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Soft Structures on Fuzzy Version of Soft INT G-Modules

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Abstract: In this paper, we introduce fuzzy version of soft int-G-modules of a vector space with respect to soft structures, which are fuzzy soft int-G-modules (IFSG-module) .These new concepts show that how a soft set affects on a G-module of a vector space in the mean of intersection, union and inclusion of sets and thus, they can be regarded as a bridge among classical sets, fuzzy soft sets and vector spaces. We then investigate their related properties with respect to soft set operations, soft image, soft pre-image, soft anti image, α-inclusion of fuzzy soft sets and linear transformations of the vector spaces. Furthermore, we give the applications of these new G-module on vector spaces.

Keywords: Soft set, IFSG-module, , fuzzy soft image, fuzzy soft anti image, α-inclusion,trivial,whole.

1.Introduction: Most of the problems in economics, engineering, medical science, environments etc. have various uncertainties. We cannot successfully use classical methods to solve these uncertainties because of various uncertainties typical for those problems. Hence some kinds of theories were given like theory of fuzzy sets [44], rough sets [17], i.e., which we can use as mathematical tools for dealing with uncertainties. However, all of these theories have their own difficulties which are pointed out in [15]. Soft set theory was introduced by Molodtsov [29] for modeling vagueness and uncertainty and it has been received much attention since Maji et al [27], Ali et al [6] and Sezgin and Atagun [34] introduced and studied operations of soft sets. Soft set theory has also potential applications especially in decision making as in [10,11,27]. This theory has started to progress in the mean of algebraic structures, since Aktas¸ and Cagman [5] defined and studied soft groups. Since then, soft substructures of rings, fields and modules [8], union soft substructures of near-rings and near-ring modules [35], normalistic soft groups [32] are defined and studied in detailed. Soft set has also been studied in the following papers [1,2,3,25,26,45].The theory of G-modules originated in the 20th century. Representation theory was developed on the basis of embedding a group G in to a linear group GL (V).

The theory of group representation (G module theory) was developed by Frobenious G [1962]. Soon after the concept of fuzzy sets was introduced by Zadeh [44] in 1965. Fuzzy subgroup and its important properties were defined and established by Rosenfeld [31] in 1971. After that in the year 2004 Shery Fernandez [36] introduced fuzzy parallels of the notions of G-modules. This study is of great importance since IFSG-modules show how a fuzzy soft set affect on a G-module of a vector space in the mean of intersection, union and inclusion of sets, so it functions as a bridge among classical sets, soft sets and vector spaces. In this paper, we introduce intersection fuzzy soft G-modules of a vector space that is abbreviated by IFSG- module and investigate its related properties with respect to fuzzy soft set operations. Then we give the application of fuzzy soft image, fuzzy soft pre image, upper α-inclusion of fuzzy soft sets, linear transformations of vector spaces on vector spaces in the mean of IFSG-modules. Moreover, we apply soft pre image, soft anti-image, lower α-inclusion of soft sets, linear transformations of vector spaces on these fuzzy soft G-modules. The work of this paper is organized as follows. In the second section as preliminaries, we give basic concepts of soft sets and fuzzy soft G-modules. In Section 3, we introduce IFSG-modules and study their characteristic properties. In Section 4, we give the applications of IFSG-modules .

2.Preliminaries: In this section as a beginning, the concepts of G-module[36] soft sets introduced by Molodsov [29] and the notions of fuzzy soft set introduced by Maji et al. [26] have been presented.

2.1 Definition [36**]:** Let G be a finite group. A vector space M over a field K (a subfield of C) is called a G-module if for every g ∈ G and m ∈ M, there exists a product (called the right action of G on M) m.g ∈ M which satisfies the following axioms.

1. $m.1_G = m$ for all $m \in M$ (1_G being the identify of G)
2. m. (g. h) = (m.g). h, m ∈ M, g, h ∈ G
3. $(k_1 m_1 + k_2 m_2). G = k_1 (m_1. g) + k_2(m_2. g)$, $k_1, k_2 \in K$, $m_1, m_2 \in M$ &

g ∈ G. In a similar manner the left action of G on M can be defined.

