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Euler Type Triple Integrals Involving Some Special Functions

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Abstract

In this paper we obtain some Euler type triple integrals involving general class of polynomials, special functions and Multivariable H–function. Further we establish some special cases. These results may find applications in solving certain problems of applied mathematics.

Keywords: *General Class of Polynomial, Special Functions, H-function, Multivariable H-function.*

1. Introduction

The H–function of r–complex variables $z_1, \dots, z_r$ was introduced by Srivastava and Panda [5]. We shall use here the following contracted form [4, p. 251–253 eqn. (C.2)–(C.8)] to denote it.

1. $H\,[z_1, \dots, z_r]$

$$= H \begin{matrix} 0, n: m_1, ; \dots ; m_r, n_r \\ p, q: p_1, q_1; \dots ; p_r, q_r \end{matrix} \left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} \left(a_j; \alpha_j', \dots, \alpha_j^{(r)}\right)_1, p: \\ \left(b_j; \beta_j', \dots, \beta_j^{(r)}\right)_1, q: \end{matrix} \begin{matrix} \left(c_j', \gamma_j'\right)_1, p_1; \dots ; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_1, p_r \\ \left(d_j', \delta_j'\right)_1, q_1; \dots ; \left(d_j^{(r)}, \delta_j^{(r)}\right)_1, q_r \end{matrix} \right]$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \Phi_1(\xi_1) \dots \Phi_r(\xi_r)\ \Psi(\xi_1, \dots, \xi_r)\ z_1, \dots, z_r\, d\xi_1, \dots, d\xi_r \quad \textbf{(1.1)}$$

where $\omega = \sqrt{-1}$

For the convergence, existence conditions and other details of the above multivariable H –function, we refer to book by Srivastava et al [4, p. 251–253, eqn. (C.2)–(C.8)].

The following results will be required to establish our main results: Vyas and Rathi [6, p. 33].

$$\int_0^1 x^{c-1}(1-x)^{-\frac{1}{2}}\,{}_2F_1\left(a, b; a+b+\frac{1}{2}; x\right) dx = \frac{\pi\Gamma(c)\,\Gamma\left(a+b+\frac{1}{2}\right)\Gamma\left(c-a-b+\frac{1}{2}\right)}{\Gamma\left(a+\frac{1}{2}\right)\,\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(c-a+\frac{1}{2}\right)\Gamma\left(c-b+\frac{1}{2}\right)} \quad \textbf{(1.2)}$$

where $Re(c) > 0, Re(2c-a-b) > -1.$

Erdélyi[1] [p. 78, eqn. (2.4) (1), vol. 1]

$$\int_0^1 \int_0^1 t^{b-1}\, r^{a-1}\,(1-t)^{c-b-1}\,(1-r)^{c-a-1}\,(1-trz)^{-c}\,dt\,dr = \frac{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)\,{}_2F_1(a, b; c; z)}{[\Gamma(c)]^2} \quad \textbf{(1.3)}$$

Where $Re(a) > 0, Re(b) > 0, Re(c-a) > 0,$
$Re(c-b) > 0$

Erdélyi [1, p. 230, eqn. (5.8.1) (2), vol. 1]

$$\int_0^1 \int_0^1 u^{\beta-1}\, v^{\beta'-1}(1-u)^{\gamma-\beta-1}\,(1-v)^{\gamma'-\beta'-1}\,(1-ux-vy)^{-\alpha}\,du\,dv = \frac{\Gamma(\beta)\Gamma(\beta')\Gamma(\gamma-\beta)\Gamma(\gamma'-\beta')F_2(\alpha, \beta, \beta', \gamma, \gamma'; x, y)}{\Gamma(\gamma)\ \Gamma(\gamma')} \quad \textbf{(1.4)}$$

Erdélyi [1, p. 230, eqn. (5.8.1), (4)]

$$\int_0^1 \int_0^1 u^{\alpha-1}\, v^{\beta-1}(1-u)^{\gamma-\alpha-1}\,(1-v)^{\gamma'-\beta-1}\,(1-ux)^{\alpha-\gamma-\gamma'+1}(1-vy)^{\beta-\gamma-\gamma'+1}(1-ux-uy)^{\gamma+\gamma'-\alpha-\beta-1}\,du\,dv = \frac{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma-\alpha)\Gamma(\gamma'-\beta)F_4(\alpha, \beta, \gamma, \gamma'; x(1-y), y(1-x))}{\Gamma(\gamma)\ \Gamma(\gamma')} \quad \textbf{(1.5)}$$

