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A Fermionic ITO Product Formula

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Abstract

We prove an Ito product formula for stochastic integral on Fermion Fock space by analogy with the same kind of integral on Boson Fock space. Because of the noncommutativity relation, the appropriate theory of stochastic intregration must distinguish between the left and right integral. We have four such possibilities.

Keywords: *stochastic integrals, Fermion Fock space,C*-algebra,inner product.*

1. Introduction

First we construct a stochastic integral on Fermion Fock space [7][2] by analogy with the same kind of integral on Boson Fock space [4], first of simple processes. We define the Fermion stochastic integral for square-integrable integrands. We present the infinitesimal operators like infinite sums, but we assume they are continuous. Because of the canonical anticommutation relation we have left, right and mixt stochastic integrals[1].
We recall, giving only the necessary details some concepts and notations.

Definition 1.1. Let F,G,H be simple processes and write

$$F=\sum_{n=0}^{\infty} F_n \chi\left[t_n,t_{n+1}\right),$$

$$G=\sum_{n=0}^{\infty} G_n \chi\left[t_n,t_{n+1}\right), H=\sum_{n=0}^{\infty} H_n\left[t_n,t_{n+1}\right)$$

$$0=t_0<t_1<\cdots<t_n\to\infty$$

The family of operators $M=\left(M(t),t\ge 0\right)$ with $D\left(M(t)\right)=h_0\otimes\varepsilon_t\otimes h^t$ defined by $M(0)=0$,

$$M(t)=M(t_n)+\left(A_L^+(t)-A_L^+(t_n)\right)F_n$$
$$+G_n\left(A_L(t)-A_L(t_n)\right)+(t-t_n)H_n$$

for $t_n<t<t_{n+1}$ is called right stochastic integral of (F,G,H) and is denoted by:

$$M(t)=\int_0^t dA_L^+F+GdA_L+Hds$$

Similarly we can define the left stochastic integral of (F,G,H) denoted by

$$N(t)=N(t_n)+\left(A_L^+(t)-A_L^+(t_n)\right)F_n$$
$$+\left(A_L(t)-A_L(t_n)\right)G_n+(t-t_n)H_n$$

For $t_n<t\le t_{n+1}$

$$N(t)=\int_0^t dA_L^+F+dA_LG+Hds$$

And the mixt stochastic integral of (F,G,H) denoted by:

$$P_1(t)=\int_0^t\left(FdA_L^+ +dA_LG+Hds\right) \text{ or}$$

$$P_2(t)=\int_0^t\left(dA_L^+F+GdA_L+Hds\right)$$

Theorem 1.2. Let F,G,H be simple processes and we assume that F has a parity ε which means that all $F(t)$ have the parity ε and let be

$$dM=dA_L^+F+GdA_L+Hdt$$

If $u,v\in h_0$ $\quad x=x_1\wedge\cdots\wedge x_r \quad x_1,\dots x_r\in H$

$$y=y_1\wedge\cdots\wedge y_w \quad y_1,\dots y_w\in H$$

Then:

$$\langle M(t)(u\otimes x), v\otimes y\rangle = \langle M(0)(u\otimes x), v\otimes y\rangle +$$
$$+\int_0^t \sum_{p\in J}\sum_{k=1}^{w}(-1)^{k-r-w+\varepsilon(F)}\left\langle (L_p\theta\otimes 1)F(\theta u\otimes x), v\otimes y_{ck}\right\rangle y_{kp}+$$
$$+\int_0^t \sum_{p\in J}\sum_{j=1}^{r}(-1)^{j-1}\left\langle G(L_p^+\otimes x_{Cj}), v\otimes y\right\rangle x_{jp}$$
$$+\int_0^t \langle H(u\otimes x), v\otimes y\rangle$$

Proof: see in [3].

