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Results On Cycle Related Hetro-Cordial Graphs

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Abstract – Let G = (V,E) be a graph with p vertices and q edges. A Hetro-Cordial labeling of a graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 0 if f(u) =f(v) or 1 if f(u) ≠ f(v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. The graph that admits a Hetro-Cordial labeling is called a Hetro-Cordial graph (HeCG). In this paper, we proved that Cycle related graphs Cycle C_n(n-odd), Double triangular snake $C_2(P_n)$, $D_2(C_n)$, Globe Gl(n), $C_n \Theta K_1$ are Hetro-Cordial Graphs.

Keywords-Hetro-Cordial Graph, Hetro-Cordial labeling.2000 Mathematics Subject classification 05C78.

I.INTRODUCTION

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {uv} of vertices in E is called edges or a line of G. In this paper, we proved that Cycle related graphs Cycle C_n (n-odd), Double triangular snake $C_2(P_n)$, $D_2(C_n)$, Globe Gl(n), $C_n \Theta K_1$ are Hetro-Cordial graphs. For graph theory terminology, we follow [2]

II.PRELIMINARIES

Let G = (V,E) be a graph with p vertices and q edges. A Hetro-Cordial labeling of a graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 0 if f(u) =f(v) or 1 if f(u) ≠ f(v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a Hetro-Cordial labeling is called a Hetro-Cordial graph (HeCG). We proved that Cycle related graphs Cycle C_n(n-odd), Double triangular snake $C_2(P_n)$, $D_2(C_n)$, Globe Gl(n), $C_n \Theta K_1$ are Hetro-Cordial graphs.

Definition 2.1

A closed path is called a cycle and a cycle of length n is denoted by C_n.

Definition 2.2

Graph obtained from a path P_n,by joining each end vertices of an edge with two isolated vertex. It is denoted by $C_2(P_n)$.

Definition 2.3

Let G be a connected graph. A graph constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G to the neighbours of the corresponding vertex v in G_2, that is for every vertex u in G_1 there exists v in G_2 such that N(u)=N(v). The resulting graph is known as shadow graph and it is denoted by $D_2(G)$.

Definition 2.4

Globe is defined as the two isolated vertex are joined by n paths of length 2. It is denoted by G(n).

Definition: 2.5

The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 points) and P_1 copies of G_2 and joining the i^{th} point of G_1 to every point in the i^{th} copy of G.

III.MAIN RESULTS

Theorem:3.1

Cycle C_n (n-odd) is Hetro-Cordial Graph.

Proof:

` Let $V(C_n) = \{[u_i : 1 \leq i \leq n]\}$ and

$E(C_n) = \{[(u_i u_{i+1}) : 1 \leq i \leq n\text{-}1] \cup [(u_1 u_n)]\}$.

Define $f : V(C_n) \rightarrow \{0,1\}$.

Case: 1

When n = 3,

The labeling is,

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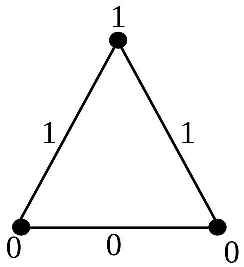

Case: 2

When $n > 3$,

The vertex labeling are ,

$$f(u_i) = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \le i \le n$$

The induced edge labeling are,

$$f^*[(u_i u_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n\text{-}1$$

$$f^*[(u_1 u_n)] = \begin{cases} 0 & n \equiv 1 \bmod 4 \\ 1 & n \equiv 3 \bmod 4 \end{cases}$$

Here, $v_f(0) = v_f(1)+1$ for $n \equiv 3 \bmod 4$,

$v_f(1) = v_f(0)+1$ for $n \equiv 1 \bmod 4$,

$e_f(0) = e_f(1)+1$ for $n \equiv 1 \bmod 4$ and

$e_f(1) = e_f(0)+1$ for $n \equiv 3 \bmod 4$.

Therefore, Cycle C_n satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) – e_f(1)| \le 1$.

Hence,Cycle C_n (n-odd) is Hetro-Cordial.

