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Scaling Group –Theoretic Technique for the Class of MHD Boundary Layer Equations Of Non-Newtonian Fluids

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Abstract

Using Scaling group transformation technique, class of similarity solution for steady, two dimensional laminar MHD boundary layer flows of incompressible non-Newtonian is derived. From the present analysis it is interesting to observe that for non-Newtonian viscoinelastic fluids of any model, which is characterized by the property that its stress and the rate of strain can be related by arbitrary continuous function, the similarity solutions exist only for the flows past 90° wedge. Numerical solution for the Prandtl-Eyring model is also discussed.

Keywords: Group-theoretic method, non-Newtonian fluids, MHD, 90^0 wedge, Prandtl-Eyring fluids

Introduction:

In fluid dynamics the effects of external magnetic field on Magnetohydrodynamic (MHD) flow over a stretching sheet are very important due to its applications in many engineering problems, such as glass manufacturing, geophysics, paper production and purification of crude oil. The flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in Magnetohydrodynamic (MHD) generators, pumps, accelerators and flowmeters and have applications in nuclear reactors, filtration, geothermal systems and others.

The interest in the outer magnetic field effect on heat-physical processes appeared seventy years ago. Research in Magnetohydrodynamic grew rapidly during the late 1950s as a result of extensive studies of ionized gases for a number of applications. Blum et al. [1] carried out one of the first works in the field of heat and mass transfer in the presence of a magnetic field. Zivojin et al. [2] investigated MHD flow of two immiscible and electrically conducting fluids between isothermal, insulated moving plates in the presence of an applied electric and inclined magnetic field. Mohammed Ibrahim and Suneetha [3] studied the effects of variable thermal conductivity and heat generation on flow of a viscous incompressible electrically conducting fluid in the presence of uniform transverse magnetic field, thermal radiation, porous medium, mass transfer and variable free stream near a stagnation point on a non-conducting stretching sheet.

Magnetohydrodynamic is the study of the motion of an electrically conducting fluid in the presence of a magnetic field. Due to the motion of an electrically conducting fluid in a magnetic field the electrical currents are induced in the fluid which produces their own magnetic field, called induced magnetic field, and these modify the original magnetic field. In addition to this the induced currents interacts with the magnetic field to produce electromagnetic forces perturbing the original motion. The two important basic effects of Magneto hydrodynamics are (1) The motion of the fluid affects the magnetic field and (2) the magnetic field affects the motion of the fluid. Boundary layer flow of an electrically conducting fluid over moving surfaces emerges in a large variety of industrial and technological applications. Wu [4] has studied the effects of suction or injection in a steady two-dimensional MHD boundary layer flow on a flat plate. Sharma et al. [5] investigated flow of an electrically conducting fluid over a flat plate.

Non-Newtonian fluids are generally divided in to two categories like viscoinelastic fluids and Viscoelastic fluids. The common feature of viscoinelastic fluids is that when at the rest they are isotropic and homogeneous and when they are subjected to a shear the resultant stress depends only on the rate of shear. However, such types of fluids show diverse behavior in response to applied stress. On the other hand Viscoelastic fluids are those which posses a certain degree of elasticity in addition to viscosity .For these fluids stress tensor is related to both instantaneous strain and the past strain history. Timol and Kalthia [6] are probably first to derive general non-Newtonian viscoinelastic fluids for two dimensional natural convection flows and three dimensional boundary layer flow respectively. Patel and Timol [7] have investigated the numerical solution of the laminar, incompressible flow of a non-Newtonian is Powell –Eyring fluid past 90^0 wedge without the influence of magnetic field. Also, Patel and Timol [8], have derived numerical solution for two-dimensional steady, laminar, incompressible MHD boundary layer flow of electrically conducting non-Newtonian Powell-Eyring fluid past 90^0 wedge using the method of satisfaction of asymptotic boundary conditions.

