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# Cosine Gear Stress Analysis with Experimental Validation, and Comparison with Involute Gear

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**Abstract**
This study focuses on analysis and evaluation of Cosine gear from stress perspective. Cosine gear prototype is manufactured with EDM wire cut process. Bending stresses and contact stresses are studied for Involute and Cosine gear of same specification. Stress analysis is conducted using Finite element method and experimentally verified with the photoelastic stress analysis. A Finite element analysis has been carried out by considering frictional sliding between teeth with the help of well established non linear models. Bending stresses and contact stresses in cosine gear are found to be about 50 % and 30 % less than involute profile gear respectively.

***Keywords:*** *Cosine gear, Involute gear, Bending stress, Contact stress, Photoelastic stress analysis, FEM.*

## 1. Introduction

Basic gear design includes tooth force analysis, bending stress calculation etc. which are functions of tooth profile. A tooth profile is the geometry of a tooth; which determines kinematic and dynamic properties of a gear drive. It is the fundamental thing that directly relates to design parameters and ultimately current stringent market requirements like strength, Transmission performance, life, quietness in operation. Some of the commonly used gear teeth profiles are Involute, Cycloid, Epicycloids, and Hypocycloid. Most commonly used among them is Involute because of simplicity in manufacturing.

There is ample amount of work done on Involute profile gears to study it in details from almost all perspectives. Mostly on reducing the bending and contact stresses. Different approaches used for profile modification are as changing root fillet shape from trochoidal to circular, profile shifting, face crowning, changing the pressure angle and combining different profile modification techniques. FEA and photo elasticity stress analysis techniques are used effectively to study the sated characteristics. There is need to analyze the Cosine gears also for stress characteristics using FEA & Experimental validation.

Bending stress evaluation in modern gear design is generally based on Lewis equation. His equation is applied with stress concentration factor, geometry factor as per AGMA standards. These factors are defined from photoelastic experiments. Lewis equation and its related coefficients do not provide a reliable solution to the wide variety of non-standard gear tooth profiles that could be considered. [12]. Hence FEA and photo elasticity can be used to analyze non standard gear tooth profiles.

## 2. Involute & Cosine Gear pair specification

As this study is focused on one to one comparison of bending stress and contact stress for Involute and cosine gear, selection of gear pair does not matter much. Because selected performance evaluation parameters are mainly driven by the tooth profile. Selected involute gear pair is from Ply board compactor machine used to compress the material to form ply board. Detailed specification and operating loading conditions are described below in table.

Table No -1 Specifications of the Gears used

| Parameter | Notation | Formula | Pinion | Gear |
|---|---|---|---|---|
| Gear Ratio | i | Zg / Zp | 1.5 | |
| No. of Teeth | Z | | 28 | 42 |
| Module (mm) | m | | 3 | 3 |
| PCD (mm) | dp /dg | m*Z | 84 | 126 |
| Addendum (mm) | ha | m | 3 | 3 |
| Dedendum (mm) | hf | 1.25*m | 3.75 | 3.75 |
| Tooth thickness (mm) | | 1.5708*m | 4.7124 | 4.7124 |
| Fillet Radius (mm) | | 0.4*m | 1.2 | 1.2 |
| Torque (N-mm) | Mt | | 21894 | |
| Speed (rpm) | N | | 1440 | |

According to the specification, Involute gear pair is modeled in Pro-E.

