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Solution of Fuzzy Game Problem Using Triangular Fuzzy Number

R. Senthil Kumar, S. Kumaraghuru
CMS College of Science &Commerce, Coimbatore
Chikkanna Government Arts College, Tirupur

Abstract

Fuzzy Game has been applied in many fields such as Operations Research, Control Theory and Management Sciences etc. In this paper, we consider some operations of triangular fuzzy numbers and a solution of Fuzzy Game Problem with triangular fuzzy numbers. The Solution of such Fuzzy games with pure strategies by minimax-maximin principle is discussed.

Keywords – Fuzzy sets, Symmetric Triangular Fuzzy numbers, Fuzzy Game problem, Fuzzy ranking, Fuzzy arithmetic.

1. Introduction.

Game theory is a mathematical theory that deals with the general features of competitive situations like some operations of triangular fuzzy numbers in a formal abstract way. It places particular emphasis on the decision making processes of the adversaries. Further, it is usually used when two or more individuals or organisations with conflicting objectives try to make decisions.

This paper will provide pointers to our algorithms, solution of fuzzy game problem with triangular fuzzy number, ranking of triangular fuzzy number based on r cut procedure. The focus in this paper is on the minimax principle which states that each competitor will act so as to minimize one player's maximum loss (or maximize one player's minimum gain). Furthermore, even if the opponent is able to use only his knowledge of the tendencies of the first player to deduce probabilities that are different from those for the optimal mixed strategy , then the opponent still can take advantage for this knowledge to reduce the expected payoff to the first player . We have analyzed the solution of such fuzzy games with pure strategies by minimax – maximin principle.

2. PRELIMINARIES

The aim of this section is to present some notations, notions and results which are useful in our further consideration.

2.1 Fuzzy numbers.

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_A : R \to [\ 0\ ,\ 1\ ]$ has the following characteristics.

(i) A is normal. It means that there exists an $x \in R$ such that $\mu_A (x) = 1$.

(ii) A is convex. It means that for every $x_1, x_2 \in R$, $\mu_A (\lambda x_1 + (1 - \lambda) x_2) \geq \min\{ \mu_A (x_1) , \mu_A (x_2) \}, \lambda \in [0 , 1]$

(iii) μ_A is upper semi-continuous.

(iv) sup (A) is bounded in R.

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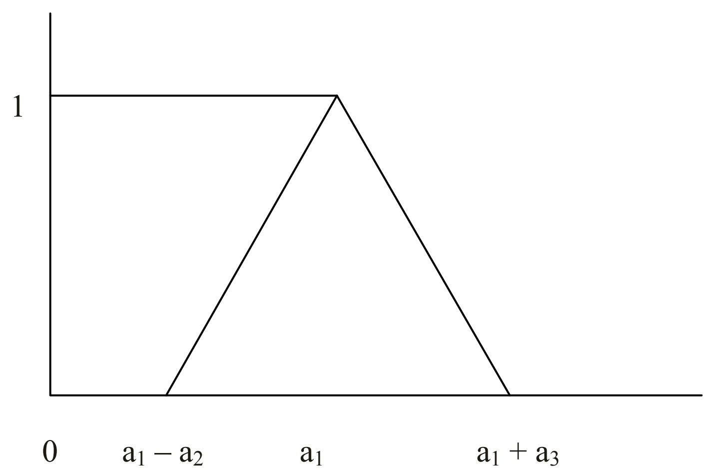
2.2 Triangular Fuzzy number

A fuzzy number A in R is said to be a triangular fuzzy number if its membership function $\mu_A : R \rightarrow [0,1]$ has the following characteristics.

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1 \quad , & x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0 \quad , & otherwise \end{cases}$$

It is denoted by $A = (a_1, a_2, a_3)$ where a_1 is Core (A), a_2 is left width and a_3 is right width. The geometric representation of triangular fuzzy number is shown in figure. The shape of the triangular fuzzy number A is usually in the form of triangle and hence it is called so.



Membership function of triangular fuzzy number

The Parametric form of a triangular fuzzy number is represented by $A = [a_1 – a_2 (1 – r) , a_1 + a_3 (1 – r)]$.

3. Ranking of Triangular Fuzzy number

Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function based on their graded means. That is, for every $A = (a_1 , a_2 , a_3) \in F(R)$, the ranking function $\Re: F(R) \rightarrow R$ by graded mean is defined as

$$\Re(A) = \left(\frac{a_1 + 4a_2 + a_3}{6} \right)$$

For any two triangular fuzzy numbers $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ in F(R), we have the following comparison.

(i) A< B If and only if R(A) < R(B).

(ii) A> B If and only if R(A) > R(B).

(iii) A≈B If and only if R(A) = R(B).

(iv) A–B = 0 if and only if R(A) - R(B) = 0.

(v) If R(A) >0 then A >0

(vi) If R(A)=0 then A≈0

4. Arithmetic operations of triangular fuzzy number based on r-cut

The fuzzy number is fully and uniquely represented by its r-cut, since the r-cut of each fuzzy number are closed interval of real numbers for all $r\in(0,1]$. This enables us to define arithmetic operations on fuzzy number in terms of arithmetic operations on their r-cut.

