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Analytic Mean Labeling of Cycle Related Graphs

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Abstract

In this paper, we investigate an analytic mean labeling for some graphs obtained by duplication of graph elements, $S'(K_{1,n})$ and tadpole.

Keywords: *Analytic Mean Labeling, Analytic Mean Graph, Duplication of a vertex, Duplication of an edge.*

1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [2]. For standard terminology and notations related to graph labeling, we refer to Gallian [1]. In [4], Tharmaraj et al. introduce the concept of an analytic mean labeling of graph. Analytic mean labeling of various types of graphs are presented in [5]. The brief summaries of definition which are necessary for the present investigation are provided below.

2. Definitions

Definition :2.1

A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph.

Definition :2.2 [3]

For a graph G, the splitting graph $S'(G)$ of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Definition :2.3 [7]

Duplication of a vertex v_k by a new edge $e = v'_k v''_k$ in a graph G produces a new graph G' such that $N(v'_k) = \{v_k, v''_k\}$ and $N(v''_k) = \{v_k, v'_k\}$.

Definition :2.4 [7]

Duplication of an edge $e = v_i v_{i+1}$ by a vertex v_k in a graph G produces a new graph G' such that $N(v_k) = \{v_i, v_{i+1}\}$.

Definition :2.5 [9]

Duplication of a vertex v_k of a graph G produces a new graph G' by adding a new vertex v'_k in such that $N(v_k) = N(v'_k)$.

Definition :2.6 [8]

Duplication of an edge $e_k = v_k v_{k+1}$ of a graph G produces a new graph G' by adding an edge $e'_k = v'_k \, v'_{k+1}$ such that $N(v'_k) = N(v_k) \cup \{v'_{k+1}\} - \{v_{k+1}\}$ and $N(v'_{k+1}) = N(v_{k+1}) \cup \{v'_k\} - \{v_k\}$.

Definition :2.7 [6]

The tadpole graph is formed by joining the end point of a path P_m to a cycle C_n. It is denoted by C_n @ P_m.

Definition :2.8 [4]

A (p,q) graph G(V,E) is said to be an analytic mean graph if it is possible to label the vertices v in V with distinct from 0,1,2,…, p−1 in such a way that when each edge e = uv is labeled with $f^*(e = uv) = \frac{|[f(u)]^2 - [f(v)]^2|}{2}$ if $|[f(u)]^2 - [f(v)]^2|$ is even and if $\frac{|[f(u)]^2 - [f(v)]^2| + 1}{2}$ if $|[f(u)]^2 - [f(v)]^2|$ is odd and the edge labels are distinct. In this case, f is called an analytic mean labeling of G. A graph with an analytic mean labeling is called an analytic mean graph.

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3. Main Results

Theorem 3.1

$S'(K_{1,n})$ is an analytic mean graph.

Proof :

Let $v_1, v_2, v_3, \ldots, v_n$ be the pendant vertices and v be the apex vertex of $K_{1,n}$ and u, u_1, u_2, u_3, …, u_n are added vertices corresponding to v, v_1, v_2, v_3, …, v_n to obtain $S'(K_{1,n})$.

Let G be the graph $S'(K_{1,n})$

Then $|V(G)| = 2n + 2$ and $|E(G)| = 3n$.

Define $f : V(G) \to \{0,1,2,\ldots, 2n+1\}$ by

$f(v) = 0$,

$f(u) = 1$,

$f(v_i) = 2i + 1$, for $1 \le i \le n$;

$f(u_i) = 2i$, for $1 \le i \le n$;

Let f^* be the induced edge labeling of f.

Then

$$f^*(vv_i) = \frac{(2i+1)^2+1}{2}, \quad \text{for } 1 \le i \le n$$

$$f^*(uv_i) = \frac{(2i+1)^2-1}{2}, \quad \text{for } 1 \le i \le n$$

$$f^*(vu_i) = \frac{(2i)^2}{2}, \quad \text{for } 1 \le i \le n$$

Then the induced edge labels are $\{2,4,5,\ldots, \frac{4n^2+4n+2}{2}\}$.