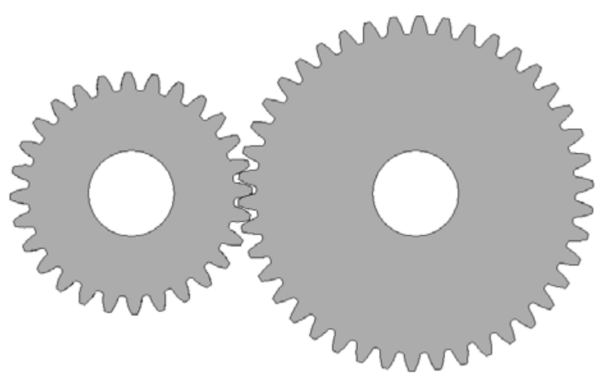

Figure 1 - CAD Model of Involute Profile Spur Gear

## 3. Generation & Manufacturing of Cosine Profile Gear

A Cosine gear as presented by the Shanming Luo, Yue Wub, Jian Wang [1]. The pinion of the drive utilizes a cosine curve as the tooth profile. It takes the zero line of the cosine curve as the pitch circle, a period of the curve as a tooth space, and the amplitude of the curve as the tooth addendum. Based on mathematical model, equations of spatial co-ordinates are programmed into excel in order to generate the point data. This point data is then exported in format that is able to read by CAD software. Figure 1.2 shows cosine tooth profile gear.

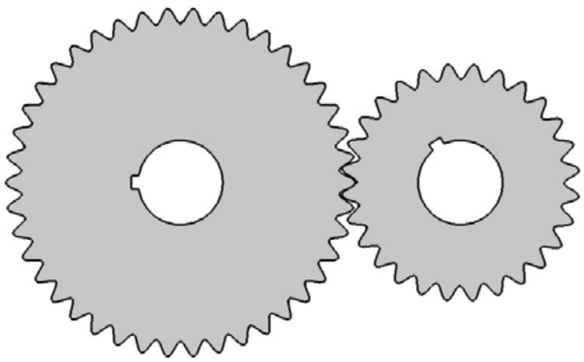

Figure 2 - CAD Model of Cosine Profile Spur Gear

As cosine gear is a non standard and new profile gear, manufacturing of the gear is by conventional gear manufacturing method seems to be difficult. There are several reasons behind it like Design of special hob for this profile itself is a big and expensive task and time taken by the specialized companies to manufacture the same. After extensive survey of gear manufacturers around the local industrial area as well as gear experts, economical and quick way to manufacture the cosine gears found to be the CNC wire cutting operation. Cosine gears are cut on standard MS plate of 25mm thickness to use them as a pattern for photo elastic model.

Figure 3 - Cosine Gear Prototypes

## 4. Stress Analysis using FEM

### 4.1 Introduction

In the practical complicated structures mathematical tools will not be sufficient to find the exact solution and sometimes, even an approximate solution. Thus, in the absence of any other convenient method to find even the approximate solution of a given real life problem, the finite element method is preferred. Moreover, in the finite element method, it will often be possible to improve or refine the approximate solution by spending more computational effort. Here Abaqus 6.10 is used to solve the problem.

### 4.2 Analysis methodology and model attributes

To simplify the problem and reduce the total degrees of freedom a single tooth of pinion and gear is considered and modelled three dimensional eight node solid continuum elements C3D8I. This element is enhanced by incompatible modes to improve the bending behavior. In addition to the displacement degrees of freedom, incompatible deformation modes are added internal to the elements. The primary effect of these degrees of freedom is to eliminate the so-called parasitic shear stresses that are observed in regular displacement elements if they are loaded in bending.
Table 2 below shows the material properties assigned for the finite element model.

Table No - 2 Material properties

| Material Properties | Value |
|---|---|
| Modulus of Elasticity (MPa) | 2.1e5 |
| Poisson's Ratio | 0.30 |
| Density (Kg/m3) | 7890 |

### 4.3 Meshing

To find the locations of the critical areas (high stress regions) of the Spur gears the analysis was just performed with reasonable element length and less number of elements and the results were obtained. Element density at the tooth root and contact area has been chosen carefully to capture the converged stress values.
Element density has kept optimum in order to control the total number of degrees of freedom and at the same time to achieve solution accuracy. Computer hard-ware available puts restrictions on model size. This factor is also considered while choosing element size. Number of iterations has performed on element size while reaching to the final size.

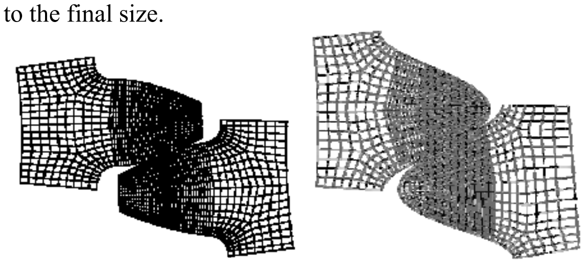

Figure 4 - Meshed models of Involute and Cosine gears

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Element size in both the models is kept same in the concern areas so that one to one comparison can be made. Model size for Involute and cosine gears is as below.