Let A and B be arbitrary fuzzy numbers with the r-cut $A=\left[\underline{A}(r),\overline{A}(r)\right]$ and $B=\left[\underline{B}(r),\overline{B}(r)\right]$. Then the arithmetic operations between A and B are denoted by

(i) $A+B=\left[\underline{A}(r)+\underline{B}(r),\overline{A}(r)+\overline{B}(r)\right]$

(ii) $A-B=\left[\underline{A}(r)-\overline{B}(r),\overline{A}(r)-\underline{B}(r)\right]$

(iii) $AB=H=\left[\underline{H}(r),\overline{H}(r)\right]$

$$\text{Where } \underline{H}(r)=\min\{\underline{A}(r)\underline{B}(r),\overline{A}(r)\overline{B}(r),\overline{A}(r)\underline{B}(r),\underline{A}(r)\overline{B}(r)\}$$

$$\overline{H}(r)=\max\{\underline{A}(r)\underline{B}(r),\overline{A}(r)\overline{B}(r),\overline{A}(r)\underline{B}(r),\underline{A}(r)\overline{B}(r)\}$$

(iv) $A/B=H=\left[\underline{H}(r),\overline{H}(r)\right]$

$$\text{Where } \underline{H}(r)=\min\{\underline{A}(r)/\underline{B}(r),\overline{A}(r)/\overline{B}(r),\overline{A}(r)/\underline{B}(r),\underline{A}(r)/\overline{B}(r)\}$$

$$\overline{H}(r)=\max\{\underline{A}(r)/\underline{B}(r),\overline{A}(r)/\overline{B}(r),\overline{A}(r)/\underline{B}(r),\underline{A}(r)/\overline{B}(r)\}$$

(v) $KA=\begin{cases}\left[K\underline{A}(r),K\overline{A}(r)\right], & \text{if } K\geq 0\\ \left[K\overline{A}(r),K\underline{A}(r)\right], & \text{if } K<0\end{cases}$

Player B				
		B_1	.....	B_n
Player A	A_1	a_{11}	......	a_{1n}
	...	....	......	......
	A_m	a_{m1}	......	a_{mn}

5. Mathematical formulation of a fuzzy game problem.

Let Player A have m strategies A_1, A_2, ...A_m and Player B have n strategies B_1,B_2, ...B_n . Here, it is assumed that each player has his choices from amongst the pure strategies. Also it is assumed that player A is always the gainer and player B is always the loser. That is, all payoff are assumed in terms of player A. Let a_{ij} be the payoff which player A gains from player B if player A chooses strategy A_i and player B chooses strategy B_j. Then the payoff matrix to player A is:

6. Procedure for solving Fuzzy Game Problem

We shall present a solution to fuzzy game problem involving strategies of the players using triangular fuzzy numbers.

Step 1: Check whether a saddle point exists in the problem. If it exists, the solution can be obtained directly. If the saddle point does not exist, go to the next step.

Step 2: Comparison of column strategies.

(a). If elements of Column A $\leq$ elements of Column B, Column A strategy dominates over column B strategy. Hence delete column B strategy from the pay off matrix.

(b). Compare each column strategy with all possible column strategies and delete inferior strategies as for as possible.

Step 3: Comparison of row strategies.

(a). If elements of Row A ≥ elements of Row B, Row A strategy dominates over Row B strategy. Hence delete Row B strategy from the pay off matrix.

(b). Compare each row strategy with all possible row strategies and delete inferior strategies as for as possible.

(c). The Game may reduce to a single cell giving information about the value of the game and optimal strategies of players. If not go to step 4.

Step 4: Dominance need not to be based on the superiority of pure strategies only. A given strategy can be dominated if it is inferior to an average of two or more other pure strategies.

Remark

When the value of the game is zero, it is called as a fair game. That is, the gain of the first player and loss of the second player will be equal to zero.

7. Numerical Examples

Consider the following fuzzy game problem

Convert the given fuzzy problem into a crisp value problem by using the measure.

Rowmin

Rowmin		B_1	B_2
3.17	A_1	3.17	3.17
6	A_2	6.83	6

Column max 6.83 6

$\underline{V} = \max(3.17, 6) = 6$

$\overline{V} = \min(6.83, 6) = 6$

Hence value of the game V= 6.

8. Conclusion

In this paper a simple method of solving fuzzy game problem (strategies of the players are all triangular fuzzy numbers) were introduced by using ranking of fuzzy numbers.

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AUTHOR DETAILS

(1) R. SENTHIL KUMAR is working as Assistant Professor in Department of Mathematics, CMS College of Science & Commerce, Chinnavedampatty, Coimbatore, Tamilnadu, India.

(2) Dr.S.KUMARAGHURU is working as Assistant Professor in Mathematics, PG and Research Department of Mathematics, Chikkanna Government Arts College, Tirupur, Tamilnadu, India.