Therefore, $S'(K_{1,n})$ is an analytic mean graph.

Example 3.1

Analytic mean labeling of $S'(K_{1,4})$ is given in figure 3.1.

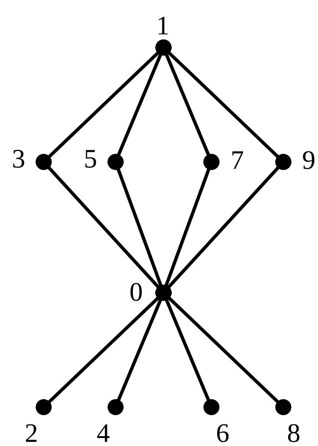

Figure 3.1

Theorem 3.2

The graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_n admits an analytic mean labeling.

Proof :

Let $v_1, v_2, \ldots, v_n$ be the vertices of cycle C_n and let G be the graph obtained by duplicating an arbitrary vertex of C_n by a new edge.

Without loss of generality let this vertex be v_1 and the edge be $e = v_1' v_1''$.

Then $|V(G)| = n+2$ and $|E(G)| = n+3$.

Define $f : V(G) \to \{0,1,2,\ldots, n+1\}$ by

For n = 3

$f(v_1) = 0$, $f(v_2) = 2$, $f(v_3) = 4$, $f(v_1') = 3$ and $f(v_1'') = 1$.

Let f^* be the induced edge labeling of f.

Then $f^*(v_1v_2) = 2$, $f^*(v_2v_3) = 6$, $f^*(v_3v_1) = 8$, $f^*(v_1' v_1) = 5$, $f^*(v_1'' v_1) = 1$ and $f^*(v_1' v_1'') = 4$.

Then the induced edge labels are {1, 2, 4, 5, 6, 8}.

For n = 4

$f(v_1) = 0$, $f(v_2) = 2$, $f(v_3) = 3$, $f(v_4) = 4$, $f(v_1') = 5$ and $f(v_1'') = 1$.

Let f^* be the induced edge labeling of f.

Then $f^*(v_1v_2) = 2$, $f^*(v_2v_3) = 3$, $f^*(v_3v_4) = 4$, $f^*(v_4v_1) = 8$, $f^*(v_1' v_1) = 13$, $f^*(v_1'' v_1) = 1$ and $f^*(v_1' v_1'') = 12$.

Then the induced edge labels are {1, 2, 3, 4, 8, 12, 13}.

For $n \ge 5$

$f(v_1') = n$

$f(v_1'') = n+1$

$f(v_i) = i - 1$, for $1 \le i \le n$

Let f^* be the induced edge labeling of f.

Then $f^*(v_iv_{i+1}) = i$, for $1 \le i \le n-1$

$f^*(v_1' v_1'') = n+1$,

$$f^*(v_nv_1) = \begin{cases} \frac{(n-1)^2}{2} & \text{if } n \text{ is odd} \\ \frac{(n-1)^2+1}{2} & \text{if } n \text{ is even} \end{cases}$$

$$f^*(v_1' v_1) = \begin{cases} \frac{n^2+1}{2} & \text{if } n \text{ is odd} \\ \frac{n^2}{2} & \text{if } n \text{ is even} \end{cases}$$

$$f^*(v_1'' v_1) = \begin{cases} \frac{(n+1)^2}{2} & \text{if } n \text{ is odd} \\ \frac{(n+1)^2+1}{2} & \text{if } n \text{ is even} \end{cases}$$

Then the induced edge labels are

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$$\{1,2,3,\ldots,\frac{(n+1)^2+1}{2} \text{ or } \frac{(n+1)^2}{2}\}.$$

Therefore, the graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_n is an analytic mean graph.

Example 3.2

Analytic mean labeling of the graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_8 is given in figure 3.2.