Theorem 1.3. Let F,G,H be simple processes, $dM = dA_L^+F + GdA_L + Hdt$

For $u\in h_0$, $x = x_1\wedge\cdots\wedge x_r$ $x_1,\dots x_r\in H$ and $v\in h_s$, $y = y_1\wedge\cdots\wedge y_w$ $y_1,\dots y_w\in H^s$

Then:

$$\langle (M(t)-M(s))(u\otimes x), v\otimes y\rangle =$$
$$\int_0^t \sum_{p\in J}\sum_{k=1}^{w}(-1)^{k-r-w+\varepsilon(F)}\left\langle (L_p\theta\otimes 1)F(\theta u\otimes x), v\otimes y_{ck}\right\rangle y_{kp}+$$
$$+\int_0^t \sum_{p\in J}\sum_{j=1}^{r}(-1)^{j-1}\left\langle G(L_p^+\otimes x_{Cj}), v\otimes y\right\rangle x_{jp}$$
$$+\int_0^t \langle H(u\otimes x), v\otimes y\rangle$$

We generalize the relations of theorem 1.2. as :

$$v\otimes(y_1\wedge\cdots\wedge y_w) = v'\otimes(y_{b+1}\wedge\cdots\wedge y_w)$$
$$= v'\otimes(y_1'\wedge\cdots\wedge y_{w-b}')$$

with $v' = v\otimes(y_1\wedge\cdots\wedge y_b)$, hence we replace k with $k-b$ and w with $w-b$ but $y_1,\cdots,y_w\in H^S$ and $v\in\varepsilon_s$. Proof: see in [3].

2. Operator's parity

Definition 2.1. We assume that h is Z_2 – graded with even and odd subspaces h_+, h_- and we denote by θ the parity operator, that is the self – adjoint unitary operator wich is 1 on h_+ and –1 on h_-.

An operator $T: k\subset h\to h$ is said to be even if $\theta T\theta = T\,\theta$ and odd if $\theta T\theta = -T$ and T has the **parity** $\varepsilon = 0$ sau $\varepsilon = 1$ if $\theta T\theta = (-1)^{\varepsilon}T$

Let H be a Hilbert space. We define the antisymmetric Fock space $\Gamma_a(H)$ over H as the linear hull of all $x_1\wedge x_2\wedge\ldots\wedge x_n, n\geq 0,\ x_i\in H$ (where for $n=0$, we have the unit element, namely 1) with the following inner product:

$$\langle x_1\wedge x_2\wedge\ldots\wedge x_n, y_1\wedge y_2\wedge\ldots\wedge y_k$$
$$=\delta_{n,k}\det\left(\langle x_i, y_i\rangle\right)_{i,j=1,\dots,n}$$

for $n=k=0$ the determinant is considered to be 1).

About this space we mention the following:

i. $\Gamma_a(H) = \bigoplus_{n\geq 0} H_\wedge^n$, where $H_\wedge^n$ is the closed linear hull of all $x_1\wedge x_2\wedge\ldots\wedge x_n,\ x_i\in H$

ii. $x_{y(1)}\wedge\ldots\wedge x_{y(n)} = \varepsilon(\gamma)x_{y(1)}\wedge\ldots\wedge x_{y(n)}$ where $\varepsilon(\gamma)$ is 1 or -1 if γ is even or odd.

If two x_i with different indexes i are equal this product is null.

iii. $\Gamma_a(H) = \left\{\sum_{n\geq 0} x_n \,\middle|\, x_n\in H_\wedge^n, \{n | x_n\neq 0\}\ finite\right\}$ is an associative algebra, with unit element 1 and $x_1\wedge\ldots\wedge x_n$ is the product of $x_1,\dots,x_n,\ x_i\in H = H_\wedge^1$, in established order.

iv. If $H\subset K$, then $\Gamma_a(H)\subset\Gamma_a(K)$.

Let be $\Gamma_a(H) = \Gamma_a(H)_+\oplus\Gamma_a(H)_-$

$$\Gamma_a(H)_+ = \bigoplus_{n\ even} H_\wedge^{\,n}$$
$$\Gamma_a(H)_- = \bigoplus_{n\ odd} H_\wedge^{\,n}$$

We denote the parity operator on $\Gamma_a(H)$ by I and

$$I(x_1\wedge\cdots\wedge x_r) = (-1)^r x_1\wedge\cdots\wedge x_r$$

Remark The operators $A(v)$ and $A^+(v)$ have the parity 1 on h.
We can write:

$$A^+(v) = 1 \otimes A^+(v)$$
$$L = L \otimes 1$$

And if $h = h_s \otimes h^s = h_0 \oplus \Gamma_a(H_s) \otimes h^s$, we can decompose

$$A_L^+(t) - A_L^+(s) = \sum_{p \in j} L_p \otimes I \otimes A^+\left(\chi_{[s,t]} e_p\right)$$

respectively

$$A_L(t) - A_L(s) = \sum_{p \in j} L_p^+ \otimes I \otimes A^+\left(\chi_{[s,t]} e_p\right)$$

