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k-Neighborhood-prime Labeling of Graphs

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Abstract

In this paper, we investigate the k-neighborhood-prime labeling of the switching of a vertex in cycle C_n, switching of a pendent vertex in path P_n, $W_n \cup T_m$, $B_{n,n}$, $K_{1,n,n}$, $D_2(K_{1,n})$ and $S'(K_{1,n})$.

Keywords: *Neighborhood-Prime Labeling, Neighborhood-Prime graph, k-Neighborhood-Prime Labeling, k-Neighborhood-Prime graph .*

1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [4]. For standard terminology and notations related to number theory we refer to Burton [2] and graph labeling, we refer to Gallian [3]. The notion of prime labeling for graphs originated with Roger Entringer and was introduced in a paper by Tout et al. [8] in the early 1980s and since then it is an active field of research for many scholars. Patel et al.[6] introduce the notion of neighborhood-prime labeling of graph and they present the neighborhood-prime labeling of various graphs in [6,7]. Ananthavalli et al. present the neighborhood-prime labeling of some special graphs in [1]. In [9], Vaidya et al. introduce the concept of k-prime labeling of graphs. Lawrence et al. introduce the notation of k-neighborhood-prime labeling and they present the neighborhood-prime labeling of $G *_B B$, where B is the book with triangular and rectangle pages, $G *_{B_{n,m}} B_{n,m}$, and k- neighborhood - prime labeling of Paths and some special graphs in [5]

Definition 1.1

Let G = (V,E) be a graph with n vertices. A function $f : V(G) \rightarrow \{1,2,3,\ldots,n\}$ is said to be a prime labeling, if it is bijective and for every pair of adjacent vertices u and v, $\gcd(f(u),f(v)) = 1$. A graph which admits prime labeling is called a prime graph.

Definition 1.2

Let G = (V,E) be a graph with n vertices. A bijective function f : $V(G) \rightarrow \{1,2,3,...,n\}$ is said to be a neighborhood-prime labeling, if for every vertex $v \in V(G)$ with $\deg(v) > 1$, $\gcd \{f(u): u \in N(v)\} = 1$. A graph which admits neighborhood-prime labeling is called a neighborhood-prime graph.

Definition 1.3

A k-prime labeling of a graph G is an injective function $f : V \rightarrow \{k, k+1,\ldots, k+|V|-1\}$ for some positive integer k that induces a function $f^+: E(G) \rightarrow N$ of the edges of G defined by $f^+(uv) = \gcd(f(u),f(v))$, $\forall e = uv \in E(G)$ such that $\gcd(f(u), f(v)) = 1$, $\forall e = uv \in E(G)$. The graph which admits a k-prime labeling is called a k-prime graph.

Definition 1.4

Let G = (V(G),E(G)) be a graph with n vertices. A bijective function $f : V(G) \rightarrow \{k, k+1, \ldots, k+n-1\}$ is said to be a k-neighborhood-prime labeling, if for every vertex $v \in V(G)$ with $\deg(v) > 1$, $\gcd \{f(u) : u \in N(v)\} = 1$. A graph which admits k-neighborhood-prime labeling is called a k-neighborhood-prime graph.

Definition 1.5

A complete biparitite graph $K_{1,n}$ is called a star and it has n+1 vertices and n edges. $K_{1,n,n}$ is the graph obtained by the subdivision of the edges of the star $K_{1,n}$.

Definition 1.6

For a graph G the splitting graph $S'(G)$ of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Definition 1.7

Bistar $B_{n,n}$ is the graph obtained by joining the center (apex) vertices of two copies of $K_{1,n}$ by an edge.

Definition 1.8

A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.9

The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.

2. Main Results

Theorem 2.1

Switching of a vertex in cycle C_n is k-neighborhood-prime graph.

Proof.

Let $v_1, v_2, \ldots, v_n$ be the successive vertices of C_n.

G_v denotes graph is obtained by switching of vertex v of $G = C_n$.

Without loss of generality let the switched vertex be v_1.

Then $|V(G_{v_1})| = n$ and $|E(G_{v_1})| = 2n - 5$.

Let p be the largest prime such that $k \le p \le k+n-1$.

Define $f : V(G_{v_1}) \to \{k,k+1,\ldots,k+n-1\}$ as follows:

$f(v_1) = p$,

Label the remaining vertices $v_2,v_3,\ldots,v_n$ by the remaining numbers from k to k+n–1 other than p.

Claim that f is a neighborhood-prime by considering the following two cases.