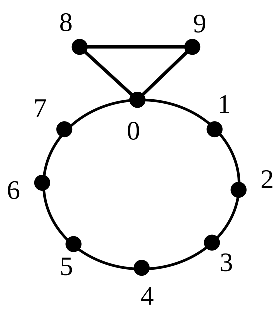

Figure 3.2

Theorem 3.3

Duplication of an arbitrary edge by a new vertex in cycle C_n produces an analytic mean graph.

Proof :

Let $v_1,v_2,\ldots,v_n$ be the vertices of cycle C_n.

Let G be the graph obtained by duplication of an arbitrary edge in C_n by a new vertex.

Without loss of generality let this edge be $e = v_nv_1$ and the vertex be v'.

Then $|V(G)| = n+1$ and $|E(G)| = n+2$.

Define $f : V(G) \to \{0,1,2,\ldots, n\}$ by

For $n = 3$

$f(v_1) = 0$, $f(v_2) = 1$, $f(v_3) = 3$ and $f(v') = 2$.

Let f^* be the induced edge labeling of f.

Then $f^*(v_1v_2) = 1$, $f^*(v_2v_3) = 4$, $f^*(v_3v_1) = 5$, $f^*(v_3v') = 3$ and $f^*(v_1v') = 2$.

Then the induced edge labels are $\{1, 2, 3, 4, 5\}$.

For $n \geq 4$

$f(v') = n$

$f(v_i) = i - 1$, for $1 \leq i \leq n$

Let f^* be the induced edge labeling of f.

Then

$f^*(v_iv_{i+1}) = i$, for $1 \leq i \leq n-1$

$f^*(v_nv') = n$,

$$f^*(v_nv_1) = \begin{cases} \frac{(n-1)^2}{2} & \text{if } n \text{ is odd} \\ \frac{(n-1)^2+1}{2} & \text{if } n \text{ is even} \end{cases}$$

$$f^*(v'v_1) = \begin{cases} \frac{n^2+1}{2} & \text{if } n \text{ is odd} \\ \frac{n^2}{2} & \text{if } n \text{ is even} \end{cases}$$

Then the induced edge labels are

$$\{1,2,3,\ldots,\frac{n^2+1}{2} \text{ or } \frac{n^2}{2}\}.$$

Therefore, the graph obtained by duplication of an arbitrary edge by a new vertex in cycle C_n is an analytic mean graph.

Example 3.3

Analytic mean labeling of the graph obtained by duplication of an arbitrary edge by a new vertex in cycle C_8 is given in figure 3.3.

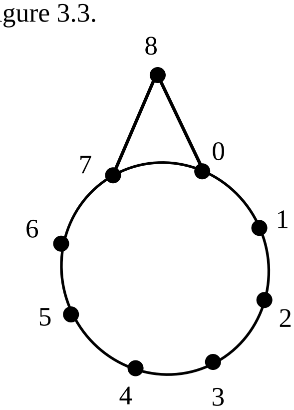

Figure 3.3

Theorem 3.4

The graph obtained by duplication of an arbitrary vertex of C_n admits an analytic mean labeling.

Proof :

Let $v_1,v_2,\ldots,v_n$ be the vertices of the cycle C_n.

Let G be the graph obtained by duplicating an arbitrary vertex of C_n.

Without loss of generality let this vertex be v_1 and the newly added vertex be v'_1.

$E(G) = \{E(C_n), e', e''\}$ where $e' = v'_1v_2$ and $e'' = v_n v'_1$.

Then $|V(G)| = n+1$ and $|E(G)| = n+2$.

Define $f : V(G) \to \{0,1,2,\ldots, n\}$ by

For $n = 3$

$f(v_1) = 0$, $f(v_2) = 1$, $f(v_3) = 3$ and $f(v'_1) = 2$.

Let f^* be the induced edge labeling of f.

Then $f^*(v_1v_2) = 1$, $f^*(v_2v_3) = 4$, $f^*(v_3v_1) = 5$, $f^*(v_3 v_1') = 3$ and $f^*(v_1' v_2) = 2$.