When we consider electrically conducting non-Newtonian fluids flowing under the influence of external magnetic field, the study becomes interesting .This is because in such situation magnetic forces produced in it could influence the motion of the fluids in significant way and hence such interaction problems have great practical applications. The problem of two-dimensional magneto hydrodynamic boundary layer equation for laminar incompressible flow past flat plate has been investigated by Rossow [9] and Greenspan et al [10].Rossow [9] has considered transverse magnetic field where as Greenspan et al [10] have considered longitudinal magnetic fields on the velocity and temperature distributions. Timol et al [11] have investigated three-dimensional magneto hydrodynamic boundary layer flow with pressure gradient and fluid injection. Similarity transformation for both steady and unsteady three-dimensional MHD boundary layer flow of purely viscous non-Newtonian fluid has been derived by Manisha et al[12].They have also derived Similarity Analysis in MHD Heat and Mass Transfer of Non-Newtonian Power Law Fluids Past a Semi-infinite Flat Plate[13]. Darji and Timol [14] have derived similarity solutions for steady, two dimensional laminar boundary layer flows of incompressible non-Newtonian viscoinelastic fluids.

So Motivated by these , we represent in the present paper Using Scaling group transformation technique ,class of similarity solution for steady ,two dimensional laminar MHD boundary layer flows of incompressible non-Newtonian is derived .From the present analysis it is interesting to observe that for non-Newtonian viscoinelastic fluids of any model , which is characterized by the property that its stress and the rate of strain can be related by arbitrary continuous function given by equation (1), the similarity solutions exist only for the flows past 90^0 wedge , as shown in Fig.1.

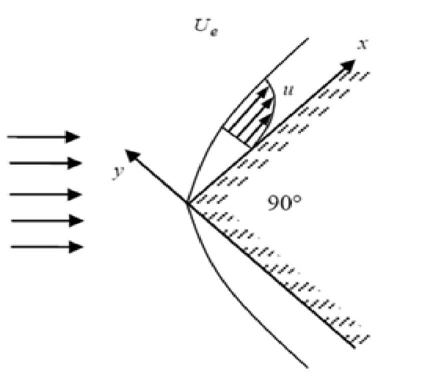

Figure 1: Schematic Diagram of flow past 90° wedge

For the derivation of the constitutive equations governing the motion of non-Newtonian fluids, the mathematical structure of stress-strain relationship, which is nonlinear, is important in functional form. This relationship may be implicit or explicit. In the present paper we have consider such relationship in the form of general arbitrary continuous function of the type

$$\tau_{yx} = \zeta\left(\frac{\partial u}{\partial y}\right) \qquad (1)$$

Here τ is the shearing stress and $\frac{\partial u}{\partial y}$ is the rate of the strain of the fluids.

Governing Equation

Under above description and following Darji and Timol [14] we can write the equation of motion for incompressible electrically conducting non-Newtonian fluid as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}\frac{\partial \tau}{\partial y} + U\frac{\partial U}{\partial x} - \frac{\sigma B_0^2}{\rho}u \qquad (3)$$

With stress-strain relationship is given by,

$$\tau_{yx} = \zeta\left(\frac{\partial u}{\partial y}\right) \qquad (4)$$

Together with boundary conditions,

$$y = 0, \qquad u(x,0) = v(x,0) = 0$$

$$y = \infty, \qquad u(x,\infty) = U(x) \qquad (5)$$

Formulation of the Problem

The above equation can be made dimenstionless using following quantities,

$$x^* = \frac{x}{L}, \qquad y^* = \frac{y}{L}(Re)^{\frac{1}{2}}$$

$$u^* = \frac{u}{U_\infty}, \qquad v^* = \frac{v}{U_\infty}(Re)^{\frac{1}{2}}$$

$$\tau_{yx}^* = \frac{\tau_{yx}}{\rho U_\infty^2}(Re)^{\frac{1}{2}}, \qquad U^* = \frac{U}{U_\infty}$$

$$R_e = \frac{U_\infty L}{v}, \qquad S^* = \frac{L}{U_\infty}S \qquad (6)$$

Where $R_e = \frac{U_\infty L}{v}$ Reynolds number and $S^*(x) = \frac{\sigma B_0^2(x)}{\rho}$ magnetic parameter