Table No – 2 Model count

| Gear Type | Nodes | Elements |
|---|---|---|
| Involute | 49090 | 45402 |
| Cosine | 44178 | 41002 |

### 4.4 Boundary and Loading conditions

As discussed in analysis methodology, single tooth interaction of gear and pinion is considered for the analysis. A small portion of the hub is taken along with gear tooth, this portion helps us to connect the tooth to the center of the pinion / gear with the help of rigid elements (BEAM). Rigid elements serve the purpose to apply the constraints at the center of the gear and apply the torque at the center of the pinion. Gear center is fixed in all 6 degrees of freedom and Pinion center is fixed in 5 degrees of freedom except 6th (Rotational Z). A torque of 21894 N-mm is applied about Z axis of pinion. Pinion will transfer the applied torque to gear through contact interaction defined between pinion and gear tooth surface.

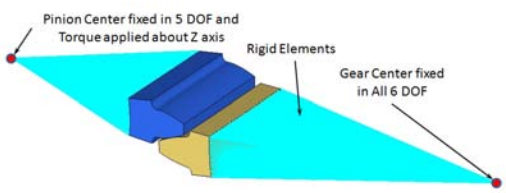


Figure 5 - Loading and Boundary conditions

Figure above shows the loading and boundary conditions for Involute profile gear, same is applied to Cosine gears also.

### 4.5 Gear tooth contact interface attributes

Choosing a proper attributes at the contact interface is a important to get good contact interface results. Contact is modelled as small sliding with friction. For normal to surface direction ‘hard’ contact pressure over closure relationship is used and for tangential surface direction Coulomb friction model is used with 0.15 as coefficient of friction. Surface to surface discretization is used to achieve smoother results.

After solving Bending stresses at tooth root, contact stresses at tooth interface and tooth deflections are studied to analyze the gears.

## 5. Experimental validation using Photo elastic Stress Analysis

Photo elasticity is an experimental technique for stress and strain analysis that is particularly useful for members having complicated geometry, complicated loading conditions or both. For such cases, analytical methods may be too cumbersome and analysis by an experimental approach may be more appropriate.

### 5.1 Typical process followed for stress analysis

For conducting photo elastic stress analysis of any component we need a prototype for mould preparation. In this case we have used metal prototypes of both the gears. Step by step procedure includes, Pattern making (Prototype) in case you don’t have, rubber mould preparation, casting of model and calibration disc using photoelastic material, loading of photo elastic model, stress freezing, slicing of the discs and fringe pattern reading using Polariscope. Material used for rubber mould is Sylartivi 11 with catalyst 27 and for photo elastic model AY-103 and hardener HY-951.

Figure 6 - Experimental setup

### 5.2 Material fringe value

The material fringe value $F_\sigma$ is defined as number of fringes produced per unit load. The material fringe value is the property of the model material for a given wavelength (λ) and thickness of the model (h). Here the circular disc of diameter 55 mm and thickness 3 mm was used to find material fringe value. This circular disc was loaded under compression by special fixture. A compressive load of 2 Kg was applied to find material fringe value. This circular disc was also subjected to same stress freezing cycle as that for the model. $F_\sigma$ found to be 0.3 N/mm for selected material.

### 5.3 Stress calculation for Involute and Cosine gear

At root of marked tooth on the slice, the isoclinic and isochromatic fringes were observed by using plane and circular Polariscope. All the values of fringe orders were noted down.

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The stresses developed in each slice at root of marked tooth point have been calculated as follows.