Again $S\,\begin{matrix} m_1, \dots, m_r \\ n_1, \dots, n_r \end{matrix}[y_1, \dots y_r]$ stands for the multivariable polynomials studied by Srivastava [6, p. 185, Eqn.(7)] which will be defined and represented in the paper in the following form:

Where $N_r' = 0, 1, \dots, r, M_r' \neq 0, (r' = 1, \dots, r)$

The coefficients $A[N_1, K_1; \dots; N_r, K_r]$ being arbitrary constants real or complex and M_r' is an arbitrary positive integer.

2. Main Integrals

$$\int_0^1 \int_0^1 \int_0^1 x^{c-1}(1-x)^{-\frac{1}{2}} {}_2F_1\left(a, b; a+b+\frac{1}{2}; x\right)$$
$$\times y^{\beta-1} z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\lambda-\alpha-1}(1-yzt)^{-\lambda}$$
$$\times S_{N_1,\dots N_r}^{M_1,\dots M_r}[y_1 x^{c_1} y^{\rho'} z^{\zeta'}(1-y)^{\mu_1-\rho'}(1-z)^{\mu_1-\zeta'}$$
$$\times (1-yzt)^{-\mu_1}, \dots, y_r x^{c_r} y^{\rho^r} z^{\zeta^r}(1-y)^{\mu_r-\rho^r}$$
$$\times (1-z)^{\mu_r-\zeta^{(r)}}(1-yzt)^{-\mu_r}]\, H[z_1\, x^{\sigma_1}\, y^{\rho_1}\, z^{\zeta_1}$$
$$\times (1-y)^{\eta_1-\rho_1}(1-z)^{\eta_1-\zeta_1}(1-yzt)^{-\eta_1}$$
$$\times, \dots, z_r\, x^{\sigma_r}\, y^{\rho_r}\, z^{\zeta_r}\, (1-y)^{\eta_r-\rho_r}(1-z)^{\eta_r-\zeta_r}$$
$$\times (1-yzt)^{-\eta_r}]\, dx\, dy\, dz$$

$$= \frac{\pi\,\Gamma\left(a+b+\frac{1}{2}\right)}{\Gamma\left(a+\frac{1}{2}\right)\Gamma\left(b+\frac{1}{2}\right)} \sum_{k_1=0}^{[N_1/M_1]}, \dots, \sum_{k_r=0}^{[N_r/M_r]} \sum_{k=0}^{\infty} \frac{t^k}{k!} \frac{(-N_1)m_1 k_1}{k_1!}$$
$$\times, \dots, \frac{(-N_r)m_r k_r}{k_r!}\, A[N_1, k_1, \dots, N_r, k_r] y_1^{k_1}, \dots, y_r^{k_r}$$
$$\times H_{p+6, q+4: p_1, q_1; \dots; p_r, q_r}^{0, n'+6: m_1', n_1'; \dots; m_r', n_r'}$$
$$\times \left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} (1-c-c_1k_1, \dots, c_rk_r; \sigma_1, \dots, \sigma_r) \\ \left(\frac{1}{2}-c-c_1k_1, \dots, c_rk_r; +a; \sigma_1, \dots \sigma_r\right) \end{matrix}\right.$$
$$\left(\frac{1}{2}-c-c_1k_1, \dots, c_rk_r; +a+b; \sigma_1, \dots, \sigma_r\right),$$
$$\left(\frac{1}{2}-c-c_1k_1, \dots, c_rk_r; +b; \sigma_1, \dots, \sigma_r\right),$$
$$\{1-(\lambda+\mu_1k_1+\dots+\mu_rk_r-a-\zeta'k_1-\dots-\zeta'k_r); \eta_1-\zeta_1, \dots, \eta_r-\zeta_r\}$$
$$\{1-(\lambda+\mu_1k_1+\dots+\mu_rk_r; n_1, \dots, n_r)$$
$$\{1-(\lambda+\mu_1k_1+\dots+\mu_rk_r-\beta-\rho'k_1-\dots-\rho^rk_r); \eta_1-\rho_1; \dots; \eta_r-\rho_r\}$$
$$\{(1-a-\zeta^1k_1-\dots-\zeta^rk_r-k; \zeta_1, \dots, \zeta_r), \eta_1-\zeta_1, \dots, \eta_r-\zeta_r\}$$
$$(1-\lambda+\mu_1k_1+\dots\mu_rk_r; n_1, \dots, n_r),$$
$$(1-\beta-\rho^1k_1-\dots-\rho^rk_r-k;\ \rho_1, \dots, \rho_r),$$
$$\left.\begin{matrix} \left(a_j; \alpha_j', \dots \alpha_j^{(r)}\right)_1, p: \left(c_j'; \gamma_j'\right)_1, p_1; \dots; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_1, p_r \\ \left(b_j; \beta_j', \dots \beta_j^{(r)}\right)_1, q: \left(d_j'; \delta_j'\right)_1, q_1; \dots; \left(d_j^{(r)}, \delta_j^{(r)}\right)_1, q_r \end{matrix}\right] \tag{2.1}$$