Sub case (i): $x = v_1$.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) has at least two consecutive integers.

Sub case (ii): $x = v_i$ for $2 \le i \le n$.

Then the gcd of the labels of vertices in N(x) is 1. Since one of the label of vertices in N(x) is p.

Thus f admits k-neighborhood-prime labeling of G.

Hence, the graph obtained by switching of a vertex in cycle C_n is k-neighborhood-prime graph.

Example 2.1

The 3-neighborhood-prime labeling of switching of a vertex in cycle C_8 is shown in figure 2.1.

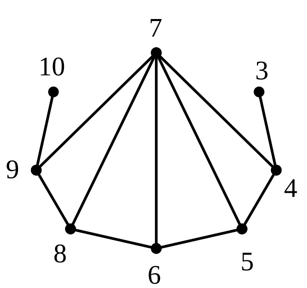

Figure 2.1

Theorem 2.2

Switching of a pendent vertex in path P_n is k-neighborhood-prime graph.

Proof.

Let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n.

The graph G is obtained by switching of a pendent vertex in path P_n. v_1 and v_n are pendent vertex of path P_n.

Without loss of generality, let the switched vertex be v_1.

Then $|V(G)| = n$ and $|E(G)| = 2n - 4$.

Let p be the largest prime such that $k \le p \le k+n-1$.

Define $f : V(G_{v_1}) \to \{k,k+1,\ldots,k+n-1\}$ as follows:

$f(v_1) = p$,

Label the remaining vertices $v_2,v_3,\ldots,v_n$ by the remaining numbers from k to k+n–1 other than p.

Claim that f is a neighborhood-prime by considering the following two cases.

Sub case (i): $x = v_1$.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) has at least two consecutive integers.

Sub case (ii): $x = v_i$ for $3 \le i \le n$.

Then the gcd of the labels of vertices in N(x) is 1. Since one of the label of vertices in N(x) is p.

Thus f admits k-neighborhood-prime labeling of G.

Hence, the graph obtained by switching of a vertex in path P_n is k-neighborhood-prime graph.

Example 2.2

The 5-neighborhood-prime labeling of switching of a pendent vertex in path P_6 is shown in figure 2.2.

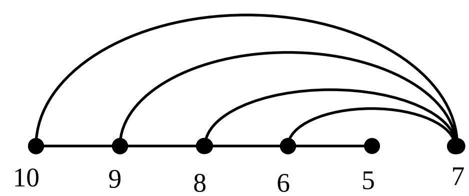

Figure 2.2

Theorem 2.3

The disconnected graph $W_n \cup T_m$ is k-neighborhood-prime graph, where $n \ge 3$ and $m \ge 2$.

Proof.

Let G be a disconnected graph $W_n \cup T_m$.

In W_n, let v be the central vertex and $v_1, v_2, \ldots, v_n$ be the vertices of C_n.

Let $w_1, w_2, \ldots, w_{m-1}, w_m, w_{m+1}, \ldots, w_{2m-1}$ be the 2m–1 vertices of T_m.

Then $|V(G)| = n+2m$ and $|E(G)| = 2n+3m-3$.

Define $f : V(P_n) \to \{k,k+1,\ldots,k+n+2m-1\}$ as follows:

Case 1: k is odd.

Let p be the largest prime such that $k+2m-1 \le p \le k+n+2m-1$.

$f(v) = p$

$f(w_j) = k+1+2(j-1), \qquad 1 \le j \le m-1$

$f(w_j) = k+2(j–1)$, $1 \le j \le m$

Label the remaining vertices $v_1, v_2, ..., v_n$ by the remaining numbers from k+2m–1 to k+n+2m–1 other than p.

Case 2: k is even.

Let p be the largest prime such that $k+2m \le p \le k+n+2m–1$.

$f(v) = p$
$f(v_1) = k$
$f(w_j) = k+2+2(j–1)$, $1 \le j \le m–1$
$f(w_j) = k+1+2(j–1)$, $1 \le j \le m$

Label the remaining vertices $v_1, v_2, ..., v_n$ by the remaining numbers from k+2m–1 to k+n+2m–1 other than p.

Claim that f is a neighborhood-prime labeling for both cases by considering the following three sub cases.

Sub case (i): x = v.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) has at least two consecutive integers.

Sub case (ii): $x = v_i$ for $1 \le i \le n$.

Then the gcd of the labels of vertices in N(x) is 1. Since one of the label of vertices in N(x) is p.

Sub case (iii): $x = w_j$ for $1 \le j \le m–1$.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) are odd consecutive integers.