2.2. Definition [36**]:**Let M and M* be G-modules. A mapping Ø: M→M* is a G-module homomorphism if

1. $Ø(k_1 m_1 + k_2 m_2) = k_1 Ø (m_1) + k_2 Ø (m_2)$
2. Ø(gm) = g Ø (m), $k_1, k_2 \in K$, $m, m_1, m_2 \in M$ & $g \in G$.

2.3. Definition [36]**:**Let M be a G-module. A subspace N of M is a G - sub module if N is also a G-module under the action of G.

Let U be a universe set, E be a set of parameters, P(U) be the power set of U and $A \subseteq E$.

2.4.Definition[29]: A pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \rightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A,G_A, H_A, respectively. For more details, we refer to [11,17,18,26,29,7].

2.5.Definition[6] :The relative complement of the soft set F_A over U is denoted by $F^r{}_A$, where $F^r{}_A : A \rightarrow P(U)$ is a mapping given as $F^r{}_A(a) = U \setminus F_A(a)$, for all $a \in A$.

2.6.Definition[6]: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted intersection of F_A and G_B is denoted by $F_A \Cup G_B$, and is defined as $F_A \Cup G_B = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.

2.7.Definition[6]: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B = (H,C)$,where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cup G(c)$.

2.8.Definition[12]: Let F_A and G_B be soft sets over the common universe U and ψbe a function from A to B. Then we can define the soft set $\psi(F_A)$ over U, where $\psi(F_A) : B \rightarrow P(U)$ is a set valued function defined by $\psi(F_A)(b) = \cup\{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if$\psi^{-1}(b) \neq \emptyset$, $= 0$ otherwise for all $b \in B$. Here, $\psi(F_A)$ is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B) : A \rightarrow P(U)$ is a set-valued function defined by $\psi^{-1}(G_B)(a) = G(\psi(a))$ for all $a \in A$. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ.

2.9.Definition[13]: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^{\star}(F_A)$ over U, where $\psi^{\star}(F_A) : B \rightarrow P(U)$ is a set-valued function defined by $\psi^{\star}(F_A)(b) = \cap\{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \emptyset$, $=0$ otherwise for all $b \in B$. Here, $\psi^{\star}(F_A)$ is called the soft anti image of F_A under ψ.

2.1.Theorem [13]: Let F_H and T_K be soft sets over U, F^r_H, T^r_K be their relative soft sets, respectively and ψ be a function from H to K. then, i) $\psi^{-1}(T^r_K) = (\psi^{-1}(T_K))^r$,
ii) $\psi(F^r_H) = (\psi^\star(F_H))^r$ and $\psi^\star(F^r_H) = (\psi(F_H))^r$.

2.10.Definition[14]: Let F_A be a soft set over U and a be a subset of U. Then upper α-inclusion of F_A, denoted by $F_A^{\supseteq\alpha}$, is defined as $F_A^{\supseteq\alpha} = \{x \in A / F(x) \supseteq \alpha\}$. Similarly, $F_A^{\subseteq\alpha} = \{x \in A \mid F(x) \subseteq \alpha\}$is called the lower α -inclusion of F_A. A nonempty subset U of a vector space V is called a subspace of V if U is a vector space on F. From now on,V denotes a vector space over F and if U is a subspace of V, then it is denoted by U < V.

3. IFSG-modules: In this section, we first define intersection fuzzy soft G-modules of a vector space, abbreviated as IFSG-modules. We then investigate its related properties with respect to soft set operations.

Let G be a non-empty set. A fuzzy subset μ on G is defined by $\mu : G \to [0,1]$ for all x ∈G.

3.1.Definition: Let G be a group. Let M be a G-module of V and A_M be a fuzzy soft set over V. Then A_M is called Intersection Fuzzy Soft G-module of V (IFSG-m), denoted by $A_M \widetilde{<}_\iota V$ if the following properties are satisfied

(IFSG-m_1) $A(ax + by) \geq A(x) \cap A(y)$

(IFSG-m_2) $A(\alpha x) \geq A(x)$, for all x, y∈ M, a,b, $\alpha \in F$.