3. An ITO Product Formula

Theorem 3.1. Let F, G, H be simple processes and $dM = dA_L^+ + GdA_L + Hdt$. We assume that all L have the same parity.
For $u, v \in h_t$, $x = x_1 \wedge \cdots \wedge x_n$ $x_i \in H^t$,
$y = y_1 \wedge \cdots \wedge y_w$ $y_i \in H^t$ we have:

$$\langle M(t)(u \otimes x), M(t)(v \otimes y)\rangle$$
$$= \langle M(0)(u \otimes x), M(0)(v \otimes y)\rangle$$
$$+ \int_0^t \Bigg(\sum_{p \in J} \langle L_p F(u \otimes x), L_p F(v \otimes y)\rangle$$
$$+ \sum_{p \in J} \sum_{k=1}^{w} \sum_{\phi \neq M} \Big(\langle M(a)(u \otimes x_{1,k,\phi}), \beta_p(\phi)(v \otimes y_{2,k,\phi})\rangle$$
$$+ \langle \beta_p(\phi)(u \otimes x_{2,k,\phi}), M(a)(v \otimes y_{1,k,\phi})\rangle \Big) \Bigg) da$$

where $\beta_p(\square)$ and $x_{i,k,\square}$ will be specificated in the followings.
Proof: Let be $u, v \in h_0$

$$x = x' \wedge x'' \quad x' = x_1 \wedge \cdots \wedge x_a \in H_S$$
$$x'' = x_{a+1} \wedge \cdots \wedge x_n \in H^S$$
$$y = y' \wedge y'' \quad y' = y_1 \wedge \cdots \wedge y_a \in H_S$$
$$y'' = y_{b+1} \wedge \cdots \wedge y_n \in H^S$$

Using the theorem 1.3. we have :

1)

$$<(A_L^+(t) - A_L^+(s))F(u \otimes x), (A_L^+(t) - A_L^+(s))F(v \otimes y)> =$$
$$= \left\langle \sum_{p \in J} \left(L_q \otimes I_{Hs}\left(\chi_{[s,t]} e_q \wedge\right)\right)(F_1 \otimes 1)(u \otimes x' \otimes x'') \right\rangle =$$
$$= \sum_{p \in J} \langle L_p F_1(u \otimes x), L_q F_1(v \otimes y)\rangle (t-s) +$$
$$+ \sum_{\substack{p,q \in J \\ k=b+1 \\ j=a+1}}^{w} \langle L_p F_1(u \otimes x'), L_q F_1(v \otimes y')\rangle$$
$$(-1)^{j-a-1} \left(\int_s^t \overline{y_{k,p}} \right) \left(\int_s^t x_{jq} \right) \langle x''_{Cj}, y''_{Ck}\rangle$$

We remark that $\langle x''_{Cj}, y''_{Ck}\rangle \neq 0$ implies $r - a - 1 = w - b - 1$ hence $a + b = r + w$.

Hence we have:

$$\sum_{p \in J} \langle L_p F_1(u \otimes x), L_q F_1(v \otimes y)\rangle (t-s) +$$
$$+ \sum_{\substack{p,q \in J \\ k=b+1 \\ j=a+1}}^{w} \langle L_p F(u \otimes x_{Cj}), L_q F(v \otimes y_{Ck})\rangle$$
$$(-1)^{r+w+k+j-1} \left(\int_s^t \overline{y_{k,p}} \right) \left(\int_s^t x_{jq} \right) =$$
$$= \sum_{p \in J} \langle L_p F_1(u \otimes x), L_q F_1(v \otimes y)\rangle (t-s) +$$
$$+ \sum_{\substack{p \in J \\ k=b+1}}^{w} (-1)^k \left(\int_s^t \overline{y_{k,p}}(a) \right)$$
$$\sum_{\substack{q \in J \\ j=a+1}}^{r} \langle L_p F(u \otimes x_{Cj}), L_q F(v \otimes y_{Ck})\rangle (-1)^{r+w+k+j-1} \left(\int_s^a x_{jq} \right) +$$
$$+ \sum_{q \in J} (-1)^{r+w+j} \int_s^a x_{jq}(a)$$
$$\sum_{p,k} \langle L_p F(u \otimes x_{Cj}), L_q F(v \otimes y_{Ck})\rangle (-1)^{k-1} \left(\int_s^t \overline{y_{k,p}} \right) da$$

Using the dominated convergence theorem we have :

$$\sum_{j,k}(-1)^{r+w+j+k-1}\int_s^t < \sum_{p\in J'}(\int_s^a \overline{y_{kp}}(b)L_pF(u\otimes x_{Cj}),$$

$$,\sum_{p\in J''}\overline{x_{jq}}(a)L_qF(v\otimes y_{Ck})da >$$

The sums $\sum_{j,k}$ and $\sum_{p\in J'},\sum_{q\in J''}$ are finite.

