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Non-Existance of String Cosmological Models in Presence of Magnetic Field in Bimetric Theory

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Abstract

In this paper, we have constructed spherically symmetric String Cosmological model which does not exist in presence of magnetic field.The magnetic field is equated to the energy density of the string ,but the field is to be absent to get an exact solution of the model. Also we have studied various physical geometrical properties of the string model in Bimetric relativity.

Keywords:Bimetric Theory, Magnetic Field, Geometric String.

1. Introduction

theory of gravitation has been proposed by Rosen [18] to remove some of the unsatisfactory features of the general theory of gravitation. This theory is running with two metric tensors i.e. a Riemannian metric tensor g_{ij}, which describes the geometry of a curved space time and thus, the gravitational field and the background metric γ_{ij}, which inters into the field equations and interacts with g_{ij}, but does not interact directly with matter. Also the postulation of γ_{ij} improves the formulism of general relativity without changing its physical content. One can regard γ_{ij} as describing the geometry that exists if matter were not present. Accordingly, at each space-time point one has two line elements

$$ds^2 = g_{ij}\, dx^i\, dx^j \tag{1}$$

and

$$d\sigma^2 = \gamma_{ij}\, dx^i\, dx^j \tag{2}$$

where ds is the interval between two neighbouring events as measured by a clock and measuring rod, the interval $d\sigma$ is an abstract or a geometrical quantity which is not directly measurable. The theory agrees with the present observational facts pertaining to general relativity. As an general relativity, the variational principle also leads to the conservation law

$$T^{ij}_{;j} = 0 \qquad (3)$$

where (;) denotes the covariant differentiation with respect to g_{ij}. Accordingly the geodesic equation of a test particle is the same as that of general relativity.

The field equations of theory of gravitation proposed by Rosen[18] are

$$N^i_j - \frac{1}{2} N\, g^i_j = -8\pi\, k\, T^i_j \qquad (4)$$

where $$N^i_j = \frac{1}{2} \gamma^{ab} \left(g^{hi}\ g_{hj;a} \right); b,$$

$$N = N^i_j = N^1_1 + N^2_2 + N^3_3 + N^4_4,$$

$$g = \det\left(g_{ij}\right), \gamma = \det\left(\gamma_{ij}\right), K = \left(\frac{g}{\gamma}\right)^{1/2}$$

and T_{ij} is the energy momentum tensor of the matter.

Israelit (4-6), Karade and Dhoble [7], Karade [8], Rosen [18-19], Yilmaz[25] are some of the eminent authors who have studied several aspects of bimetric theory of gravitation. In particularly, Mahanty and Sahoo [11] and Mahanty et. al. [12] have established the non existence of anisotropic spatially homogeneous Bianchi type cosmological models in bimetric theory when the source of gravitation is governed by either perfect fluid or mesonic perfect fluid. Reddy [13] have discussed the non existence of anisotropic spatially homogeneous Bianchi type-1 cosmological model in bimetric theory of gravitation in case of cosmic strings and Reddy and Venkateswarly [15] have shown the non-existence of anisotropic Bianchi type-1 perfect fluid models in Rosen's bimetric theory. Sahoo [20] have studies spherical symmetric string cosmological models in bimetric theory.

In bimetric theory, according to the Rosen [18], the background metric tensor γ_{ij} should not be taken as describing an empty universe but it should rather be chosen on the basis of cosmological consideration. Hence the Rosen proposed that the metric γ_{ij} be taken as the metric tensor of a universe in which perfect cosmological principle holds. In accordance with this principle, the large scale structure of the Universe presents the same aspect from every where in space and at all times. The fact, however, is that while taking the matter actually present in the universe, this principle is not valid on small scale structure due to irregularities in the matter distribution and also not valid on large scale structure due to the evolution of the matter. Therefore, we adopt the perfect cosmological principle as the guiding principle. It does not apply to g_{ij} and the matter in the universe but to the metric γ_{ij}. Hence γ_{ij} describes a space time of constant curvature. Also it is interesting to note that magnetic field plays a significant role in cosmological models. Some cosmologist suggested in the cosmological solution for dust and electromagnetic field that during the evolution of the universe, the matter was in high ionized state and smoothly coupled with magnetic field and consequently forms a neutral matter as a result of universe expansion.

The relativists use various symmetries to get physically viable information from the complicated structure of the field equations in Einstein's theory of relativity and it is very difficult to obtain their exact solutions. The involvement of symmetry namely spherical or Cylindrical or plane reduces the number of gravitational potentials g_{ij} and thus helps one in simplifying the field equations to some extent.

In this paper, spherical symmetric metric is studied in bimetric theory with cosmic strings coupled to an electromagnetic field and it is observed that electromagnetic filed does not contribute to the energy momentum tenson in this theory. Hence singularity free geometric string is obtained.

2. Cosmological Model and Field Equations

Considered the spherical symmetric metric in the form

$$ds^2 = e^{\mu}\, dt^2 - e^{\lambda}\, dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \tag{5}$$

Where λ, μ are functions of cosmic time 't' only.