Sub case (iv): $x = w_j$ for $m \le j \le 2m–1$.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) has at least two consecutive integers.

Therefore, f is a k-neighborhood-prime labeling for both cases.

Thus $W_n \cup T_m$ is k-neighborhood-prime graph, where $n \ge 3$ and $m \ge 2$.

Example 2.3

The 4-neighborhood-prime labelings of paths $W_4 \cup T_4$ is shown in Figure 2.3.

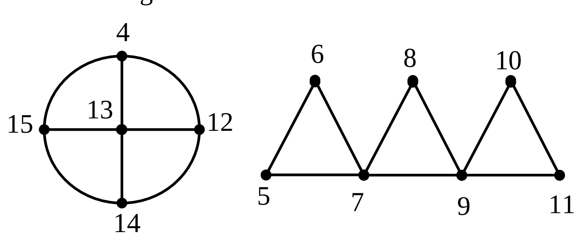

Figure 2.3

Theorem 2.4

The $B_{n,n}$ is k-neighborhood-prime graph, where $n \ge 2$.

Proof.

Let $B_{n,n}$ be a graph with vertex set $\{u,v,u_i,v_i, 1\le i \le n\}$, where u_i, v_i are pendant vertices.

Let G be the graph $B_{n,n}$.

The vertex set $V(G) = \{u, w, u_i, v_i : 1 \le i \le n\}$ and the edge set $E(G) = \{uw, uu_i, vv_i, :1 \le i \le n\}$.

Then $|V(G)| = 2n+2$ and $|E(G)| = 2n+1$.

Define $f : V(G) \rightarrow \{k,k+1,\ldots,k+2n+1\}$ as follows.

$f(u_i) = k+i–1$ for $1\le i \le n$
$f(u) = k+n+1$,
$f(v) = k+n$,
$f(v_i) = k+n+1+i$, for $1\le i \le n$

Claim that f is a neighborhood-prime labeling by considering the following two cases.

Sub case (i): x = u, v.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) has consecutive integers.

Sub case (ii): $x = u_i, v_i$ for $1 \le i \le n$.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) are consecutive integers.

Thus f admits k-neighborhood-prime labeling of G.

Hence, the $B_{n,n}$ is k-neighborhood-prime graph, where $n \ge 2$.

Example 2.4

The 5-neighborhood-prime labeling of $B_{4,4}$ is shown in figure 2.4.

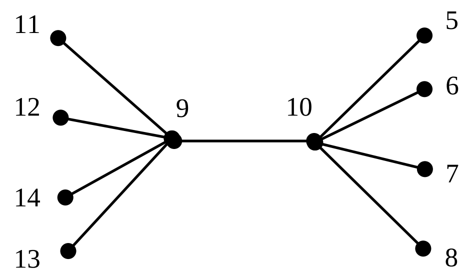

Figure 2.4

Theorem 2.5

The graph $K_{1,n,n}$ is k-neighborhood-prime graph.

Proof

Let G be a $K_{1,n,n}$. Let $V(G) = \{v, v_i, u_i : 1 \le i \le n\}$ and $E(G) = \{vv_i,v_iu_i : 1 \le i \le n\}$.

Then $|V(G)| = 2n+1$ and $|E(G)| = 2n$.

Let p be the largest prime such that $k+n \le p \le k+2n$.

Define $f : V(G) \rightarrow \{k,k+1,\ldots,k+2n\}$ as follows:

$f(v) = p$,
$f(v_i) = k+i–1$, for $1\le i \le n$

Label the remaining vertices $u_1,u_2,...,u_n$ by the remaining numbers from k+n to k+2n other than p.

Claim that f is a neighborhood-prime by considering the following two cases.

Sub case (i): x = v.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) are consecutive integers.

Sub case (ii): $x = v_i$ for $1 \le i \le n$.

Then the gcd of the labels of vertices in N(x) is 1. Since one of the label of vertices in N(x) is p.

Thus f admits k-neighborhood-prime labeling of G.

Hence, the graph $K_{1,n,n}$ is k-neighborhood-prime graph.

Example 2.5

The 8-neighborhood-prime labeling of $K_{1,6,6}$ is shown in figure 2.5.