Now $J'\uparrow J$, $J''\uparrow J$ and the integrand $\sum_{q\in J'}\overline{x_{jq}}(a)L_q$ converges to $\sum_{q\in J}\overline{x_{jq}}(a)L_q$.

$$| < \sum_{p\in J}\left(\int_s^a \overline{y_{kp}}\right)L_pF(u\otimes x_{Cj}),\sum_{q\in J}\overline{x_{jq}}(a)L_qF(v\otimes y_{Ck}) > da \,| \le$$

$$\le \left\|\sum_{p\in J}\left(\int_s^a y_{kp}\right)L_p\right\| \|F\|_{x_{Cj}}\|u\| \left\|\sum_{q\in J}\overline{x_{jq}}L_q\right\| \|F\|_{y_{Ck}}\|v\| \le$$

$$\le M\left\|\sum_p L_p^+L_p\right\|^{1/2}\left(\sum_p\left(\int_s^a y_{kp}\right)^2\right)^{1/2}\left\|\sum_q L_q^+L_q\right\|^{1/2}$$

$$\left(\sum_q x_{jq}^2(a)\right)^{1/2}$$

Now we have :

$$\left\langle\left(A_L^+(t)-A_L^+(s)\right)F(u\otimes x),\left(A_L^+(t)-A_L^+(s)\right)F(v\otimes y)\right\rangle =$$

$$=\sum_p\left\langle L_pF(u\otimes x),L_pF(v\otimes y)\right\rangle(t-s)+$$

$$+\sum_{p,k}(-1)^{k+\varepsilon(F)}\int_s^t \overline{y_{kp}}(a)\left\langle L_pF(u\otimes x),\right.$$

$$\left.\left(A_L^+(a)-A_L^+(s)\right)(\theta\otimes 1)F(\theta v\otimes y_{Ck})\right\rangle da+$$

$$+\sum_{q,j}(-1)^{j+\varepsilon(F)}\int_s^t x_{jq}(a)\left\langle\left(A_L^+(a)-A_L^+(s)\right)(\theta\otimes 1)F\left(\theta u\otimes x_{Cj}\right)\right.$$

$$\left.L_pF(v\otimes y)\right\rangle da$$

We assume that all L_p have the same parity $\varepsilon(L)$. Then θ of $\left(A_L^+(a)-A_L^+(s)\right)$ moves in front using $(-1)^{\varepsilon(L)}$, then through $\langle\theta a,b\rangle=\langle a,\theta b\rangle$ in front of L_p, then in the right and $(-1)^{\varepsilon(L)}$ disappears.

We assume that u and v have opposite parities, that is $\theta u=(-1)^{\varepsilon(u)}u$.

Then $L_pF(u\otimes x)$ L_p θ F(u ⊗ x) has the parity $\varepsilon(L)+\varepsilon(F)+\varepsilon(u)+r$, and $\left(A_L^+(a)-A_L^+(s)\right)F(\theta v\otimes y_{Ck})$ has the parity $\varepsilon(L)+\varepsilon(F)+1+\varepsilon(v)+w-1$.

For

$$\left|\left\langle L_pF(u\otimes x),\left(A_L^+(a)-A_L^+(s)\right)(\theta\otimes 1)F(\theta v\otimes y_{Ck})\right\rangle\right| \ne 0$$

$$\varepsilon(L)+\varepsilon(F)+\varepsilon(u)+r=\varepsilon(L)+1+\varepsilon(F)+\varepsilon(v)+w-1$$

or $\varepsilon(v)+w$

Hence L_p must have the same parity.

We generalize to u,v arbitrary writing $u=u_+ +u$ and $v=v_+ +v$.