### 5.3.1 Involute gear Specimen Sample Calculations

Calculations are presented for one slice,

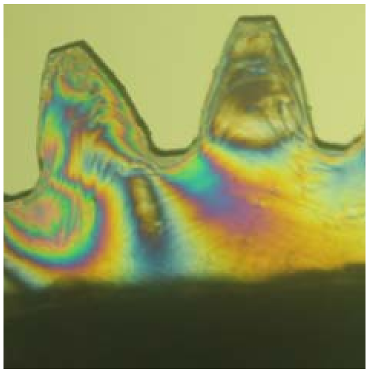

Figure 7 - Fringe pattern for Involute gear

Fringe order calculation,

$$N = n \pm (\gamma/180)$$
$$N = 2 + (234 / 180)$$
$$N = 3.3$$

We have material fringe value = 0.3 N/mm and Slice thickness = 2.1 mm

$$\sigma_1 - \sigma_2 = \frac{N \times F_\sigma}{t}$$

$$\sigma_1 = \frac{3.3 \times 0.3}{2.1}$$

$$\sigma_1 = 0.4714 \text{ N/mm}^2$$

Now scaling the stress values from model to prototype by following equation

$$\sigma_p = \sigma_m \times \left(\frac{T_p}{T_m} \times \frac{h_m}{h_p} \times \frac{L_m}{L_p}\right)$$

Where,
$T_p$= Torque on prototype
$T_m$= Torque on model
$\sigma_p$= Stresses produced in prototype
$\sigma_m$= Stresses produced in model
As model and prototype have same dimensions, $h_m$=$h_p$ and $L_m$=$L_p$. Therefore the equation becomes

$$\sigma_p = \sigma_m \times \left(\frac{T_p}{T_m}\right)$$

Torque on prototype = 21894 N-mm.
(P = 3 KW, N = 1440 rpm)
Torque on model = (1.5 × 9.81 × 42) = 618 N-mm.

$$\sigma_p = 0.4714 \times \left(\frac{21894}{618}\right)$$

$$\sigma_p = 16.70 \text{ MPa}$$

Same calculations are done for the Cosine gear.

### 5.3.2 Cosine gear Specimen Sample Calculations

Fringe order calculation,

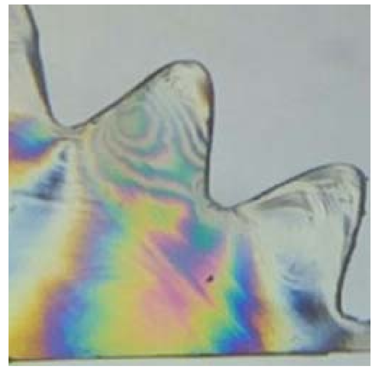

Figure 8 - Fringe pattern for Cosine gear

$$N = n \pm (\gamma/180)$$
$$N = 2 + (175/ 180)$$
$$N = 2.97$$

We have material fringe value = 0.3 N/mm and Slice thickness = 3 mm.

$$\sigma_1 = \frac{2.97 \times 0.3}{3}$$

$$\sigma_1 = 0.297 \text{ N/mm}^2$$

now scaling the stresses from model to prototype.
We have,
Torque on prototype = 21894 N-mm.
(P = 3 KW, N = 1440 rpm)
Torque on model = (2.0 × 9.81 × 42) = 824 N-mm.

$$\sigma_p = 0.297 \times \left(\frac{21894}{824}\right)$$

$$\sigma_p = 7.89 \text{ MPa}$$

## 6. Results and Discussion

In this section, results obtained from FEM are presented and analyzed to draw inferences from them. As discussed Tooth deflection, bending stresses and contact stresses are plotted for Involute gear and Cosine gear side by side to facilitate easy comparison.
Figure 9 shows the resultant deformation for both the gears. Deformation contours clearly shows that cosine gear is having more bending strength than the involute gear and deforms less than the involute gear for specific load. Tooth deformation is one of the causes which directly relate to the transmission error in gear systems,

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increase in transmission error increases vibrations and noise.