Then the induced edge labels are {1, 2, 3, 4, 5}.

For $n \geq 4$

$f(v_1') = n$

$f(v_i) = i - 1, \quad \text{for } 1 \leq i \leq n$

Let f^* be the induced edge labeling of f.

Then

$f^*(v_iv_{i+1}) = i, \quad \text{for } 1 \leq i \leq n-1$

$f^*(v_n v_1') = n,$

$$f^*(v_nv_1) = \begin{cases} \frac{(n-1)^2}{2} & \text{if } n \text{ is odd} \\ \frac{(n-1)^2+1}{2} & \text{if } n \text{ is even} \end{cases}$$

$$f^*(v_1' v_2) = \begin{cases} \frac{n^2-1}{2} & \text{if } n \text{ is odd} \\ \frac{n^2}{2} & \text{if } n \text{ is even} \end{cases}$$

Then the induced edge labels are

$$\{1,2,3,\ldots, \frac{n^2-1}{2} \text{ or } \frac{n^2}{2}\}.$$

Therefore, the graph obtained by duplication of an arbitrary vertex of C_n is an analytic mean graph.

Example 3.4

Analytic mean labeling of the graph obtained by duplication of an arbitrary vertex in cycle C_8 is given in figure 3.4.

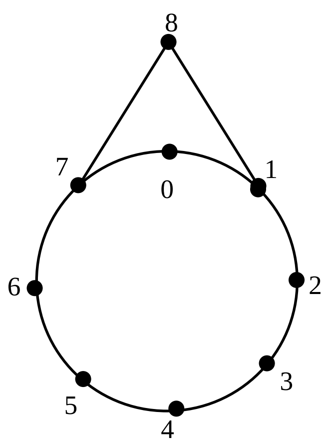

Figure 3.4

Theorem 3.5

The graph obtained by duplication of an arbitrary edge in C_n admits an analytic mean labeling.

Proof :

Let $v_1,v_2,\ldots,v_n$ be the vertices of the cycle C_n.

Let G be the graph obtained by duplicating an arbitrary edge of C_n.

Without loss of generality let this edge be $e_1 = v_1v_2$ and the newly added edge be $e_1' = v_1' v_2'$.

$E(G) = \{E(C_n), e_1', e', e''\}$ where $e' = v_2' v_3$ and $e'' = v_n v_1'$.

Then $|V(G)| = n+2$ and $|E(G)| = n+3$.

Define $f : V(G) \to \{0,1,2,\ldots, n+1\}$ by

For n = 3

$f(v_1) = 1$, $f(v_2) = 3$, $f(v_3) = 0$, $f(v_1') = 2$ and $f(v_2') = 4$.

Let f^* be the induced edge labeling of f.

Then $f^*(v_1v_2)= 4$, $f^*(v_2v_3) = 5$, $f^*(v_3v_1) = 1$, $f^*(v_1' v_n) =2$, $f^*(v_2' v_3) = 8$ and $f^*(v_1' v_2') = 6$.

Then the induced edge labels are {1, 2, 4, 5, 6, 8}.

For n = 4

$f(v_1) = 2$, $f(v_2) = 3$, $f(v_3) = 1$, $f(v_4) = 0$, $f(v_1') = 4$, and $f(v_2') = 5$.

Let f^* be the induced edge labeling of f.

Then $f^*(v_1v_2) = 3$, $f^*(v_2v_3) = 4$, $f^*(v_3v_4) = 1$, $f^*(v_4v_1) = 2$, $f^*(v_1' v_4) = 8$, $f^*(v_2' v_3) = 12$ and $f^*(v_1' v_2') = 5$.

Then the induced edge labels are {1, 2, 3, 4, 5, 8, 12}.

For $n \geq 5$

$f(v_1') = n$

$f(v_2') = n+1$

$f(v_i) = i - 1, \quad \text{for } 1 \leq i \leq n$

Let f^* be the induced edge labeling of f.