For example, Hetro-Cordial labeling of cycle C_5 is shown in the fig 3.2

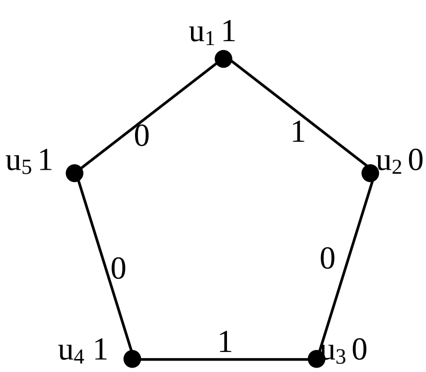

fig 3.2: C_5

Theorem:3.3

$D_2(C_n)$ is Hetro-Cordial Graph.

Proof:

Let $V(D_2(C_n)) = \{[u_i, v_i : 1 \le i \le n]\}$ and

$E(D_2(C_n)) = \{[(u_i u_{i+1}) \cup (u_i v_{i+1}) \cup (v_i u_{i+1}) \cup (v_i v_{i+1}) : 1 \le i \le n\text{-}1]$

$\cup[(u_1 u_n) \cup (v_1 v_n) \cup (u_1 v_n) \cup (v_1 u_n)]\}$.

Define $f : V(D_2(C_n)) \rightarrow \{0,1\}$.

The vertex labeling are ,

$f(u_i) = 0 \quad 1 \le i \le n$

$f(v_i) = 1 \quad 1 \le i \le n$

The induced edge labeling are,

$f^*[(u_i u_{i+1})] = 0 \quad 1 \le i \le n\text{-}1$

$f^*[(v_i v_{i+1})] = 0 \quad 1 \le i \le n\text{-}1$

$f^*[(u_i v_{i+1})] = 1 \quad 1 \le i \le n\text{-}1$

$f^*[(v_i u_{i+1})] = 1 \quad 1 \le i \le n\text{-}1$

$f^*[(u_1 u_n)] = 0$

$f^*[(v_1 v_n)] = 0$

$f^*[(v_1 u_n)] = 1$

$f^*[(u_1 v_n)] = 1$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1)$ for all n.

Therefore, $D_2(C_n)$ satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, $D_2(C_n)$ is Hetro-Cordial.

For example, Hetro-Cordial labeling of $D_2(C_5)$ and $D_2(C_6)$ are shown in the fig 3.4 and fig 3.5 respectively.

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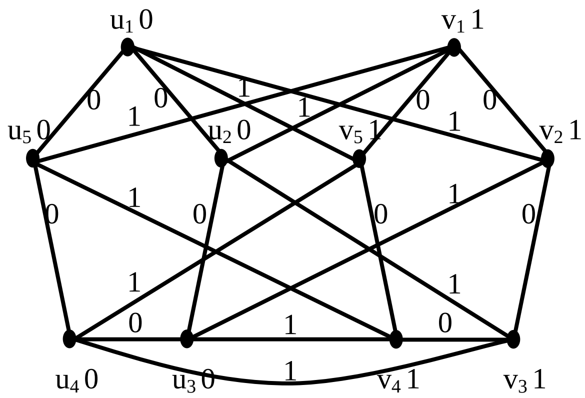

fig 3.4: $D_2(C_5)$

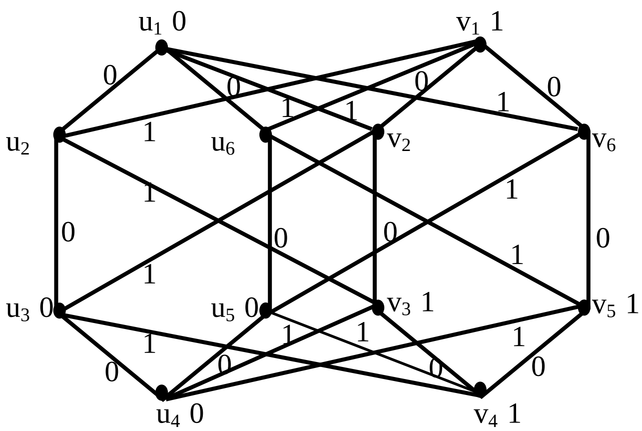

fig 3.5: $D_2(C_6)$

Theorem:3.6

Globe Gl(n) (n-even) is Hetro-Cordial Graph.