Since $\int_s^t \overline{y_{kp}}=0$, $k\le b$

$\int_s^t x_{jq}=0$, $j\le a$

using theorem 1.3 we can consider $u,v\in h_s$ and

$x=x_1\wedge\cdots\wedge x_r;\quad x_i\in H^S$

$y=y_1\wedge\cdots\wedge y_w;\quad y_i\in H^S$

Hence:

$$\left\langle\left(A_L^+(t)-A_L^+(s)\right)F(u\otimes x),\left(A_L^+(t)-A_L^+(s)\right)F(v\otimes y)\right\rangle =$$

$$=\int_s^t\sum_{p,k}(-1)^{k+\varepsilon(F)+r+w}\overline{y_{kp}}(a)\left\langle L_p\theta F(\theta u\otimes x),\right.$$

$$\left.\left(A_L^+(a)-A_L^+(s)\right)F(u\otimes y_{Ck})\right\rangle da+$$

$$+\int_s^t\sum_{p,j}(-1)^{j+\varepsilon(F)+r+w}x_{jq}(a)\left\langle\left(A_L^+(a)-A_L^+(s)\right)F(u\otimes x_{Cj}),\right.$$

$$\left.L_p\theta F(\theta v\otimes y)\right\rangle da$$

In the same manner we obtain the value of:

2) $\left\langle\left(A_L^+(t)-A_L^+(s)\right)F(u\otimes x),G\left(A_L(t)-A_L(s)\right)(v\otimes y)\right\rangle$

3) $\left\langle G\left(A_L(t)-A_L(s)\right)(u\otimes x),G\left(A_L(t)-A_L(s)\right)(v\otimes y)\right\rangle$

4) $\left\langle\left(A_L(t)-A_L(s)\right)F(u\otimes x),H(t-s)(v\otimes y)\right\rangle$

5) $\left\langle G\left(A_L(t)-A_L(s)\right)F(u\otimes x),H(t-s)(v\otimes y)\right\rangle$

6) $\left\langle H(t-s)(u\otimes x),H(t-s)(v\otimes y)\right\rangle$

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We summarise these formulas in the following way .We consider three possibilities, exactly F,G,H and if ϕ is one of them, we define :

$$\gamma_t(F)=\left(A_L^+(t)-A_L^+(s)\right)F,$$

$$\gamma_t(G)=G\left(A_L^+(t)-A_L^+(s)\right),\quad \gamma_t(H)=H(t-s)$$

with fixed s. We consider 2 of them ϕ_1 and ϕ_2 and we have:

$$\left\langle \gamma(\phi_1)(u\otimes x),\gamma(\phi_2)(v\otimes y)\right\rangle =$$

$$=\int_s^t\Bigg(\delta_{F,\phi_1}\delta_{F,\phi_2}\sum_p\left\langle L_pF(u\otimes x),L_pF(v\otimes y)\right\rangle$$

$$+\sum_{k,p}\left\langle \gamma_a(\phi_1)\left(u\otimes x_{1,k,\phi_2}\right),\alpha_{kp}(\phi_2)\beta_p(\phi_2)\left(v\otimes y_{2,k,\phi_2}\right)\right\rangle$$

$$+\left\langle \alpha_{kp}^*(\phi_1)\beta_p(\phi_1)\left(u\otimes x_{2,k,\phi_1}\right),\gamma_a(\phi_2)\left(v\otimes y_{1,k,\phi_1}\right)\right\rangle\Bigg)da$$

where:

$$\beta_p(F)=L_p\theta F\theta,\quad \beta_p(G)=GL_p^+,\quad \beta_p(H)=H\delta_{p.o},$$

$$x_{1,k,F}=x_{Ck},\; x_{1,k,G}=x_{1,k,H}=(-1)^{k+r+w+\varepsilon(F)}x,$$

$$x_{2,k,G}=(-1)^{k-1}x_{Ck},\; x_{2,k,H}=x\delta_{k1},$$

$$\alpha_{kp}(F)=\overline{x_{kp}}(a),\; \alpha_{kp}(G)=y_{kp}(a),$$

$$\alpha_{kp}^*(H)=1,\; \alpha_{kp}^*(F)=\overline{y_{kp}}(a),\; \alpha_{kp}^*(G)=x_{kp}(a)$$

We recall that $p\in N$ and when $\phi_2=H$ we have only one term:

We consider $F=\sum_{n\geq 0}F_n\chi_{[tn,tn+1]}$ where $0=t_0<\ldots<t_n<\ldots$ and $F_n=F_{n,1}\otimes 1$ relative to $h=h_{tn}\otimes h^{tn}$ and similarly $G=\sum_{n\geq 0}G_n\chi_{[tn,tn+1]}$ and $H=\sum_{n\geq 0}H_n\chi_{[tn,tn+1]}$

We define

$$M(t)=M(0)+\sum_{n=0}^{b}\left(A_L^+(s_{n+1})-A_L^+(s_n)\right)F_n$$

$$+G_n\left(A_L(s_{n+1})-A_L(s_n)\right)+H(s_{n+1}-s_n)$$

for $t_b\leq t<t_{b+1}$ and $s_i=t_i$ for $i=0,\ldots,b,s_{b+1}=t$

Then $M(t)=M(t_b)\sum_\phi\gamma_b(\phi)$ where in γ_b we replace s with t_b.