Then

$f^*(v_iv_{i+1}) = i, \quad \text{for } 1 \leq i \leq n-1$

$f^*(v_n v_1') = n$

$f^*(v_1' v_2') = n+1,$

$$f^*(v_nv_1) = \begin{cases} \frac{(n-1)^2}{2} & \text{if } n \text{ is odd} \\ \frac{(n-1)^2+1}{2} & \text{if } n \text{ is even} \end{cases}$$

$$f^*(v_2' v_1) = \begin{cases} \frac{n^2+2n-3}{2} & \text{if } n \text{ is odd} \\ \frac{n^2+2n-2}{2} & \text{if } n \text{ is even} \end{cases}$$

Then the induced edge labels are

$$\{1,2,3,\ldots, \frac{n^2+2n-3}{2} \text{ or } \frac{n^2+2n-2}{2}\}.$$

Therefore, the graph obtained by duplication of an arbitrary edge in C_n is an analytic mean graph.

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Example 3.5

Analytic mean labeling of the graph obtained by duplication of an arbitrary edge in cycle C_8 is given in figure 3.5.

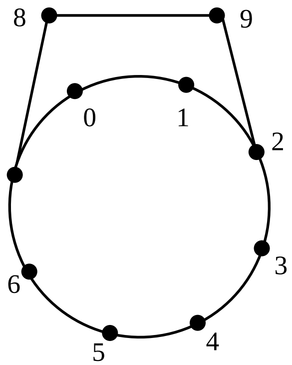

Figure 3.5

Theorem 3.6

The tadpole C_n @ P_m is an analytic mean graph.

Proof :

Let $v_1, v_2, \ldots, v_m$ be the vertices of path P_m and $v_m, v_{m+1}, \ldots, v_{m+n-1}$ be the vertices of cycle C_n.

Tadpole G = C_n @ P_m has m+n−1 vertices and m+n−1 edges.

Then $|V(G)| = m+n-1$ and $|E(G)| = m+n-1$.

Define $f : V(G) \to \{0,1,2,\ldots, m+n-2\}$ by

$f(v_i) = i - 1, \quad \text{for } 1 \le i \le m+n-1$

Let f^* be the induced edge labeling of f.

Then

$f^*(v_i v_{i+1}) = i, \quad \text{for } 1 \le i \le m+n-2$

$$f^*(v_{m+n-1} v_1) = \begin{cases} \dfrac{n^2 + 2nm - 2m - 4n + 4}{2} & \text{if } m+n \text{ is odd} \\ \dfrac{n^2 + 2nm - 2m - 4n + 3}{2} & \text{if } m+n \text{ is even} \end{cases}$$

Then the induced edge labels are $\{1,2,3,\ldots, \dfrac{n^2 + 2nm - 2m - 4n + 3}{2} \text{ or } \dfrac{n^2 + 2nm - 2m - 4n + 4}{2}\}$.

Therefore, tadpole C_n @ P_m is an analytic mean graph.

Example 3.6

Analytic mean labeling tadpole graph C_4 @ P_5 is given in figure 3.6.

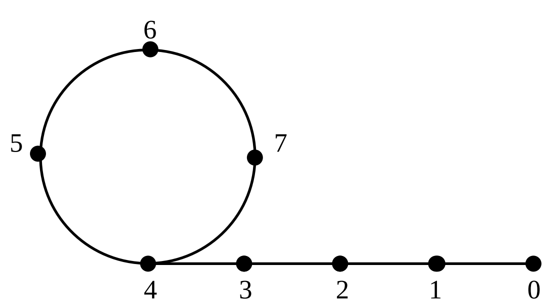

Figure 3.6

4. Conclusions

In this paper, we prove an analytic mean labeling of $S'(K_{1,n})$, the graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_n, the graph obtained by duplication of an arbitrary edge by a new vertex in cycle C_n, the graph obtained by duplication of an arbitrary vertex by of C_n, the graph obtained by duplication of an arbitrary edge of C_n and tadpole.

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