Proof:

Let $V(Gl(n)) =\{[u,v,u_i : 1\leq i\leq n]\}$ and

$E(Gl(n)) = \{[(uu_i) \cup (vu_i) : 1\leq i\leq n]\}$.

Define $f : V(Gl(n))\rightarrow\{0,1\}$.

The vertex labeling are ,

$f(u) = 0$

$f(v) = 1$

$f(u_i) = 0 \qquad 1 \leq i \leq \frac{n}{2}$

$f(u_i) = 1 \qquad \frac{n}{2}+1 \leq i \leq n$

The induced edge labeling are,

$f^*[(uu_i)] = 0 \qquad 1 \leq i \leq \frac{n}{2}$

$f^*[(uu_i)] = 1 \qquad \frac{n}{2}+1 \leq i \leq n$

$f^*[(vu_i)] = 1 \qquad 1 \leq i \leq \frac{n}{2}$

$f^*[(vu_i)] = 0 \qquad \frac{n}{2}+1 \leq i \leq n$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1)$ for all n.

Therefore, Globe Gl(n) satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, Globe Gl(n) n-even is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of Globe Gl(4) is shown in the following fig 3.7

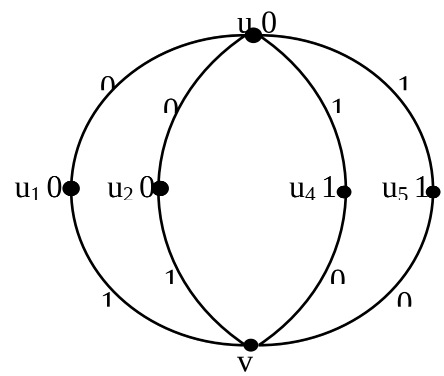

fig 3.7:

Theorem:3.8

Globe Gl(n) n-odd is Hetro-Cordial.

Proof:

Let $V(Gl(n)) =\{[u,v,u_i : 1\leq i\leq n]\}$ and

$E(Gl(n)) = \{[(uu_i) \cup (vu_i) : 1\leq i\leq n]\}$.

Define $f : V(Gl(n))\rightarrow\{0,1\}$.

The vertex labeling are ,

$f(u) = 0$

$f(v) = 1$

$f(u_i) = 0 \qquad 1 \leq i \leq \frac{n-1}{2}$

$f(u_i) = 1 \qquad \frac{n+1}{2} \leq i \leq n$

The induced edge labeling are,

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$f^*[(uu_i)] = 0 \quad 1 \le i \le \frac{n-1}{2}$

$f^*[(uu_i)] = 1 \quad \frac{n+1}{2} \le i \le n$

$f^*[(vu_i)] = 1 \quad 1 \le i \le \frac{n-1}{2}$

$f^*[(vu_i)] = 0 \quad \frac{n+1}{2} \le i \le n$

Here, $v_f(1) = v_f(0)+1$ for all n and

$e_f(0) = e_f(1)$ for all n.

Therefore, Globe Gl(n) satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, Globe Gl(n) n-odd is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of Globe Gl(5) is shown in the following fig 3.9

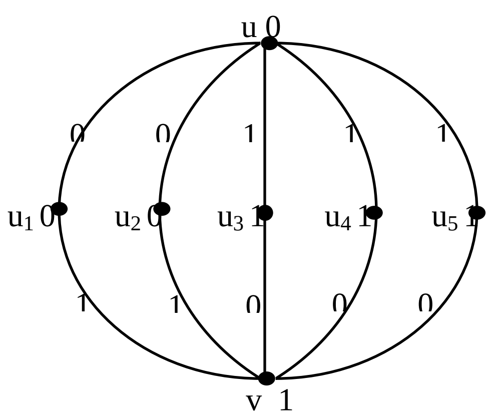

fig 3.9: Gl(5)

Theorem:3.10

Double triangular snake $C_2(P_n)$ is a Hetro-Cordial Graph.

Proof:

Let $V(C_2(P_n)) = \{[u_i: 1 \le i \le n], [v_i, w_i : 1 \le i \le n-1]\}$ and

$E(C_2(P_n)) = \{[(u_i v_i) \cup (u_i w_i) \cup (u_{i+1} v_i) \cup (u_{i+1} w_i) \cup (u_i u_{i+1}): 1 \le i \le n-1]\}$

Define $f: V(C_2(P_n)) \to \{0,1\}$.