where

$Re(c+c_1k_1+\dots+c_rk_r+\sigma_1\xi_1+\dots+\sigma_r\xi_r) > 0$

$Re\{2(c+c_1k_1+\dots+c_rk_r+\sigma_1\xi_1+\dots+\sigma_r\xi_r)-a-b\} > -1$

$Re(\alpha+\zeta^1k_1+\dots+\zeta^rk_r+\zeta_1\xi_1+\dots+\zeta_r\xi_r) > 0$

$Re(\beta+\rho^1k_1+\dots+\rho^rk_r+\rho_1\xi_1+\dots+\rho_r\xi_r) > 0$

$$Re(\lambda-\alpha+(\mu_1-\zeta^1)k_1+\dots+(\mu_r-\zeta^r)k_r+(\eta_1-\zeta_1)\xi_1 + \dots + (\eta_r-\zeta_r)\xi_r) > 0$$

$$Re(\lambda-\beta+(\mu_1-\rho^1)k_1+\dots+(\mu_r-\rho^r)k_r+(\eta_1-\rho^1)_1 + \dots + (\eta_r-\rho^r)\xi_r) > 0$$

1.

$$\int_0^1 \int_0^1 \int_0^1 x^{c-1}(1-x)^{-\frac{1}{2}} {}_2F_1\left(a, b; a+b+\frac{1}{2}; x\right)$$
$$\times y^{\beta-1} z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\mu-\alpha-1}(1-uy-vz)^{-n}$$

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$$\times S^{M_1,\dots M_r}_{N_1,\dots N_r}[y_1 x^{c_1} y^{\rho'} z^{\zeta'} (1-y)^{e'-\rho'}(1-z)^{t'-\zeta'}$$

$$\times (1-uv-vz)^{-\omega'},\dots, y_r x^{c_r} y^{\rho^r} z^{\zeta^r}(1-y)^{e^r-\rho^r}$$

$$\times (1-z)^{t^r-\zeta^r}(1-uy-vz)^{-\omega^r}]\, H[z_1\, x^{\sigma_1}\, y^{\rho_1}\, z^{\zeta_1}$$

$$\times (1-y)^{\eta_1-\rho_1}(1-z)^{t_1-\zeta_1}(1-uy-vz)^{-n_1}$$

$$\times,\dots, z_r\, x^{\sigma_r}\, y^{\rho_r}\, z^{\zeta_r}\,(1-y)^{\eta_r-\rho_r}(1-z)^{t_r-\zeta_r}$$

$$\times (1-uy-vz)^{-n_r}]\, dx\, dy\, dz$$

$$= \frac{\pi\,\Gamma\left(a+b+\frac{1}{2}\right)}{\Gamma\left(a+\frac{1}{2}\right)\Gamma\left(b+\frac{1}{2}\right)} \sum_{k_1=0}^{[N_1/M_1]},\dots,\sum_{k_r=0}^{[N_r/M_r]}\sum_{k,m=0}^{\infty}\frac{u^k v^m}{k!\,m!}$$

