ISSN 2348 – 7968

Compare Polynomial interpolation Her mite With Lagrange and Solving NON-Linear Boundary Value Problems With Her mite

Dr. Habib Molaei[1], Hamid Baghban[2]

[1]DEPARTMENT OF MATHEMATICS TECHNICALAND VOCATINAL UNIVERSITY GAZI-TABATABAEI ,P.O.BOX 57169-33950,URMIA,IRAN &UNIVERSITY COLLEGE OF SCIENCE AND TECHNOLOGY ELM O FANN URMIA, P.O.BOX 57351-33746 URMIA
E-mail: Habibmolaei@Gmail.com

[2]UNIVERSITY COLLEGE OF SCIENCE AND TECHNOLOGY ELM O FANN URMIA, P.O.BOX 57351-33746 URMIA
E-mail: Hamid.baghban66@yahoo.com

Abstract

In this Paper, I presented basic notions of Her mite Interpolations, Lagrange Interpolations and their error analysis with their approximation and Illustrative examples are provided to demons rate the efficiency and simplicity of the proposed method in solving this type of boundary Value Problems.

Mathematics Subject Classification (2000): 41A05, 65L 10

Keywords: Her mite Interpolation, Lagrange Interpolation

1. Introduction

Lagrange interpolation is one of the most elementary tools in numerical analysis, and it is used to compute the Values of a function at non-nodal positions according to the Values of the function at given nodes. While Her mite interpolation can be taken as an extension to Lagrange interpolation, primarily by using the Values of the function and it is derivatives at given nodes it constricts a polynomial such that at those nodes the polynomial is equal to the approximate function. Illustrative examples are provided to demons rate the efficiency and simplicity of the proposed method in solving this type of boundary Value Problems, using mat lab and more specifically utilizes the embedded function interpolation, solve and find Root.

2. Her mite Interpolation

Her mite Interpolation can be formulated by

$$H_m(x) = \sum_{k=0}^{m} \mathrm{M_k}\,(x) f\,(x_k) + \sum_{k=0}^{m} \mathrm{N_k}\,(x) f'(x_k) + E(x) \qquad (1)$$

Is now sought, with $M_k(x) and\ N_k(x)$ being Polynomials of degree (2m+1) to yield sufficient degrees of freedom to
Satisfy the fitting of f and f'. Given data points:

$$\{(x_j, f(x_j), f'(x_j)\}_{j=0}^{m}$$

we wish to construct a polynomial Such that $H_m \in P_{2m+1}$

$$H_m(x_j) = f(x_j) \qquad (2)$$

$$and \qquad H'_m(x_j) = f'(x_j) \qquad (3)$$

we seek degree (2m+1) polynomials $\{M_k\}_{k=0}^{m}\ and\ \{N_k\}_{k=0}^{m}$ Such that

$$M_k(x_j) = \delta_{kj} N_k(x_j) = 0 \qquad (4)$$

Where, $\delta_{kj} = \begin{cases} 0 & k \neq j \\ 1 & k = j \end{cases}$

Is the kronecker symbol.to give

$M_k(x_j) = \begin{cases} 0 & k \neq j \\ 1 & k = j \end{cases}$, $M_k'(x_j) = 0$ *for* $j = 0, \ldots m$

$N_k(x_j) = 0$ *for* $0,1, \ldots m$, $N_k'(x_j) = \begin{cases} 0 & k \neq j \\ 1 & k = j \end{cases}$ (5)

It is clear that $[l_i(x)]^2$ is a polynomial of degree 2m which comes close to satisfying some of the conditions in (4) and (5) . Where we recall the Lagrange basis polynomials used for the standard interpolation problem,

$$l_k(x) = \prod_{j=0, j\neq k}^{m} \frac{x - x_j}{x_{k-x_j}} \quad (6)$$

Consider the definitions

$$M_k(x) = r_k(x).[l_k(x)]^2 \quad (7)$$

& $N_k(x) = S_k(x).[l_k(x)]^2$ (8)

With r_k and s_k being linear functions of x , the required degree of polynomial is attained and four degrees of freedom remained to complete the satisfaction of (4) and (5) .
Indeed this conditions result the relations

$r_k(x_k) = 1$, $r_k'(x_k) + 2l_k'(x_k) = 0$ (9)
and $S_k(x_k) = 0$, $s_k'(x_k) = 1$ (10)
The linear from which satisfy these conditions are

$r_k(x) = 1 - 2l_k'(x_k)(x - x_k)$ (11)
$S_k(x) = x - x_k$ (12)
Which give Hermits interpolation formula as

$$H_m(x) = \sum_{k=0}^{m} M_k(x) f(x_k) + \sum_{k=0}^{m} N_k(x) f'(x_k) + E(x) \quad (13)$$

with $M_k(x) = [1 - 2l_k'(x_k)(x - x_k)]\,[l_k(x)]^2$ (14)
and $N_k(x) = (x - x_k)\;[l_k(x)]^2$ (15)
Note that since $l_k \in P_m$, we have M_k , $N_k \in P_{2m+1}$ and as in the formula Lagrange the problem of determining $E(x)$ remains. A similar approach to the Lagrange case yields a suitable error from. An error bound for Her mite interpolation is provided by the expression:

$$E(x) = \frac{f^{(2m+2)}(\xi)}{(2m+2)!} \prod_{j=0}^{m}(x - x_j)^2 \quad (16)$$

Where, $f \in C^{2m+2}[x_0, x_m]$,$H_m \in P_{2m+1}$ Such that
$H_m(x_j) = f(x_j)$ *and* $H_m'(x_j) = f'(x_j)$ for $j = 0, \ldots m$ and say at $\xi \in [x_0, x_m]$.