Substitute these quantities in equation (1) to (5) and dropping the asterisk, for simplicity

We get,

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \qquad (7)$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{\partial}{\partial y^*}\left(\tau^*_{y^*x^*}\right) + U^* \frac{\partial U^*}{\partial x} - S^*(x)\, u^* \qquad (8)$$

With stress-strain relationship is given by,

$$\zeta\left(\tau^*_{yx}, \frac{\partial u^*}{\partial y^*}\right) = 0 \qquad (9)$$

Introducing stream function ψ such that,

$$u^* = \frac{\partial \psi^*}{\partial y^*}, \qquad v^* = -\frac{\partial \psi^*}{\partial x^*} \qquad (10)$$

Equation of continuity (7) gets satisfied identically , equation (8 – 9) becomes

$$\frac{\partial \psi^*}{\partial y^*}\frac{\partial^2 \psi^*}{\partial x^* \partial y^*} - \frac{\partial \psi^*}{\partial x^*}\frac{\partial^2 \psi^*}{\partial y^{*2}} = \frac{\partial}{\partial y^*}\left(\tau^*_{yx}\right) + U^* \frac{\partial U^*}{\partial x} - S^*(x)\frac{\partial \psi^*}{\partial y^*} \qquad (11)$$

$$\zeta\left(\tau^*_{yx}, \frac{\partial^2 \psi^*}{\partial y^{*2}}\right) = 0 \qquad (12)$$

With boundary conditions

$$y = 0, \qquad \frac{\partial \psi}{\partial y}(x,0) = \frac{\partial \psi}{\partial x}(x,0) = 0$$

$$y = \infty, \qquad \frac{\partial \psi}{\partial y}(x,y) = U(x) \qquad (13)$$

Methodology and Solution of the Problem

By using scaling linear group transformation

$$\bar{x}^* = P^{\alpha_1} x^*, \qquad \bar{y}^* = P^{\alpha_2} y^*$$

$$\bar{\psi}^* = P^{\alpha_3} \psi^*, \qquad \bar{\tau}^*_{yx} = P^{\alpha_4} \tau^*_{yx}$$

$$\bar{U}^* = P^{\alpha_5} U^*, \qquad \bar{S}^* = P^{\alpha_6} S^* \qquad (14)$$

Where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ and P are Constants for the dependent and independent variables. From equation (14) one obtains

$$\left(\frac{\bar{x}^*}{x^*}\right)^{\frac{1}{\alpha_1}} = \left(\frac{\bar{y}^*}{y^*}\right)^{\frac{1}{\alpha_2}} = \left(\frac{\bar{\psi}^*}{\psi^*}\right)^{\frac{1}{\alpha_3}} = \left(\frac{\bar{\tau}^*_{yx}}{\tau^*_{yx}}\right)^{\frac{1}{\alpha_4}} = \left(\frac{\bar{U}^*}{U^*}\right)^{\frac{1}{\alpha_5}} = \left(\frac{\bar{S}^*}{S^*}\right)^{\frac{1}{6}} = P \qquad (15)$$

Introducing the linear transformation, given by equation (15), into the Eqs. (11-12) results in

$$P^{2\alpha_3 - 2\alpha_2 - \alpha_1} \frac{\partial \bar{\psi}^*}{\partial \bar{y}^*}\frac{\partial^2 \bar{\psi}^*}{\partial \bar{x}^* \partial \bar{y}^*} - P^{2\alpha_3 - 2\alpha_2 - \alpha_1} \frac{\partial \bar{\psi}^*}{\partial \bar{x}^*}\frac{\partial^2 \bar{\psi}^*}{\partial \bar{y}^{*2}}$$

$$= P^{\alpha_4 - \alpha_2} \frac{\partial}{\partial \bar{y}^*}\left(\bar{\tau}^*_{yx}\right) + P^{2\alpha_5 - \alpha_1} \bar{U}^* \frac{\partial U^*}{\partial \bar{x}^*} - P^{\alpha_6 + \alpha_3 - \alpha_2} \frac{\partial \bar{\psi}^*}{\partial \bar{x}^*} \qquad (16)$$