Example: Let G = {1,-1} , M = R^4 over R. Then M is a G-module.

Define A on M by,

$$A(x) = \begin{cases} 1, & \text{if } x_i = 0 \ \forall i. \\ 0.5, & \text{if atleast } x_i \neq 0. \end{cases}$$

Where $x=\{x_1, x_2, x_3, x_4\}$; $x_i \in R$. Then A is a fuzzy soft G-Module.

3.1.Proposition: If $A_M \widetilde{<}_\iota V$, then $A(0v) \supseteq A(x)$ for all x ∈ M.

Proof: Since A_M is an IFSG-module of V, then $A(ax+by) \supseteq A(x) \cap A(y)$ for all x,y∈ M and since (M,+) is a group, if we take ay=-ax then, for all x ∈ M,

$$A(ax\text{-}ax) = A(0_V) \supseteq A(x) \cap A(x) = A(x).$$

3.2.Proposition: If $A_{M_1} \widetilde{<}_\iota V$ and $B_{M_2} \widetilde{<}_\iota V$, then $A_{M_1} \cap B_{M_2} \widetilde{<}_\iota V$.

Proof: Since M_1 and M_2 are G-modules of V, then $M_1 \cap M_2$ is a G-module of V. By definition 2.6, let $A_{M_1} \cap B_{M_2}$ = (A, M_1) ∩ (B, M_2) = (T, $M_1 \cap M_2$), where

T(x) = A(x) ∩ B(x) for all x∈ $M_1 \cap M_2 \neq \emptyset$. Then for all x,y∈ $M_1 \cap M_2$ and $\alpha \in$ F.

(IFSG-m_1) T(ax+by) = A(ax+by) ∩ B(ax+by) ⊇ (A(x) ∩ A(y)) ∩ (B(x) ∩ B(y))

= (A(x) ∩ B(x)) ∩ (A(y) ∩ B(y)) = T(x) ∩ T(y),

(IFSG-m_2) T(αx) = A(αx) ∩ B(αx) ⊇ A(x) ∩ B(x) = T(x) .

There fore $A_{M_1} \cap B_{M_2} = T_{M_1 \cap M_2} \lessapprox_{\iota}$ V.

3.2.Definition: Let (A, M_1) and (B, M_2) be two IFSG-modules of V_1 and V_2 respectively, the product of IFSG-modules (A, M_1) and (B, M_2) is defined as (A, M_1) × (B, M_2) = (Q, $M_1 \times M_2$), where Q(x,y) = A(x) × B(y) for all (x,y) ∈ $M_1 \times M_2$.

3.1.Theorem: If $A_{M_1} \lessapprox_{\iota}$ V and $B_{M_2} \lessapprox_{\iota}$ V, then $A_{M_1} \times B_{M_2} \lessapprox_{\iota} V_1 \times V_2$.

Proof: Since M_1 and M_2 are G-modules of V_1 and V_2 respectively, then $M_1 \times M_2$ is a

G-module of $V_1 \times V_2$. By definition3.2, let

$A_{M_1} \times B_{M_2}$ = (A, M_1) × (B, M_2)
= (Q, $M_1 \times M_2$), where Q(x,y) = A(x) × B(y) for all (x,y) ∈ $M_1 \times M_2$.
Then for all (x_1, y_1), (x_2, y_2) ∈ $M_1 \times M_2$ and (α_1, α_2) ∈ F×F,
(IFSG-m_1) $Q\{(ax_1, by_1) + (ax_2, by_2)\} = Q\ (ax_1 + ax_2\ , by_1 + by_2\ )$
$= A\ (ax_1 + ax_2) \times B(by_1 + by_2)$
$\supseteq (A\ (x_1) \cap A\ (x_2)) \times (B\ (y_1) \cap B\ (y_2)$
$= Q(x_1, y_1) \cap Q(x_2, y_2)$
(IFSG-m_2) $Q\ ((\alpha_1, \alpha_2)(\ x_1, y_1)) = Q\ (\alpha_1 x_1 + \alpha_2 y_1)$
$= A\ (\alpha_1 x_1) \times B\ (\alpha_2 y_2) \supseteq A\ (x_1) \cap B\ (y_2) = Q(x_1, y_1)$.
Hence $A_{M_1} \times B_{M_2} = Q_{M_1 \times M_2} \lessapprox_{\iota} V_1 \times V_2$.