The vertex labeling are ,

$f(u_i) = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1, & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \le i \le n$

$f(v_i) = 0 \quad 1 \le i \le n-1$

$f(w_i = 1 \quad 1 \le i \le n-1$

The induced edge labeling are,

$f^*[(u_i u_{i+1})] = \begin{cases} 1 & i \equiv 1 \bmod 2 \\ 0 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n-1$

$f^*[(u_i v_i)] = \begin{cases} 1 & i \equiv 2,3 \bmod 4 \\ 0 & i \equiv 0,1 \bmod 4 \end{cases} \quad 1 \le i \le n-1$

$f^*[(u_i w_i)] = \begin{cases} 1 & i \equiv 0,1 \bmod 4 \\ 0 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \le i \le n-1$

$f^*[(u_{i+1} v_i)] = \begin{cases} 1 & i \equiv 1,2 \bmod 4 \\ 0 & i \equiv 0,3 \bmod 4 \end{cases} \quad 1 \le i \le n-1$

$f^*[(u_{i+1} w_i)] = \begin{cases} 1 & i \equiv 0,3 \bmod 4 \\ 0 & i \equiv 1,2 \bmod 4 \end{cases} \quad 1 \le i \le n-1$

Therefore, Double triangular snake $C_2(P_n)$ satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, Double triangular snake $C_2(P_n)$ is a Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of Double triangular snake $C_2(P_5)$ and $C_2(P_4)$ are shown in the following fig 3.11 and fig 3.12 respectively.

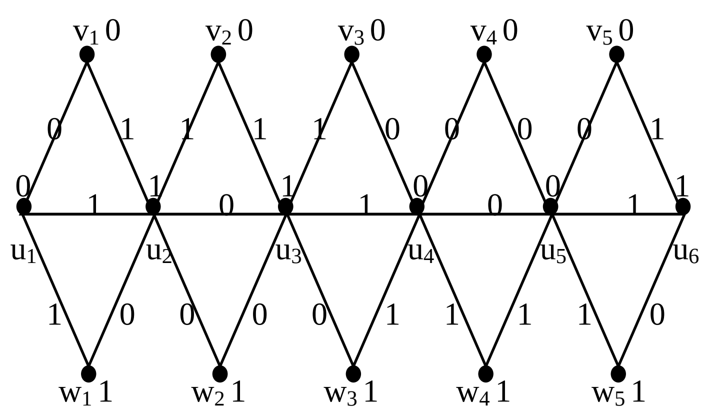

fig 3.11: $C_2(P_5)$

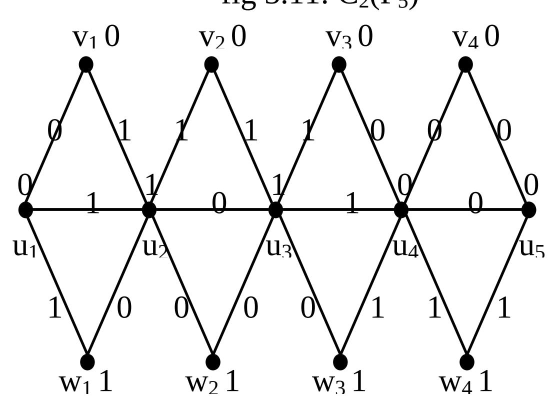

fig 3.12: $C_2(P_4)$

Theorem:3.13

$C_n \Theta K_1$ (n-even) is a Hetro-Cordial Graph.

Proof:

Let $V(C_n \Theta K_1) = \{[u_i, v_i : 1 \le i \le n]\}$ and

$E(C_n \Theta K_1) = \{[(u_i u_{i+1}): 1 \le i \le n-1] \cup [(u_1 u_n)] \cup [(u_i v_i): 1 \le i \le n]\}$.

Define $f: V(C_n \Theta K_1) \rightarrow \{0,1\}$.

The vertex labeling are ,

$$f(u_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n$$

The induced edge labeling are,

$f^*[(u_i u_{i+1})] = 1 \quad 1 \le i \le n-1$

$f^*[(u_1 u_n)] = 1$

$f^*[(u_i v_i)] = 0 \quad 1 \le i \le n$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1)$ for all n.