And

$$\zeta\left(P^{\alpha_4} \tau^*_{yx}, P^{\alpha_3 - 2\alpha_2} \frac{\partial^2 \psi^*}{\partial y^{*2}}\right) = 0 \qquad (17)$$

The differential equation are completely invariant to the proposed linear transformation, the following coupled algebraic equations are obtained

$$2\alpha_3 - 2\alpha_2 - \alpha_1 = \alpha_4 - \alpha_2 = 2\alpha_5 - \alpha_1 = \alpha_6 + \alpha_3 - \alpha_2 \qquad (18)$$

$$\alpha_3 - 2\alpha_2 = 0 \qquad (19)$$

$$\alpha_4 = 0 \qquad (20)$$

By solving above equations with $\alpha_1 \neq 0$, we get

$$\frac{\alpha_2}{\alpha_1} = \frac{1}{3}, \quad \frac{\alpha_3}{\alpha_1} = \frac{2}{3}, \quad \frac{\alpha_4}{\alpha_1} = 0, \quad \frac{\alpha_5}{\alpha_1} = \frac{1}{3}, \quad \frac{\alpha_6}{\alpha_1} = -\frac{2}{3} \qquad (21)$$

Introducing equation (21) into equation (15) result in

$$\eta = \frac{y^*}{x^{*\frac{1}{3}}}, \qquad \psi^* = f(\eta)\, x^{*\frac{2}{3}}, \qquad U^* = G(\eta)\, x^{*-\frac{1}{3}}$$

$$\tau^*_{yx} = H(\eta) \quad \text{and} \quad S(x) = S_0\, x^{*-\frac{2}{3}} \qquad (22)$$

With the boundary conditions, Eqs (13) becomes

$$\eta = 0, \qquad f(0) = f'(0) = 0$$

$$\eta \to \infty, \qquad f'(\infty) = 1 \qquad (23)$$

Introducing equations (22) in equation (11)-(13), we get following similarity equation

$$f'^2(\eta) - 2f(\eta)f''(\eta) - 3H'(\eta) + \eta G(\eta)\, G'(\eta) - G^2(\eta) + S_0\left(f'(\eta)\right) = 0 \qquad (24)$$

But U^* is independent of y, $G(\eta)$ must be constant .Therefore $G(\eta)$ assume Unity

i.e. $G(\eta) = 1$, $G'(\eta) = 0$ And also assume that $S_0 = 0$

$$f'^2(\eta) - 2f(\eta)f''(\eta) - 3H'(\eta) - 1 = 0 \qquad (25)$$

With the boundary conditions,

$$\eta = 0, \quad f(0) = f'(0) = 0$$
$$\eta \to \infty, \quad f'(\infty) = 1 \tag{26}$$

And the Stress-Strain functional relationship is given by,

$$\zeta(H, f'') = 0 \tag{27}$$

Numerical Solution:

Numerical solution is generated for non-Newtonian Prandtl-Eyring fluids flowing over a 90^0 wedge under the influence of transverse magnetic field. Non-Newtonian fluid models based on functional relationship between shear-stress and rate of the strain, shown by equation (1).Most of the research work is so far carried out on power-law fluid model, this is because of its mathematical simplicity.

Mathematically, the Prandtl-Eyring model can be written as (Bird et al [15]. Skelland [16])

$$\tau = A \sinh^{-1}\left(\frac{1}{B}\frac{\partial u}{\partial y}\right) \tag{28}$$

Introducing the dimensionless quantities in to equation (28) and then substituting it in to the equation (25), we get

$$F''' = \frac{1}{\alpha}\left(F'^2 - 2FF'' - 1\right)\left(1 + \beta F''^2\right)^{\frac{1}{2}} \tag{29}$$

Where $\alpha = \frac{A}{\mu B}$; $B = \frac{\rho U_\infty^3}{\mu L B^2}$ are dimensionless numbers and can referred as flow parameters.