3.3.Definition: Let $A_{M_1}$and B_{M_2} be two IFSG-module's of V. If $M_1 \cap M_2 = \{0_V\}$, then the sum of IFSG-module's $A_{M_1}$and B_{M_2} is defined as $A_{M_1} + B_{M_2} = T_{M_1 + M_2}$ where
T(ax+by) = A(x)+B(y) for all ax+by ∈ $M_1 + M_2$.

3.2.Theorem: If $A_{M_1} \lessapprox_{\iota}$ V and $B_{M_2} \lessapprox_{\iota}$ V where $M_1 \cap M_2 = \{0_V\}$, then $A_{M_1} + B_{M_2} \lessapprox_{\iota}$ V.
Proof: Since M_1& M_2 are G-modules of V, then $M_1 + M_2$ is a G-modules of V. By definition:3.3, let $A_{M_1} + B_{M_2}$ = (A, M_1) + (B, M_2) = (T, $M_1 + M_2$), where
T(ax+by) = A(x)+B(y) for all ax+by ∈ $M_1 + M_2$. It is obvious that since $M_1 \cap M_2 = \{0_V\}$, then the sum is well defined. Then for all $ax_1 + by_1, ax_2 + by_2 \in M_1 + M_2$ and α ∈F,

$T\ ((ax_1 + by_1)+(\ ax_2 + by_2)) = T((ax_1 + ax_2)+(\ by_1 + by_2))$

$$= A(a(x_1 + x_2)) + B(b(y_1 + y_2))$$
$$\supseteq (A(x_1) \cap A(x_2)) + (B(y_1) \cap B(y_2))$$
$$= (A(x_1) + B(y_1)) \cap (A(x_2) + B(y_2))$$
$$= T(ax_1 + by_1) \cap T(ax_2 + by_2)$$

$$T(\alpha(x_1 + y_1)) = T(\alpha x_1 + \alpha y_1)$$
$$= A(\alpha x_1) + B(\alpha y_1) \supseteq A(x_1) + B(y_1)$$
$$= T(x_1 + y_1)$$

Thus, $A_{M_1} + B_{M_2} \widetilde{<}_i V$.

3.4.Definition : Let A_M be an IFSG-module of V. Then,
(i) A_M is said to be trivial if $A(x) = \{0_V\}$ for all $x \in M$.
(ii) A_M is said to be whole if $A(x) = V$ for all $x \in M$.

3.3.Proposition: Let A_{M_1} and B_{M_2} be two IFSG-modules of V, then
(i) If A_{M_1} and B_{M_2} are trivial IFSG-modules of V, then $A_{M_1} \cap B_{M_2}$ is a trivial IFSG -module of V.
(ii) If A_{M_1} and B_{M_2} are whole IFSG-modules of V, then $A_{M_1} \cap B_{M_2}$ is a whole IFSG -module of V.
(iii) If A_{M_1} is a trivial IFSG-module of V and B_{M_2} is a whole IFSG-modules of V, then then $A_{M_1} \cap B_{M_2}$ is a trivial IFSG -module of V.
(iv) If A_{M_1} and B_{M_2} are trivial IFSG-modules of V where $M_1 \cap M_2 = \{0_V\}$, then $A_{M_1} + B_{M_2}$ is a trivial IFSG-module of V.
(v) If A_{M_1} and B_{M_2} are whole IFSG-modules of V where $M_1 \cap M_2 = \{0_V\}$, then $A_{M_1} + B_{M_2}$ is a whole IFSG-module of V.
(vi) If A_{M_1} is a trivial IFSG-module of V and B_{M_2} is a whole IFSG-modules of V where $M_1 \cap M_2 = \{0_V\}$, then $A_{M_1} + B_{M_2}$ is a whole IFSG-module of V.