The background metric for above metric is

$$d\sigma^2 = dt^2 - dr^2 - \left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \tag{6}$$

The energy momentum tensor for cosmic string as Letelien [10] and Satchel [22] coupled with electromagnetic field is written as

$$T_j^i = T_{j\ string}^i + T_{j\ mag}^i \tag{7}$$

where $$T_{j\ string}^i = \rho u^i u_j - \lambda x^i x_j \tag{8}$$

together with $$u^i u_i = 1 = -x^i x_i \ and \ u^i x_i = 0 \tag{9}$$

and $$E^{i}_{j\ mag} = -F_{jn}\,F^{in} + \frac{1}{4}\,F_{ab}\ F^{ab}\ g^{i}_{j} \qquad (10)$$

where $E^{i}_{j\ mag}$ is lectromagnetic energy tensor,

F^{i}_{j} is the electromagnetic field tensor,

u^{i} is the four velocity vector of string,

x^{i} represents a direction of anisotropy is the direction of strings,

ρ is the energy density of the strings

and λ is the tension density of the strings.

In the co-moving co-ordinate system, the equation [8] derives

$$T^{1}_{1\ strings} = \lambda,\ T^{4}_{4\ strings} = \rho\,,$$

where the string is taken in radial direction and

$$T^{i}_{j\ stong} = 0 \quad for\ i, j = 2{,}3, \text{ and for } i \neq j\,.$$

The electromagnetic field is considered to be along the radial direction. So that the only non-vanishing component of electromagnetic field tensor $F_{ij}\ is\ F_{23}$.

The first set of Maxwell's equation

$$F_{[ij,k]} = F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \qquad (11)$$

leads to the result $F_{23} = cons\tan t = A(say)$ and

$$F^{23} = g^{22}\ g^{23}\ F_{23} = \frac{A\cos ec^{2}\theta}{r^{4}}\,.$$

Then

$$E^{1}_{1\ mag} = \eta\,,\ E^{2}_{2\ mag} = 3\eta,\ E^{3}_{3} = 3\eta,\ E^{4}_{4} = -\eta\,,$$

where $$\eta = \frac{-A^{2}}{2r^{4}\sin^{2}\theta} \qquad (12)$$

The Rosen's field equations(4) for the metric (5) and (6) with the help of equation (7) becomes

$$\lambda_{44} - \mu_{44} = -32\,\pi k(\lambda + \eta), \qquad (13)$$

$$\lambda_{44} + \mu_{44} = 96\ \pi k \eta\ , \tag{14}$$

$$\lambda_{44} - \mu_{44} = 32\ \pi k (\rho - \eta), \tag{15}$$

where suffix 4 denotes ordinary differentiation with respect to t.

Solving equation (13), (14) and (15) we get

$$\mu_{44} = 16 \pi k (4\eta + \lambda),$$

and $$\lambda_{44} = 16 \pi k (2\eta + \lambda + 2\rho). \tag{16}$$

For Geometric String Model: $\rho = \lambda$

For this case, solving equation (16), we get

$$\left.\begin{aligned} \mu_{44} &= 16 \pi k (4\eta + \rho) \\ \lambda_{44} &= 16 \pi k (2\eta + 3\rho) \end{aligned}\right\}. \tag{17}$$

If $\eta = \rho$, then equation (17) reduces to

$$\lambda_{44} = \mu_{44} = 80\ \pi k \eta\ . \tag{18}$$

Taking $\eta = 0$, so that the magnetic field is absent.Then

$$\lambda_{44} = \mu_{44} = 0$$

$$\Rightarrow \mu = \lambda = c_1 t + c_2$$

where c_1, c_2 are arbitrary constants.

Then the geometric string model is designed as

$$ds^2 = e^{c_1 t + c_2}\ dt^2 - e^{c_1 t + c_2}\ dr^2 - r^2 \left(d\theta^2 + \sin^2\theta\ d\phi^2 \right). \tag{19}$$

3. Properties of the Models:

The following properties are discussed for the model (19):

(i). the spatial volume $V = e^{c_1 t + c_2}\ r^2 \sin\theta$ increase as t increases and hence the universe may flow up at infinite.

(ii). Since acceleration $\dot{v}_i = 0$, so that the string is geodesic in nature.

(iii). The anisotropy $|\sigma|$ is defined as

$$\sigma^2 = \frac{1}{12}\left[\left(\frac{g_{11,4}}{g_{11}} - \frac{g_{22,4}}{g_{22}}\right)^2 + \left(\frac{g_{22,4}}{g_{22}} - \frac{g_{33,4}}{g_{33}}\right)^2 + \left(\frac{g_{33,4}}{g_{33}} - \frac{g_{44,4}}{g_{44}}\right) + \left(\frac{g_{44,4}}{g_{44}} - \frac{g_{11,4}}{g_{11}}\right)\right]^2$$

Since $\sigma \neq 0$ *and* $\underset{t\to\infty}{Lim} \frac{\sigma}{\theta} \neq 0$, so that the universe is anisotropic in nature through out the evolution.

(iv). Since the verticity tensor $w_{ij} = 0$, that the model is non-ratating in nature.

(v). $R_{ij} \neq \frac{1}{4} R g_{ij}$ and hence the model is not an Einstein space.

4. Conclusion:

A four dimensional cosmological model in presence of cosmic string and magnetic field in Bimetric relativity has been constructed. Here we investigated that for existence of the geometric string model $(\rho = \lambda)$, the magnetic field (η) is to be absent and the string energy density (ρ) is equated to the magnetic field. Hence $\rho = 0$. The model is geodesic, anisotropic and does not satisfy the property of Einstein's space.

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