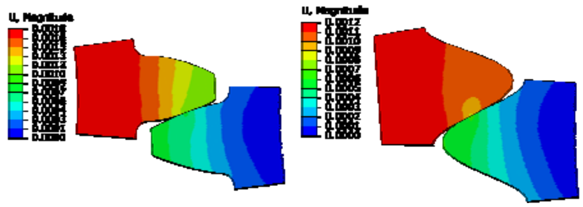

Figure 9 - Resultant deformation - Involute and Cosine Gear

Regarding contact stresses, Maximum von Mises stress in involute profile gear is 119 MPa whereas in cosine gear is 89 MPa. There is reduction of about 25 % in von Mises stress. As discussed in earlier section, shear stresses induced in gears are considerable and can't be neglected. Max. shear stress in involute profile gear is 132 MPa and in cosine gear is 92 MPa. There is reduction of about 30 % in Tresca stresses beneath the contact surface. Figure 10 shows von Mises stress plots.

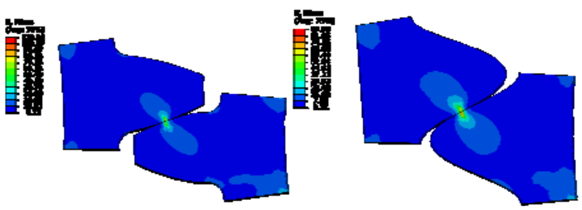

Figure 10 - von Mises stress - Involute Gear and Cosine Gear

Bending stresses at the root of the tooth of Involute profile gear is in the range of 18 MPa to 19MPa while in Cosine profile gear it is in the range of 8 to 9 MPa. There is approximately 50 % reduction in bending stresses. On the compression side of the tooth same scenario has been observed. Figure 11 shows the maximum principal stress.

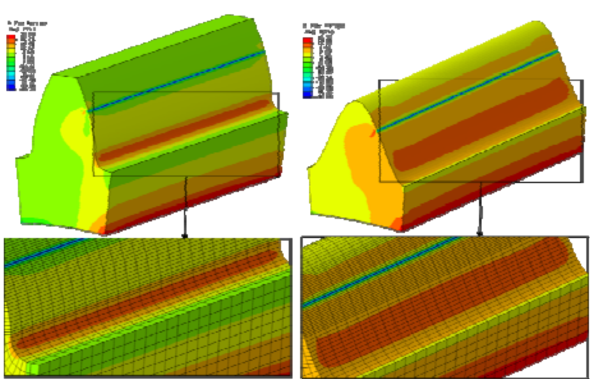

Figure 11 - Max. Principal Stress - Involute Gear and Cosine Gear

Let us have a look on stress comparison for Involute and Cosine gear from all the methods. Table 3 shows a comparative study for all the stated measures.

Table No –3 Stress Comparison - Involute

| Parameter | Involute | | |
|---|---|---|---|
| | FEA | Analytical | Experimental |
| Max. Principal (Bending)Stress (MPa) | 19 | 19.68 | 16.7 |
| Max Shear Stress (MPa) | 132 | | |
| Max von MIses Stress (MPa) | 119 | | |

Table No –3 Stress Comparison - Cosine

| Parameter | Cosine | | |
|---|---|---|---|
| | FEA | Analytical | Experimental |
| Max. Principal (Bending)Stress (MPa) | 9 | - | 7.89 |
| Max Shear Stress (MPa) | 92 | | |
| Max von MIses Stress (MPa) | 89 | | |

After examination of the stress values obtained from FEA, Analytical and Experimental for Involute profile spur gear, we can say that there is marginal difference between FEA and Analytical stress values. But looking at the FEA to experimental values, for Involute 12 % and for Cosine 12.3 % of variation exists which is acceptable.

## 7. Conclusion and Future work

Reduction in bending stress is basically because of the tooth profile at the root of the gear. Fundamentally there is not fillet radius joining the tooth profile and dedendum circle in cosine gear so resisting cross sectional area in other words section modulus is more in cosine gear. On the basis of numerical results, contact stresses in cosine gear are reduced by 25 % and Maximum shear stresses beneath the contact surface are reduced by 30 %. Basically reduction in contact stresses and shear stresses is happened because of the radius of curvature at the pitch point which is less in involute tooth profile and large in cosine tooth profile. There is future work can be done on the transmission error calculation and its impact on noise and vibration characteristics. Also dynamic stress analysis needs to conduct for variable loading and cost effective manufacturing method for mass production of cosine gears.

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