Proof: The proof is easily seen by definition2.6, definition3.3, definition3.4, theorem3.1 and theorem3.3.

3.4.Proposition: Let A_{M_1} and B_{M_2} be two IFSG-modules of V_1 and V_2 respectively. Then
(i) If A_{M_1} and B_{M_2} are trivial IFSG-modules of V_1 and V_2 respectively, then $A_{M_1} \times B_{M_2}$ is a trivial IFSG -module of $V_1 \times V_2$.
(ii) If A_{M_1} and B_{M_2} are whole IFSG-modules of V_1 and V_2 respectively, then $A_{M_1} \times B_{M_2}$ is a whole IFSG -module of $V_1 \times V_2$.

Proof: The proof is easily seen by definition3.2, definition3.4 and theorem3.3

4. Applications of IFSG-modules:

In this section, we give the applications of soft image, soft pre image, upper α-inclusion of fuzzy soft sets and linear transformation of vector spaces on vector space with respect to IFSG-modules.

4.1.Theorem: If $A_M \widetilde{<}_\iota V$, then $M_G = \{x\in M \,/\, A(x) = A(0_V)\}$ is a G-module of M .
Proof: It is obvious that $0_V \in M_G$ and $\emptyset \neq M_G \subseteq M$. We need to show that $ax+by \in M_G$ and $\alpha x \in M_G$ for all x, y$\in M_G$ and $\alpha \in F$, which means that $A(ax+by) = A(0_V)$ and $A(\alpha x) = A(0_V)$ have to be satisfied. Since x, y$\in M_G$ and A_M is an IFSG-Module of V, then $A(x) = A(y) = A(0_V)$, $A(ax+by) \supseteq A(x) \cap A(y) = A(0_V)$, $A(\alpha x) \supseteq A(x) = A(0_V)$ for all x, y$\in M_G$ and $\alpha \in F$. Moreover, by Proposition3.1, $A(0_V) \supseteq A(ax+by)$ and $A(0_V) \supseteq A(\alpha x)$ which completes the proof.

4.2.Theorem: Let A_M be a fuzzy soft set over V and α be a subset of V such that $A(0_V) \supseteq \alpha$. If A_M is an IFSG-module of V, then $A_M^{\supseteq\alpha}$ is a G-module of V.
Proof:

Since $A(0_V) \supseteq \alpha$, then $0_V \in A_M^{\supseteq\alpha}$ and $\emptyset \neq A_M^{\supseteq\alpha} \subseteq V$. Let $x,y \in A_M^{\supseteq\alpha}$, then $A(x) \supseteq \alpha$ and $A(y) \supseteq \alpha$. We need to show that $x + y \in A_M^{\supseteq\alpha}$ $nx \in A_M^{\supseteq\alpha}$ for all $x, y \in A_M^{\supseteq\alpha}$ and n∈F. Since A_M is an IFSG-module of V, it follows that

$$A(ax + by) \supseteq A(x) \cap A(y) \supseteq \alpha \cap \alpha = \alpha.$$

Furthermore, $A(nx) \supseteq A(x) \supseteq \alpha$, which completes the proof.

4.3.Theorem: Let A_M and T_W be fuzzy soft sets over V, where M and W are G-modules of γ and Ψ be a linear isomorphism from M to W. If A_M is an IFSG-Module of V, then so is $\Psi(A_M)$.
Proof: Let $w_1, w_2 \in W$. Since Ψ is a surjective linear transformation. Then there exists $m_1, m_2 \in M$ such that $\Psi(m_1) = w_1$, $\Psi(m_2) = w_2$. Then

$$\begin{aligned}(\Psi(A_M))(aw_1 + bw_2) &= \cup\{A(m): m \in M,\ \Psi(m) = aw_1 + bw_2\}\\ &= \cup\{A(m): m \in M,\ m = \Psi^{-1}(aw_1 + bw_2)\}\\ &= \cup\{A(m): m \in M,\ m = \Psi^{-1}(\Psi(aw_1 + bw_2)) = am_1 + bm_2\}\\ &= \cup\{A(am_1 + bm_2) : m_i \in M, \Psi(m_i) = w_i, i = 1,2\}\\ &\supseteq \cup\{A(m_1) \cap A(m_2) : m_i \in M, \Psi(m_i) = w_i, i = 1,2\}\\ &= (\cup A(m_1): m_1 \in M, \Psi(m_1) = w_1) \cap (\cup A(m_2): m_2 \in M, \Psi(m_2) = w_2)\\ &= (\Psi(A_M))(w_1) \cap (\Psi(A_M))(w_2)\end{aligned}$$

