W_{n_1}, W_{n_2}, …, W_{n_m} and an apex vertex of a fan Fan_n and $< K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)} >$.

References

[1] I. Cahit, "Cordial graphs: A weaker version of graceful and harmonious graphs", Ars Combinatoria, Vol 23, 1987, pp. 201-207.
[2] David M. Burton, Elementary Number Theory, Second Edition, Wm. C. Brown Company Publishers, 1980.
[3] J. A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics, 16, # DS6, 2013.
[4] F. Harary, Graph Theory, Addison-Wesley, Reading, Mass, 1972.
[5] S. Murugesan, D. Jayaraman and J. Shiama, "Square Divisor Cordial Graphs", International Journal of Computer Applications, Vol 64, No. 22, 2013, pp.1-4.
[6] S. K. Vaidya and N. H. Shah, On Square Divisor Cordial Graphs, J. Sci. Res. Vol 6, No. 3, 2014, pp. 445-455.
[7] R. Varatharajan, S. Navanaeethakrishnan and K. Nagarajan, "Divisor cordial graphs", International J. Math. Combin., Vol 4, 2011, pp. 15-25.