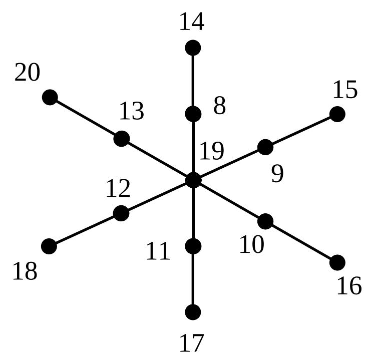

Figure 2.5

Theorem : 2.6

$D_2(K_{1,n})$ is k-neighborhood-prime graph, where $n \ge 2$.

Proof.

Consider two copies of $K_{1,n}$.

Let v, v_1, v_2, …, v_n be the vertices of the first copy of $K_{1,n}$ and $v', v'_1, v'_2, \dots v'_n$ be the vertices of the second copy of $K_{1,n}$ where v and v' are the respective apex vertices.

Let G be $D_2(K_{1,n})$.

Then |V(G)| = 2n+2 and |E(G)| = 2n+1.

Define $f : V(G) \to \{k, k+1, \dots, k+2n+1\}$ as follows.

$f(v) = k$,

$f(v') = k+1$,

$f(v_i) = k+1+i$ for $1 \le i \le n$

$f(v'_i) = k+n+1+i$, for $1 \le i \le n$

Claim that f is a neighborhood-prime labeling by considering the following two cases.

Sub case (i): $x = v, v'$.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) have consecutive integers.

Sub case (ii): $x = v_i, v'_i$ for $1 \le i \le n$.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) are consecutive integers.

Thus f admits k-neighborhood-prime labeling of G.

Hence, the $D_2(K_{1,n})$ is k-neighborhood-prime graph, where $n \ge 2$.

Example 2.6

The 9-neighborhood-prime labeling of $D_2(K_{1,4})$ is given in Figure 2.6.

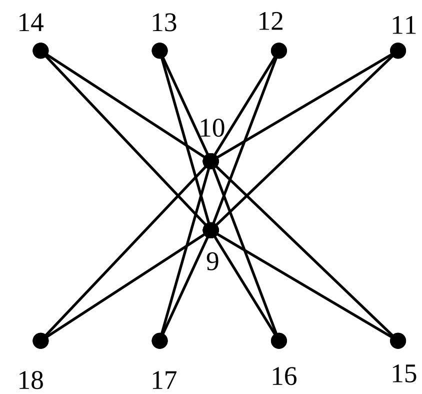

Figure 2.6

Theorem : 2.7

The graph $S'(K_{1,n})$ is k-neighborhood-prime graph, where $n \ge 2$.

Proof.

Let $v_1, v_2, v_3, \dots, v_n$ be the pendant vertices and v be the apex vertex of $K_{1,n}$ and u, u_1, u_2, u_3, …, u_n are added vertices corresponding to v, v_1, v_2, v_3, …, v_n to obtain $S'(K_{1,n})$.

Let G be the graph $S'(K_{1,n})$

Then |V(G)| = 2n+2 and |E(G)| = 3n.

Define $f : V(G) \to \{k, k+1, \dots, k+2n+1\}$ as follows.

$f(v) = k$,

$f(v_i) = k+1+i$ for $1 \le i \le n$

$f(u) = k+1$,

$f(u_i) = k+n+1+i$, for $1 \le i \le n$

Claim that f is a neighborhood-prime labeling by considering the following two cases.

Sub case (i): x = v, u.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) have consecutive integers.

Sub case (ii): $x = u_i$ for $1 \le i \le n$.

Then the gcd of the labels of vertices in N(x) is 1. Since the label of vertices in N(x) are consecutive integers.

Thus f admits k-neighborhood-prime labeling of G.

Hence, the $D_2(K_{1,n})$ is k-neighborhood-prime graph, where $n \ge 2$.

Example 2.7

The 7-neighborhood-prime labeling of $D_2(K_{1,4})$ is given in Figure 2.7.

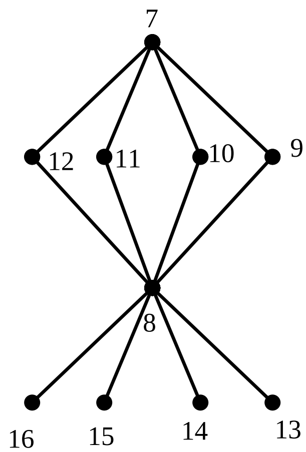

Figure 2.7

4. Conclusions

In this paper, In this paper, we present investigate the k-neighborhood-prime labeling of the switching of a vertex in cycle C_n, switching of a pendent vertex in path P_n, $W_n \cup T_m$, $B_{n,n}$, $K_{1,n,n}$, $D_2(K_{1,n})$ and $S'(K_{1,n})$.

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