Now let $\alpha \in F$ and $w\in W$. Since Ψ is a surjective linear transformation, there exits $\tilde{m} \in M$ such that $\Psi(\tilde{m}) = w$. Then

$$\begin{aligned}(\Psi(A_M))(\alpha w) &= \cup\{A(m): m \in M, \Psi(m) = \alpha w\}\\ &= \cup\{A(m): m \in M, m = \Psi^{-1}(\alpha w)\}\\ &= \cup\{A(m): m \in M, m = \Psi^{-1}(\Psi(\alpha\tilde{m})) = \alpha\tilde{m}\}\\ &= \cup\{A(\alpha\tilde{m}): \alpha\tilde{m} \in M, \Psi(\tilde{m}) = w\}\\ &= (\Psi(A_M))(w)\end{aligned}$$

Hence, $\Psi(A_M)$ is an IFSG –module of V.

4.4.Theorem: Let A_M and T_W be fuzzy soft sets over V, where M and W are G-modules of γ and Ψ be a linear isomorphism from M to W. If T_W is an IFSG-Module of V, then so is $\Psi^{-1}(T_W)$.

Proof: Let $m_1, m_2 \in M$. Then

$$\Psi^{-1}(T_W)(am_1+bm_2) = T(\Psi(am_1+bm_2))$$
$$= T(\Psi(am_1)+\Psi(bm_2))$$
$$\supseteq T(\Psi(m_1)) \cap T(\Psi(m_2))$$
$$= (\Psi^{-1}(T_W))(m_1) \cap (\Psi^{-1}(T_W))(m_2)$$

Now let $\alpha \in F$ and $m \in M$. Then,

$$\Psi^{-1}(T_W)(\alpha m) = T(\Psi(\alpha m))$$
$$= T(\alpha\Psi(m))$$
$$\supseteq T(\Psi(m)) = \Psi^{-1}(T_W)(m)$$

Hence $\Psi^{-1}(T_W)$ is an IFSG –module of V.

4.5.Theorem: Let V_1 and V_2 be two vector spaces and $(A_1, M_1) \precsim_\iota V_1$, $(A_2, M_2) \precsim_\iota V_2$.
If $f : M_1 \to M_2$ is a linear transformation of vector spaces, then

(i) f is surjective, then $(A_1, f^{-1}(M_2)) \precsim_\iota V_1$,

(ii) $(A_2, f(M_1)) \precsim_\iota V_2$,

(iii) $(A_1, \text{kerl} f) \precsim_\iota V_1$.

Proof: (i) Since $M_1 < V_1$, $M_2 < V_2$ and $f : M_1 \to M_2$ is a surjective linear transformation, then it is clear that $f^{-1}(M_2) < V_1$. Since $(A_1, M_1) \precsim_\iota V_1$ and $f^{-1}(M_2) < M_1$, $A_1(ax+by) \supseteq A(x) \cap A(y)$ and $A_1(\alpha x) \supseteq A(x)$ for all $x,y \in f^{-1}(M_2)$ and $\alpha \in F$. Hence $(A_1, f^{-1}(M_2)) \precsim_\iota V_1$.

(ii) Since $M_1 < V_1$, $M_2 < V_2$ and $f : M_1 \to M_2$ is a vector space linear transformation, then $f(M_1) < V_2$. Since $f(M_1) \subseteq M_2$, the result is obvious by definition3.1.

(iii) Since $\text{kerl} f < V_1$ and $\text{kerl} f \subseteq M_1$, the rest of the proof is clear by definition3.1.

