

Optimal line based on AHP and Kruskal Algorithm

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Abstract

In order to determine the optimal route of drug delivery route and send the drug to the patient timely. Use AHP to select and optimize, get the weight relationship between the two cities who need distribution of vaccines, then use the Kruskal algorithm to calculate the optimal path for drug delivery. For example, select 6 node regions in Sierra Leone to establish line network, according to the above model to determine the weight value among six node line and set up the weight matrix, then get the optimal line that combine with the forthcoming delivery of the drug to optimize and calculate rational drug delivery systems.

Keywords: Analytic Hierarchy Process, Kruskal Algorithm, Optimal Route.

1. Introduction

The shortest path problem is one of the fundamental questions of network optimization, and has become the city road traffic, operational command automation, network communications, urban planning, and many other network optimization sub-problems. Domestic and foreign scholars have done a lot of research in this area. For example, famous American mathematician, originator of dynamic programming R. Bellman^[1] study this issue in 1958; Dutch computer scientist E.W. Dijkstra^[2] made the famous Dijkstra algorithm to solve this problem in 1959; Cherkassky^[3] made theoretical analysis and experimental evaluation on Bellman-Ford-Moore algorithm, Dijkstra algorithm, Incremental Graph and so on 17 kinds shortest path algorithm in 1996. Based on different characteristics by random network is found, all the algorithms are not able to maintain our competitive edge under any circumstances; Lu Feng^[4] for the same question in order to limit the distance between the source point, target point to construct an elliptical limit the search area, and on this basis, proposed a circumscribed rectangle of the ellipse to limit the search area, simplifying a lot of the product and open computing in the elliptical model; Wang Hui^[5] used regular network to calculate the function of the relative position between long and short axis and the target point and between long and short axis and the source ellipse when studying oval model, and the method combined with the hierarchical search method is applied to study the shortest path problem in Washington Baltimore area. Italian scholars M. Dorigo, Maniezzo^[6] proposed ant colony algorithm to simulate the true foraging behavior of ant

colonies in nature and formed a simulation evolutionary algorithms in the 1990s. They took full advantage of the similarities between the process of ant colony foraging and the traveling salesman problem, solving the TSP problem well; Holland^[7] in Millikan University learned from biological natural selection and the mechanisms of genetic evolution and developed a global optimization adaptive probabilistic search algorithm; Zou Ligng^[8] used stochastic Dijkstra algorithm to solve the problems in the initial population of genetic algorithm in dynamic networks shortest path problem.

Genetic algorithm and ant colony algorithm has been widely used to solve all kinds of the shortest path problem, but the results of such intelligent control algorithm is not accurate enough and the viewpoint is not comprehensive enough. it is necessary to use AHP to determine weights between the nodes line and then use Kruskal algorithm to solve optimal route.

2. An Optimized Model for Drug Delivery

In order to obtain the best way to deliver drugs, use BP neural network and AHP to optimize and select, getting weights relationship between every two cities. use a_{ij} to represent the weights relationship, getting weight matrix as follows.

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1k} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2k} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{kk} & \cdots & a_{kn} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nk} & \cdots & a_{nn} \end{pmatrix}$$

we can use the aforementioned weight matrix, with the minimum spanning tree method in graph theory, to calculate a country's optimal transport routes. For example, the following drug delivery in the country of Sierra Leone, creating the following optimization model.

3. The Establishment of Optimization Model Based on the and the Analytic Hierarchy Process

First selected a typical infected area in Sierra Leone, and use 1-6 to number them respectively. As shown in Figure 1,

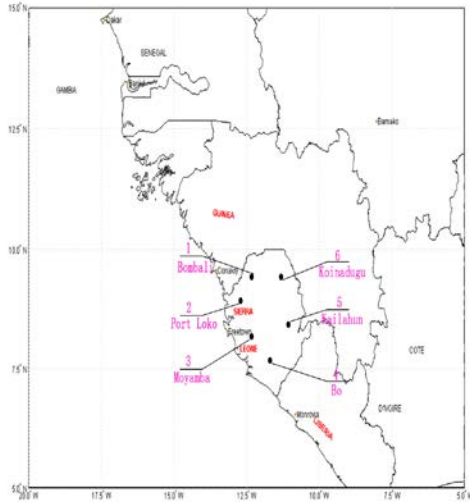


Fig.1 The typical region that are infected in this country

Analytic Hierarchy Process^[9-10] is a quantitative and qualitative analysis of multiobjective decision, quantifying the experience of policy-makers, which is more practical when the target (factor) structure is complex and the lack of necessary data. First, establish a relationship among goal, rule, scheme and layers. Optimal route as Target layer, Disease status C_1 and Mode of transport C_2 and Medicinal properties C_3 and Geographical environment C_4 as Guidelines layer, a_{ij} as Scheme layer, Hypothesis: a_{ij} represents weight the i area to the j area. i, j are all the code of the country, and $i, j \in [1, 6]$. Among them "1" represents Bombali, "2" represents Port Loko, "3" represents Moyamba, "4" represents Bo, "5" represents Kailahun, "6" represents Koinadugu. The first relationship is shown below in Figure 2.