Therefore, $C_n \Theta K_1$ satisfies the conditions $| v_f(0) - v_f(1) | \le 1$ and $| e_f(0) - e_f(1) | \le 1$.

Hence, $C_n \Theta K_1$ (n-even) is a Hetro-Cordial graph.

For example, Hetro-Cordial labeling of $C_6 \Theta K_1$ is shown in the following fig 3.14

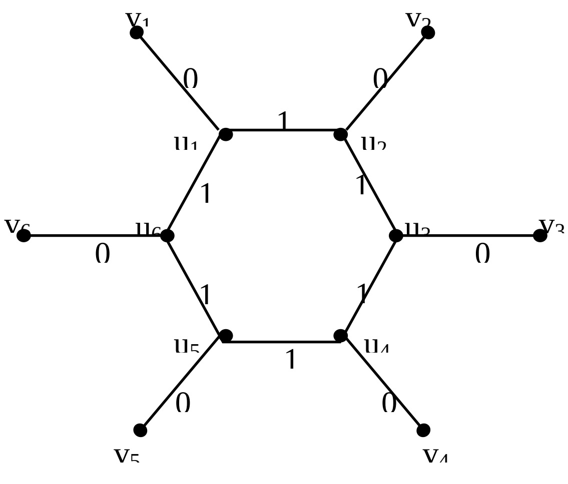

fig 3.14: $C_6 \Theta K_1$

Theorem:3.15

$C_n \Theta K_1$ (n-odd) is a Hetro-Cordial Graph.

Proof:

Let $V(C_n \Theta K_1) = \{[u_i, v_i : 1 \le i \le n]\}$ and

$E(C_n \Theta K_1) = \{[(u_i u_{i+1}): 1 \le i \le n-1] \cup [(u_1 u_n)] \cup [(u_i v_i): 1 \le i \le n]\}$.

Define $f: V(C_n \Theta K_1) \rightarrow \{0,1\}$.

The vertex labeling are ,

$$f(u_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n$$

$$f(v_1) = 1$$

$$f(v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 2 \le i \le n$$

The induced edge labeling are,

$f^*[(u_i u_{i+1})] = 1 \quad 1 \le i \le n-1$

$f^*[(u_1 u_n)] = 0$

$f^*[(u_1 v_1)] = 1$

$f^*[(u_i v_i)] = 0 \quad 2 \le i \le n$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1)$ for all n.

Therefore, $C_n \Theta K_1$ satisfies the conditions $| v_f(0) - v_f(1) | \le 1$ and $| e_f(0) - e_f(1) | \le 1$.

Hence, $C_n \Theta K_1$ (n-odd) is a Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of $C_5 \Theta K_1$ is shown in the following fig 3.16

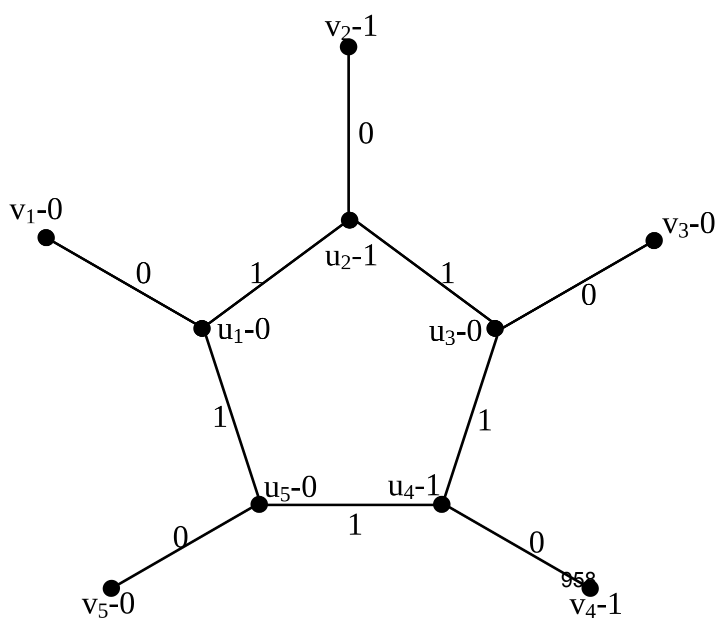

fig 3.16: $C_5 \Theta K_1$

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