We consider $\theta=M$, $\gamma_b(M)=M(s)$,then $M(t_b)$ and we remark that the formula $\left\langle \gamma_b(\phi_1)(u\otimes x),\gamma_b(\phi_2)(v\otimes y)\right\rangle$ is still valid with the following conventions: $\beta_p(M)=0,\; x_{1,k,M},\; x_{2,k,M}$ and $\alpha_{kp}(M)$ arbitrary and we add also the term: $\delta_{M,\phi_1}\delta_{M,\phi_2}\left\langle M(s)(u\otimes x),M(s)(v\otimes y)\right\rangle$

For $M(t)=\sum_\phi\gamma_b(\phi)$ we deduce:

$$\left\langle M(tb)(u\otimes x),M(tb)(v\otimes y)\right\rangle+$$

$$\int_{t_b}^t\Bigg(\sum_p\left\langle L_pF(u\otimes x),L_pF(v\otimes y)\right\rangle$$

$$+\sum_{k,p,\phi\neq M}\left(\left\langle M(a)\left(u\otimes x_{1,k,\phi}\right),\alpha_{kp}(\phi)\beta_p(\phi)\left(v\otimes y_{2,k,\phi}\right)\right\rangle\right)$$

$$+\left\langle \alpha_{kp}^*(\phi)\beta_p(\phi)\left(u\otimes x_{2,k,\phi}\right),M(a)\left(v\otimes y_{1,k,\phi}\right)\right\rangle\Bigg)da$$

Where in the place of F,G,H we have F_b,G_b,H_b

Summing with $n=0$ to a we obtain:

$$\left\langle M(t)(u\otimes x),M(t)(v\otimes y)\right\rangle=$$

$$=\left\langle M(0)(u\otimes x),M(0)(v\otimes y)\right\rangle$$

$$+\int_0^t\Bigg(\sum_p\left\langle L_pF(u\otimes x),L_pF(v\otimes y)\right\rangle$$

$$+\sum_{k,p,\phi\neq M}\left(\left\langle M(a)\left(u\otimes x_{1,k,\phi}\right),\alpha_{kp}(\phi)\beta_p(\phi)\left(v\otimes y_{2,k,\phi}\right)\right\rangle\right)$$

$$+\left\langle \alpha_{kp}^*(\phi)\beta_p(\phi)\left(u\otimes x_{2,k,\phi}\right),M(a)\left(v\otimes y_{1,k,\phi}\right)\right\rangle\Bigg)da$$

Hence:

$$\left\langle M(t)(u\otimes x), M(t)(v\otimes y)\right\rangle$$
$$=\left\langle M(0)(u\otimes x), M(0)(v\otimes y)\right\rangle$$
$$+\int_0^t\Bigg(\sum_{p\in J}\left\langle L_pF(u\otimes x), L_pF(v\otimes y)\right\rangle$$
$$+\sum_{p\in J}\sum_{k=1}^{w}\sum_{\phi\neq M}\Big(\left\langle M(a)(u\otimes x_{1,k,\phi}), \beta_p(\phi)(v\otimes y_{2,k,\phi})\right\rangle$$
$$+\left\langle \beta_p(\phi)(u\otimes x_{2,k,\phi}), M(a)(v\otimes y_{1,k,\phi})\right\rangle\Big)\Bigg)da$$

4. Conclusions

The need to build the non-commutative Markov processes was given by the evolution of probabilities in quantum mechanics. e build these processes on antisymmetric Fock space where we do not have exponential commutative vectors and where the commutative property does not occur between operators describing disjoint time intervals[5]. For this reason the processes are obtained as solutions of stochastic integral equations. This mathematical model creates the possibility to construct physical processes as stochastic integral equations solutions, being at the same time a new method of proving that certain processes are noncommutative. The model may be used in diffusion processes.

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