$$\times \frac{(-N)m_1 k_1}{k_1!}\cdots\frac{(-N)m_r k_r}{k_r!}\, A[N_1,k_1,\dots N_r,k_r]$$

$$\times H^{0,n'+7\,:\,m_1',n_1';\dots,m_r',n_r'}_{p+7\,,q+5\,:\,p_1,q_1;\dots,p_r,q_r}$$

$$\times \left[\begin{matrix} z_1\\ \vdots \\ z_r\end{matrix}\right| \begin{matrix}(1-c-c_1k_1;\dots,c_rk_r;\sigma_1,\dots,\sigma_r),\\ \left(\frac{1}{2}-c-c_1k_1;\dots,c_rk_r;+a;\sigma_1,\dots,\sigma_r\right),\end{matrix}$$

$$\times \begin{matrix}\left(\frac{1}{2}-c-c_1k_1;\dots,c_rk_r;+a+b;\sigma_1,\dots,\sigma_r\right),\\ \left(\frac{1}{2}-c-c_1k_1;\dots,c_rk_r;+b;\sigma_1,\dots,\sigma_r\right),\end{matrix}$$

$$\times \begin{matrix}(1-\lambda-(e^1-\rho^1)k_1-\cdots-(e^r-\rho^r)k_r;\eta_1-\rho_1;\dots;\eta_r-\rho_r),\\ (1-\mu-(t^1-\zeta^1)k_1-\cdots-(t^r-\zeta^r)k_r+\alpha;t^1-\zeta^1,\dots,t^r-\zeta^r),\end{matrix}$$

$$\times \begin{matrix}(1-n-\omega^1k_!-\cdots-\omega^r k_r k-m;n_1;\dots;n_r),\\ (1-n-\omega^1k_!-\cdots-\omega^r k_r;n_1;\dots;n_r),\end{matrix}$$

$$\times \begin{matrix}(1-\beta-\rho^1k_1-\cdots-\rho^r k_r-k;\rho_1;\dots;\rho_r),\\ (1-\beta-\rho^1k_1-\cdots-\rho^r k_r-k;n_1;\dots;n_r),\end{matrix}$$

$$\times \begin{matrix}(1-\alpha-\zeta^1k_1-\cdots-\zeta^r k_r k-m;\zeta_1;\dots;\zeta_r),\\ (1-\eta-t^1k_1-\cdots-t^r k_r-m;t_1;\dots;t_r),\end{matrix}$$

$$\times \left.\begin{matrix}(a_j;\alpha_j',\dots,\alpha_j^{(r)})1,p:(c_j';\gamma_j')1,p_1;\dots;(c_j^{(r)},\gamma_j^{(r)})1,p_r\\ (b_j;\beta_j',\dots,\beta_j^{(r)})1,q:(d_j';\delta_j')1,q_1;\dots;(d_j^{(r)},\delta_j^{(r)})1,q_r\end{matrix}\right]$$

(2.2)

where

$$Re(c+c_1k_1+\cdots+c_rk_r+\sigma_1\xi_1+\cdots+\sigma_r\xi_r)>0$$

$$Re\{2(c+c_1k_1+\cdots+c_rk_r+\sigma_1\xi_1+\cdots+\sigma_r\xi_r)-a-b\}>-1$$

$$Re(\alpha+\zeta^1k_1+\cdots+\zeta^r k_r+\zeta_1\xi_1+\cdots+\zeta_r\xi_r)>0$$

$$Re(\beta+\rho^1k_1+\cdots+\rho^r k_r+\rho_1\xi_1+\cdots+\rho_r\xi_r)>0$$

$$Re\{(\lambda+e^1k_1+\cdots+e^{(r)}k_r+\eta_1\xi_1+\cdots+\eta_r\xi_r) - (\beta+\rho^1k_1+\cdots+\rho^{(r)}k_r+\rho_1\xi_1+\cdots+\rho_r\xi_r)\}>0$$

$$Re\{(\mu+t^1k_1+\cdots+t^{(r)}k_r+t_1\xi_1+\cdots+t_r\xi_r) - (\alpha+\zeta^1k_1+\cdots+\zeta^{(r)}k_r+\zeta_1\xi_1+\cdots+\zeta_r\xi_r)\}>0$$

2.