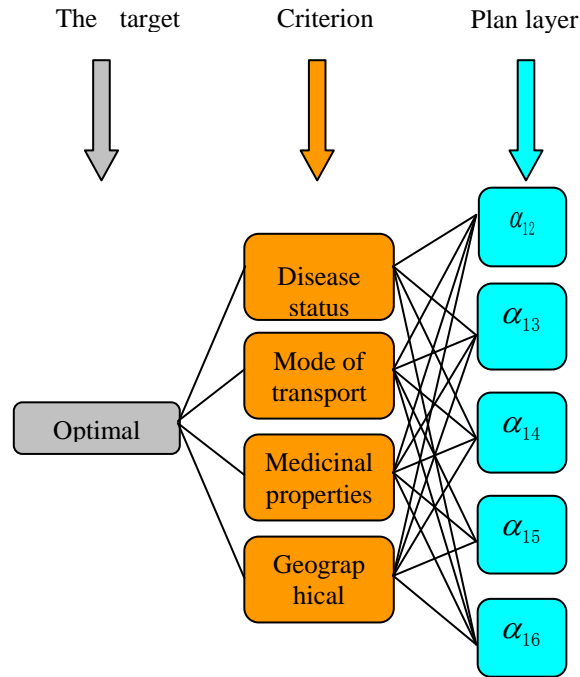


Fig.2 The relationship among goal, rule, scheme and layers

The so-called judgment matrix in the form of a matrix to describe each level of the relative importance of each element relative to its parent element. In order to compare every two factors among various factors to obtain the quantification judgment matrix, introduces 1~9 scales, as shown in table 1.

Tab. 1 1-9 scale tables

scaling a_{ij}	definitions
1	i and j factors are equally important factors
3	i factor is slightly more important than j factor
5	i factor more important than j factor
7	i factor are very important than j factor
9	i factor important absolutely than j factor
2, 4, 6, 8	For above judgment between compound state correspondence scale value
reciprocal	If compare i factor with the j factor, obtains the judgment value is,
	$a_{ji} = 1/a_{ij}, a_{ii} = 1$

First solving judgment matrix, according to above principle, refers to 1~9 scale establishments, and according to the expert and author's experience and the reference massive literature, obtains under five criteria two to two comparison matrices, as shown below:

$$\begin{pmatrix} 1 & 3 & 9 & 5 \\ \frac{1}{3} & 1 & 7 & 3 \\ \frac{1}{9} & \frac{1}{7} & 1 & \frac{1}{3} \\ \frac{1}{5} & \frac{1}{3} & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 5 & 1 & 5 \\ \frac{1}{5} & 1 & 1 & \frac{1}{5} & 1 \\ \frac{1}{5} & 1 & 1 & \frac{1}{5} & 1 \\ \frac{1}{5} & 1 & 1 & \frac{1}{5} & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{5} & \frac{1}{7} & \frac{1}{3} \\ 3 & 1 & \frac{1}{2} & \frac{1}{5} & 2 \\ 5 & 2 & 1 & \frac{1}{3} & 3 \\ 7 & 5 & 3 & 1 & 5 \\ 3 & \frac{1}{2} & \frac{1}{3} & \frac{1}{5} & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{5} & \frac{1}{7} & \frac{1}{3} \\ 3 & 1 & \frac{1}{3} & \frac{1}{5} & \frac{1}{2} \\ 5 & 3 & 1 & \frac{1}{2} & 2 \\ 7 & 5 & 3 & 1 & 5 \\ 3 & 2 & \frac{1}{2} & \frac{1}{5} & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & \frac{1}{5} & \frac{1}{3} \\ \frac{1}{2} & 1 & 2 & \frac{1}{7} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{2} & 1 & \frac{1}{3} & 3 \\ 5 & 7 & 3 & 1 & 5 \\ 3 & 5 & \frac{1}{3} & \frac{1}{5} & 1 \end{pmatrix}$$

Use MATLAB to carry on the computation, to obtain the corresponding maximum eigenvalue and eigenvector results were as follows:

$$\lambda_{\max}^{(0)} = 4.0876,$$

$$\omega^{(0)} = (-0.8885 \quad -0.4202 \quad -0.0699 \quad -0.1708)^T;$$

$$\lambda_{\max}^{(1)} = 5.0000,$$

$$\omega^{(1)} = (0.6868 \quad 0.1374 \quad 0.1374 \quad 0.6868 \quad 0.1374)^T;$$

$$\lambda_{\max}^{(2)} = 5.1438,$$

$$\omega^{(2)} = (0.0796 \quad 0.2232 \quad 0.3888 \quad 0.8762 \quad 0.1582)^T;$$

$$\lambda_{\max}^{(3)} = 5.1438,$$

$$\omega^{(3)} = (0.0796 \quad 0.1582 \quad 0.3888 \quad 0.8762 \quad 0.2232)^T;$$

$$\lambda_{\max}^{(4)} = 6.4635,$$

$$\omega^{(4)} = (-0.2494 \quad -0.1522 \quad -0.2594 \quad -0.8618 \quad -0.3236)^T$$

Testing consistency index :

$$CI = \frac{\lambda_{\max} - n}{n - 1}$$

λ_{\max} is the biggest characteristic value of comparison matrix, n is the matrix order. The smaller of the CI value is, the closer of the judgment matrix is in quite the same. On the other hand, the degree of the judgment matrix is the greater.