4.1.Corollary: Let $(A_1, M_1) \precsim_\iota V_1$, $(A_2, M_2) \precsim_\iota V_2$. If $f : M_1 \to M_2$ is a linear transformation, then $(A_2, \{0M_2\}) \precsim_\iota V_2$.

Proof: By theorem:4.5, (iii) $(A_1, \text{kerl} f) \precsim_\iota V_1$, then $(A_2, f(\text{kerl} f)) = (A_2, \{oM_2\}) \precsim_\iota V_2$,
By theorem4.5 (ii).

Conclusion: Throughout this paper, we have dealt with IFSG-modules of a vector space. We have investigated their related properties with respect to soft set operations Furthermore, we have derived some applications of IFSG-modules with respect to soft image, soft pre image, soft anti image, α-inclusion of soft sets and linear transformations of vector spaces. Further study could be done for fuzzy soft sub structures of different algebras.

REFERENCES

[1] S. Abdullah and N. Amin, Analysis of S-box Image encryption based on generalized fuzzy

soft expert set, Nonlinear Dynamics, (2014),dx.doi.org/10.1007/s11071-014-1767-5.

[2] S. Abdullah, M. Aslam and K. Ullah, Bipolar fuzzy soft sets and its applications in decision making problem, J. Intell. Fuzzy Syst. 27:2, 729-742 (2014).

[3] S. Abdullah, M. Aslam and K. Hila, A new generalization of fuzzy bi-ideals in semigroups and its Applications in fuzzy finite state machines, Multiple Valued Logic and Soft Computing, 27:2, 599-623 (2015).

[4] U. Acar, F. Koyuncu and B. Tanay, Soft sets and soft rings,Comput. Math. Appl. 59, 3458-3463 (2010).

[5] H. Aktas.and N. C.a˘gman, Soft sets and soft groups, Inform.Sci. 177, 2726-2735 (2007).

[6] M.I. Ali, F. Feng, X. Liu, W.K. Min and M. Shabir, On somenew operations in soft set theory, Comput. Math. Appl. 57,1547-1553 (2009).

[7] M. I. Ali, M. Shabir, M. Naz, Astructures of soft setsassociated with new operations, Comput. Math. Appl. 61,2647-2654 (2011).

[8] A.O. Atag¨un and A. Sezgin, Soft substructures of rings, fields and modules, Comput. Math. Appl. 61 (3), 592-601 (2011).

[9] K.V. Babitha and J.J. Sunil, Soft set relations and functions,Comput. Math. Appl. 60 (7), 1840-1849 (2010).

[10] N. C.a˘gman and S. Engino˘glu, Soft matrix theory and its decision making, Comput. Math. Appl. 59, 3308-3314 (2010).

[11] N. C.a˘gman and S. Engino˘glu, Soft set theory and uni-int decision making, Eur. J. Oper. Res. 207, 848-855 (2010).

[12] N. C¸ a˘gman, F. C¸ ıtak and H. Aktas¸, Soft int-groups and its applications to group theory, Neural Comput. Appl. 21, 151-158 (2012).

[13] N. C¸ a˘gman, A. Sezgin and A.O. Atag¨un, Soft uni-groups and its applications to group theory, (submitted).

[14] N. C¸ a˘gman, A. Sezgin and A.O. Atag¨un, α-inclusions andtheir applications to group theory, (submitted).

[15] Charles W. Curties and Irving Reiner, Representation Theory of finite groups and Associative Algevras, INC, (1962).

[16] F. Feng, Y. B. Jun, X. Zhao, Soft semirings, Comput. Math.Appl. 56, 2621-2628 (2008).

[17] F. Feng, X.Y. Liu, V. Leoreanu-Fotea, Y.B. Jun, Soft sets andsoft rough sets, Inform. Sci. 181 (6), 1125-1137 (2011).

[18] F. Feng, C. Li, B. Davvaz and M. I. Ali, Soft sets combinedwith fuzzy sets and rough sets: a tentative approach, SoftComput. 14 (6), 899-911 (2010).

[19] George J. Klir and Bo Yuan, Fuzzy sets and Fuzzy logic, Prentice-Hall of India (2000).