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-\frac{1}{2}}\,{}_2F_1\left(a,b;a+b+\frac{1}{2};x\right)$$

$$\times y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\alpha-1}(1-z)^{\mu-\beta-1}(1-uy)^{\alpha-\lambda-\mu+1}$$

$$\times (1-vz)^{\beta-\lambda-\mu+1}(1-uy-vz)^{\lambda+\mu-\alpha-\beta-1}$$

$$\times S^{M_1,\dots M_r}_{N_1,\dots N_r}[y_1 x^{\sigma'} y^{\rho'} z^{\zeta'}(1-y)^{\eta'-\rho'}(1-z)^{t'-\zeta'}$$

$$\times (1-uy)^{\rho'-\eta'-t'}(1-vz)^{\zeta'-\eta'-t'}$$

$$\times (1-uy-vz)^{\eta'+t'-\rho'-\zeta'},\dots, y_r x^{\sigma^{(r)}} y^{\rho^{(r)}} z^{\zeta^{(r)}}$$

$$\times (1-y)^{\eta^{(r)}-\rho^{(r)}}(1-z)^{t^{(r)}-\zeta^{(r)}}\,(1-uy)^{\rho^{(r)}-\eta^{(r)}-t^{(r)}}$$

$$\times (1-vz)^{\zeta^{(r)}-\eta^{(r)}-t^{(r)}}(1-uy-vz)^{\eta^{(r)}+t^{(r)}-\rho^{(r)}-\zeta^{(r)}}]$$

$$\times H[z_1\, x^{\sigma_1}\, y^{\rho_1}\, z^{\zeta_1}\,\,(1-y)^{\eta_1-\rho_1}(1-uy)^{\rho_1-\eta_1-t_1}$$

$$\times (1-vz)^{\zeta_1-\eta_1-t_1}(1-uy-vz)^{\eta_1+t_1-\rho_1-\zeta_1},\dots,$$

$$\times z_r\, x^{\sigma_r}\, y^{\rho_r}\, z^{\zeta_r}\,\,(1-y)^{\eta_r-\rho_r}(1-uy)^{\rho_r-\eta_r-t_r}$$

$$\times (1-vz)^{\zeta_r-\eta_r-t_r}(1-uy-vz)^{\eta_r+t_r-\rho_r-\zeta_r}]\, dx\, dy\, dz$$

$$= \frac{\pi\,\Gamma\left(a+b+\frac{1}{2}\right)}{\Gamma\left(a+\frac{1}{2}\right)\Gamma\left(b+\frac{1}{2}\right)} \sum_{k_1=0}^{[N_1/M_1]},\dots,\sum_{k_r=0}^{[N_r/M_r]}\sum_{k,m=0}^{\infty}\frac{u^k(1-v)^m}{k!}$$

$$\times \frac{v^m(1-u)^m}{m!}.\frac{(-N)m_1k_1}{k_1!}\cdots\frac{(-N)m_rk_r}{k_r!}$$

$$\times A[N_1,k_1,\dots N_r,k_r]\times y_1^{k_1},\dots,y_r^{k_r}$$

$$\times H^{0,n'+6\,:\,m'_1,n'_1;\,\dots,\,m'_r,n'_r}_{p+6,\,q+4\,:\,p_1,q_1;\,\dots,\,p_r,q_r}$$

$$\times\left[\begin{matrix} z_1\\ \vdots \\ z_r\end{matrix}\left|\begin{matrix}(1-c-\sigma^1k_1;\dots,\sigma^{(r)}k_r;\sigma_1,\dots,\sigma_r),\\ \left(\frac{1}{2}-c-\sigma^1k_1;\dots,\sigma^{(r)}k_r;+a;\sigma_1,\dots,\sigma_r\right),\end{matrix}\right.\right.$$

$$\times\begin{matrix}\left(\frac{1}{2}-c-\sigma^1k_1;\dots,\sigma^{(r)}k_r;+a+b;\sigma_1,\dots,\sigma_r\right),\\ \left(\frac{1}{2}-c-\sigma^1k_1;\dots,\sigma^{(r)}k_r;+b;\sigma_1,\dots,\sigma_r\right),\end{matrix}$$