Table11 RI Value

n	1	2	3	4	5	6	7
RI	0	0	0.58	0.90	1.12	1.24	1.32

Therefore $CR = \frac{CI}{RI} = 0.0325 < 0.1$. Representatives that

A degree of inconsistency is in allowable range, can be substituted for A feature vector weight vector at this time. The calculation results are as follows:

$$\eta^{(1)} = (0.2500 \quad 0.0924 \quad 0.1309 \quad 0.4326 \quad 0.0941)^T$$

Diagrams are shown in Fig. 3 below for the second time to set up:

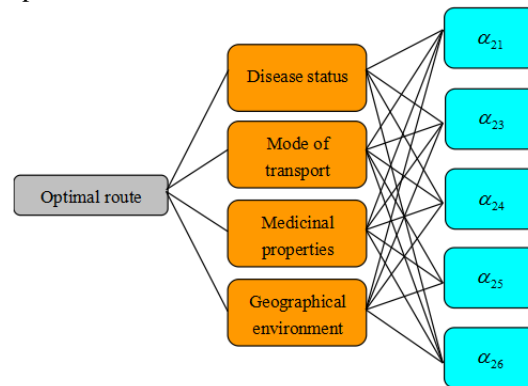


Fig.3 The Hierarchical structure

Repeat the above steps:

$$\eta^{(1)} = (0.2451 \quad 0.0677 \quad 0.1059 \quad 0.3631 \quad 0.2182)^T$$

Again on 3, 4, 5, and 6 as a starting point to repeat the above four steps, the final comprehensive between each weight, get the following weight matrix:

$$\begin{pmatrix} 0.2500 & 0.2451 & 0.3358 & 0.3726 & 0.3480 & 0.2473 \\ 0.0924 & 0.0671 & 0.0979 & 0.1413 & 0.3407 & 0.2509 \\ 0.1309 & 0.1059 & 0.1255 & 0.1328 & 0.0949 & 0.1566 \\ 0.4326 & 0.3631 & 0.1552 & 0.1018 & 0.0735 & 0.1686 \\ 0.0941 & 0.2181 & 0.2856 & 0.2515 & 0.1429 & 0.1766 \end{pmatrix}$$

Note: the matrix is used for road weight value in the following graph.

4. The Choice of Optimal Route based on the Kruskal algorithm

The best drug transport path is achieved according to the Kruskal algorithm^[11]. The algorithm is as follows:

- (i) Choose $e_1 \in E(G)$, make $f(t, s) = -\delta S(t)I(t)$.
- (ii) If e_1, e_2, \dots, e_i are chosen, then choose e_{i+1} from $E(G) - \{e_1, e_2, \dots, e_i\}$, and make
 - ① $G[\{e_1, e_2, \dots, e_i, e_{i+1}\}]$ is no ring, and
 - ② $w(e_{i+1}) = \max$
- (iii) until choose e_{v-1} .

Because the country needed drugs need to be imported from abroad, and the country only in the Port Loko has international airport (" 2 ") region, thus set a starting point for the Port Loko, and between cities has obtained all the weights in the 5.4.1 (in this paper, the larger the weights of the road the best), the path can be obtained by Kruskal algorithm as shown in Fig. 4.

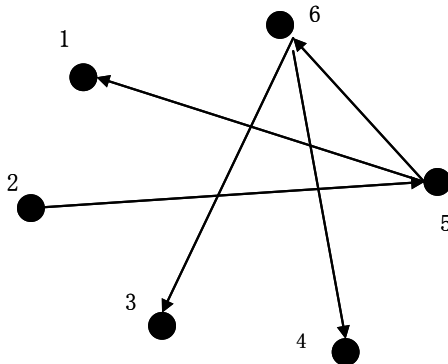


Fig. 4 Optimal Route

The adjacency matrix of the above is

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

It can be known that transport routes of Fig. 9 are the best drug transport routes. And a huge mass of data confirm that the above results are close to the actual results. Therefore, the model has good accuracy.

5. Conclusions

Use AHP to select and optimize, get the weight relationship between the two cities who need distribution of vaccines, then use the Kruskal algorithm to calculate the optimal path for drug delivery. For example, select 6 node regions in Sierra Leone to establish line network, according to the above model to determine the weight value among six node line and set up the weight matrix, then get the optimal line that combine with the forthcoming delivery of the drug to optimize and calculate rational drug delivery systems. It is Port Loko to Kailahun to Bombali; Port Loko to Kailahun to Koinadugu to Moyamba; Port Loko to Kailahun to Koinadugu to Bo Three best routes.

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