[20] John N. Mordeson and D.S. Malik, Fuzzy Commutaive Algebra, World scientific publishing (1998).

[21]Y.B. Jun, Soft BCK/BCI-algebras, Comput. Math. Appl. 56,1408-1413 (2008).

[22] Y.B. Jun, K.J. Lee and J. Zhan, Soft *p*-ideals of soft BCIalgebras,Comput. Math. Appl. 58, 2060-2068 (2009).

[23] O. Kazancı, S¸ . Yılmaz and S. Yamak, Soft sets and softBCH-algebras, Hacet. J. Math. Stat. 39 (2), 205-217 (2010).

[24] X. Ma and J. Zhan, Applications of a new soft set to h-hemi regular hemi rings via (M,N)-SI-h-ideals, J. Intell. Fuzzy Syst. 26, 2515-2525 (2014).

[25] X. Ma and J. Zhan, Characterizations of hemi regular hemi rings via a kind of new soft union sets, J. Intell. Fuzzy Syst. 27, 2883-2895 (2014).

[26] P.K.Maji, R. Biswas and A.R. Roy, Soft set theory, Comput.Math. Appl. 45, 555-562 (2003).

[27] P.K. Maji, A.R. Roy and R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44,1077-1083 (2002).

[28] P.Majumdar and S.K. Samanta, On soft mappings, Comput.Math. Appl. 60 (9), 2666-2672 (2010).

[29] D. Molodtsov, Soft set theory-first results, Comput. Math.Appl. 37, 19-31 (1999).

[30] J. N. Mordeson and D. S Malik., Fuzzy Commutative Algebra, World scientific publishing, (1998).

[31] A. Rosenfeld, Fuzzy groups, J.M. Anal. and Appli. 35,512 - 517 (1971).

[32] A. Sezgin and A. O. Atag¨un, Soft groups and normalisticsoft groups, Comput. Math. Appl. 62 (2), 685-698 (2011).

[33] A. Sezgin, A. O. Atagun, N. Cagman, Union soft substructures of near-rings and N-groups, Neural Comput.Appl. 21, 133-143 (2012).

[34] A. Sezgin and A.O. Atagun, On operations of soft sets, Comput. Math. Appl. 61 (5), 1457-1467 (2011).

[35] A. Sezgin, A.O. Atag¨un and E. Ayg¨un, A note on soft near-rings and idealistic soft near-rings, Filomat 25 (1), 53-68(2011).

[36] Shery Fernandez, Ph.D. thesis “A study of fuzzy g-modules” April 2004.

[37] Shery Fernadez, Fuzzy G-modules and Fuzzy Representations, TAJOPAM 1, 107-114 (2002).

[38] Souriar Sebastian and S. Babu Sundar, On the chains of level subgroups of homomorphic images and pre-images of Fuzzy subgroups, Banyan Mathematical Journal 1,25- 34 (1994).

[39] Souriar Sebastian and S. Babu Sundar,Existence of fuzzy subgroups of every level cardinality upto H0, Fuzzy sets and Systems, 67, 365-368(1994).

[40] Souriar Sebastian and S. Babu Sundar,Generalisations of some results of Das, Fuzzy sets and Systems, 71 , 251-253(1995).

[41] Souriar Sebastian and S. Babu Sundar, Fuzzy groups and group homomorphisms, Fuzzy sets and Systems, 81, 397-401(1996).

[42] Souriar Sebastian and Thampy Abraham, 'A fundamental theorem of Fuzzy Representations' TAJOPAM (To appear).

[43]Thampy Abraham and Souriar Sebastian,Fuzzification of Cayley's and Lagrange's Theorems , J. Comp. and Math. Sci, Vol. 1(1), 41-46 (2009).

[44] L.A. Zadeh, Fuzzy sets, Information Control 8, 338 - 353 (1965).

[45] J. Zhan, N. Cagman, A. Sezgin Sezer, Applications of soft union sets to hemi rings via SU-h-ideals, J. Intell. Fuzzy Syst.26, 1363-1370 (2014).

[46] Y. Zou and Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowled-Based Syst. 21, 941-945 (2008).

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