3. Compare method^s of Her mite and Lagrange Interpolations

I will solve a problem by using Her mite and Lagrange Interpolation methods, to show that which is closed to exact value or sensible at error analysis.

Example: Assume
$f(x) = \log(x).$ *and* $f(2) = 0.3010$, $f'(2) = 0.217147$, $f(3) = 0.4771$, $f'(3) = 0.144765$, $f(4) = 0.6021$, $f'(4) = 0.108574$
Evaluate the interpolations of f (2.4).

Solution:
1) By using Lagrange Interpolations from 6 gives
$L\,(2.4) = \frac{(0.6)(1.6)}{(-1)(-2)} f(2) + \frac{(0.4)(-1.6)}{1(-1)} f(3) + \frac{(0.4)(-1.6)}{2(1)} f(4) = 0.3776$
With an actual error

$E(x) = f(x) - L(x) = f(2.4) - L(2.4) = 0.0026$ *from* 16
$E(x) \leq \frac{0.4(-0.6)(-1.6)}{6} \frac{1}{4l_{n10}} = 0.0070$
This is a valid bound.

2) By using Her mite Interpolations:

$$l_x(x) = \prod_{j=0\ ,j\neq k}^{m} \frac{(x-x_j)}{(x_k-x_j)} = \frac{(x-x_.)(x-x_1)\dots(x-x_m)}{(x_k-x_.)(x_k-x_1)\dots(x_k-x_m)}$$

Gives

$l'(2) = -\frac{3}{2}$, $l_1'(3) = 0$, $l_2'(4) = \frac{3}{2}$

$M_0(x) = (3x-5)[l_0(x)]^2$

$M_1(x) = [l_1(x)]^2$

$M_2 = (13-3x)[l_2(x)]^2$

$N_0(X) = (x-2)[l_0(X)]^2$

$N_1(x) = (x-3)[l_1(x)]^2$

$N_2(x) = (x-4)[l_2(x)]^2$

Hence the interpolated value x=2.4 is given by

$$H_m(2.4) = 0.50688 * 0.301030 + 0.4096 * 0.477121 + 0.08352 * 0.602060 + 0.09216 * 0.217147 - 0.24576 * 0.144765 - 0.02304 * 0.108574 = 0.380232$$

And the associated actual error is $E(x) = f(x) - H_m(X) = f(2.4) - H_m(2.4) = 0.0000209$

On the other hand , Hermit interpolation[s] error formula gives us :

$$E(x) = (x-x_.)^2(x-x_1)^2(x-x_2)^2 \ \frac{f^{(6)}(\xi)}{6!} = (0.4)^2(0.6)^2(1.6)^2\frac{0.814}{6!} = 0.001667$$

As I said Her mite Interpolation is more better than Lagrange interpolation to give more sensible result. Because it is more sensible in error analysis.

4. The method for BVPS

We consider described by the following 2^{nd} order differential equation:

$$y'' = f(x, y, y') \quad , \quad 0 \le x \le 1 \qquad (17)$$

Associated with 2-generally also boundary conditions of the from:

$$y(0) = G \quad , \quad y(1) = \sum_{i=1}^{N-2} Qy(\xi_i) \qquad (18)$$

$$or \quad y(0) = \sum_{i=1}^{N-2} Gy(\xi_i) \quad , \quad y(1) = Q \qquad (19)$$

Where $Q, G \in \mathbb{R}$, $\xi_i \in (0,1)$, $\forall_i = 1,2,\dots N-2$

The solve command embedded in mat lab , for the solution of such a higher order differential equation requires a sufficient set of boundary conditions and the range to solve for as in plats .

Example:

Consider the nonlinear 2^{nd} order BVPS:

$$y'' + \frac{x^2(1-x)}{2}y' + y^2 = x^3 + 2$$

Such that

$$y(0) = 0$$

$$y(1) = \sum_{i=0}^{4} \frac{1}{i+1} y\left(\frac{i}{5}\right) + 0.708667$$

Withe exact solution $y(x) = x^2$

Using Taylor polynomials, We have :

$y(0) = 0$, $Y(1) = 1.000000848$, $y'(0) = 0.000000956399$, $y'(1) = 2.000000469$, $y\left(\frac{1}{5}\right) = 0.04000019122$, $y\left(\frac{2}{5}\right) = 0.1600003815$, $y\left(\frac{3}{5}\right) = 0.3600005687$, $y\left(\frac{4}{5}\right) = 0.6400007488$

Then using these data in equation (13), we get:

$$H_3(x) = -0.0000002708684265x^3 + 1.0000000163x^2 + 0.00000009564047771x$$

Acknowledgement

Acknowledging the author of articles on differential equations and other sciences is the optimal control.

Reference

[1] Wang H Y, cui F, Wang X H. Explicit representations for local lagrangian numerical differentiation, Act Math silica, 2007, 23
[2] Wang X H lai MJ, yang SJ. An the divided differences of the remainder in polynomial interpolation, J Approx. X Theory, 2004,127
[3] Howell K.B, or denary differential equation, springer, USA, 2009
[4] Stoer, Inter diction to Numerical Analysis, 2002