$$\times\begin{matrix}(1-\lambda-\eta^1k_1-\dots-\eta^{(r)}k_r;\rho^1k_1+\dots+\rho^{(r)}k_r+\alpha;\eta_1-\rho_1,\dots,\eta_r-\rho_r)\\ (1-\mu-(t^1-\zeta^1)k_1-\dots-(t^r-\zeta^r)k_r+\beta;t^1-\zeta^1,\dots,t^r-\zeta^r),\end{matrix},$$

$$\times\begin{matrix}(1-\alpha-\rho^1k_1-\dots-\rho^rk_r-k-m;\rho_1;\dots;\rho_r),\\ (1-\lambda-\eta^1k_1-\dots-\eta^rk_r+\rho^1k_1+\dots+\rho^rk_r-k;\eta_1;\dots;\eta_r),\end{matrix}$$

$$\times\begin{matrix}(1-\beta-\zeta^1k_1-\dots-\zeta^rk_r-k;\zeta_1;\dots;\zeta_r),\\ (1-\mu-(t^1-\zeta^1)k_1-\dots-(t^r-\zeta^r)k_r-m;t_1;\dots;t_r),\end{matrix}$$

$$\left.\times\begin{matrix}(a_j;\alpha'_j,\dots,\alpha_j^{(r)})1,p:(c'_j;\gamma'_j)1,p_1;\dots;(c_j^{(r)},\gamma_j^{(r)})1,p_r\\ (b_j;\beta'_j,\dots,\beta_j^{(r)})1,q:(d'_j;\delta'_j)1,q_1;\dots;(d_j^{(r)},\delta_j^{(r)})1,q_r\end{matrix}\right]$$

(2.3)

Where

$$Re\left(c+\sigma^1k_1+\dots+\sigma^{(r)}k_r+\sigma_1\xi_1+\dots+\sigma_r\xi_r\right)>0$$

$$Re\left\{2\left(c+\sigma^1k_1+\dots+\sigma^{(r)}k_r+\sigma_1\xi_1+\dots+\sigma_r\xi_r\right)-a-b\right\}>-1$$

$$Re\left(\alpha+\rho^1k_1+\dots+\rho^{(r)}k_r+\rho_1\xi_1+\dots+\rho_r\xi_r\right)>0$$

$$Re\left(\beta+\zeta^1k_1+\dots+\zeta^{(r)}k_r+\zeta_1\xi_1+\dots+\zeta_r\xi_r\right)>0$$

$$Re\left\{\left(\lambda+(\eta^1-\rho^1)k_1+\dots+\left(\eta^{(r)}-\rho^{(r)}\right)k_r+\eta_1\xi_1+\dots+\eta_r\xi_r\right)-\left(\alpha+\rho^1k_1+\dots+\rho^{(r)}k_r+\rho_1\xi_1+\dots+\rho_r\xi_r\right)\right\}>0$$

$$Re\left\{\left(\mu+(t^1-\zeta^1)k_1+\dots+(t^{(r)}-\zeta^{(r)})k_r+t_1\xi_1+\dots+t_r\xi_r\right)-\left(\beta+\zeta^1k_1+\dots+\zeta^{(r)}k_r+\zeta_1\xi_1+\dots+\zeta_r\xi_r\right)\right\}>0$$

3. Proof of (2.1):

We first express H– function involved in the left hand side of (2.1) in terms of Mellin–Barnes contour integral with the help of (1.1) and general class of multivariable polynomial involved in (2.1) in summation form given then interchange the order of integration and summation, we get the right hand side of (2.1) with the help of (1.2), (1.3), and (1.1), (2.2) and (2.3) with the help of results (1.4) and (1.5).

4. Special Cases:

Our main results provide unifications and extensions of various results. For the sake of illustration, we mention the following few special cases:

i. If we take $r=1$, we get the known results obtained by Garg, Kumar and Shakeeluddin [3].

ii. If we take $a=-x$, $b=g+n$ in ${}_2F_1(a,b;a+b+\frac{1}{2};x)$ and using the relationship [2].

$${}_2F_1\left(a,b;\ a+b+\tfrac{1}{2};\ x\right)=\ {}_2F_1\left(-n,g+n;g+\tfrac{1}{2};x\right)_n\,p^{g,g+\frac{1}{2}}(x)$$

we get the results involving Jacobi polynomial.

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