A numerical solution of (29) is obtained using Method of Satisfaction of Asymptotic Boundary Condition (MSABC) due to Nachtsheim and Swigert [17] with the boundary condition (26).The detail of this technique is recently presented by Patel and Timol [18] and hence same is not repeated here. Controlling the non-dimensional numbers α and then for $\beta = 5 \times 10^3$; $\beta = 5 \times 10^4$; $\beta = 5 \times 10^5$ the velocity profile and the slope of velocity profile are generated.(see figure-2 and figure-3)

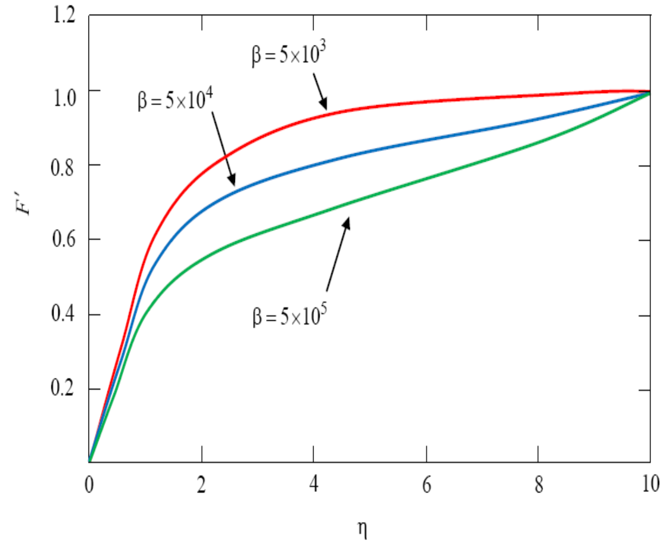

Figure-2: Velocity profile

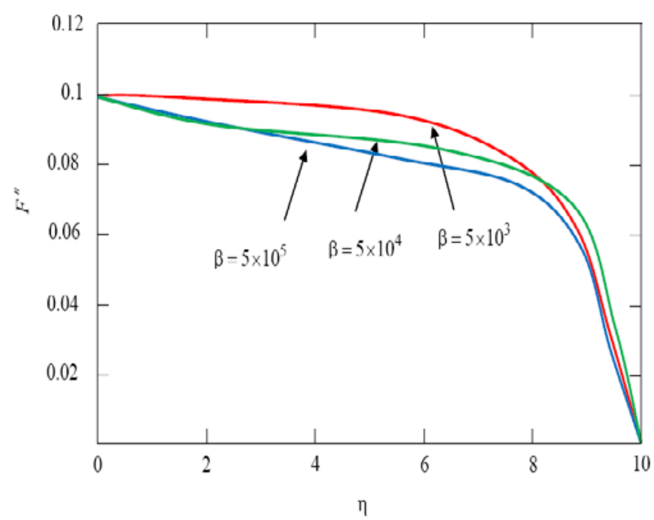

Figure-3: Slope of velocity profiles

From the figure (2), it is clear that both α and β have great influence on the velocity of the Prandtl-Eyring fluids. Here for fix value of α the velocity of fluid is increases rapidly and approaches to one as β increases. The slope of velocity profile in figure (3) is found always decreases fast an approaches to zero as β increases. Figure (2) and (3) are plotted in terms of dimensionless parameters and hence they represent behavior of all Prandtl Eyring fluids

Conclusion:

The similarity solutions laminar incompressible boundary layer equations of all non-Newtonian viscoinelastic fluids that are characterized by the property that its shearing stress is related to the rate of strain by some arbitrary continuous functions as shown in equation (1), is derived. It is interesting to note that the Scaling group transformation technique is applied to derive proper similarity transformations for the non linear partial differential equation with the stress-strain functional relationship condition, governing the flow under consideration. It is to be observing that similarity solutions for all non-Newtonian fluids exist only for the flow past 90^0 wedge only. The present similarity equation is solved by MSABC for the case of Prandtl-Eyring fluid model.

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