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To the Qualitative Properties of Solution of System Equations not in Divergence Form

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Abstract In this paper the properties of solutions of nonlinear systems of parabolic equations not in divergence form

$$|x|^n \frac{\partial u}{\partial t} = u^{\gamma_1} \nabla\left(|\nabla u|^{p-2} \nabla u\right) + |x|^n v^{q_1},$$
$$|x|^n \frac{\partial v}{\partial t} = v^{\gamma_2} \nabla\left(|\nabla v|^{p-2} \nabla v\right) + |x|^n u^{q_2},$$

are studied. In this work used: method of nonlinear splitting, known previously for non-linear parabolic equations and systems of equations in divergence form, asymptotic theory and asymptotic methods based on different transformations. Constructed asymptotic representation of self-similar solutions of nonlinear parabolic systems of equations not in divergence form, depending on the value in the system of the numerical parameters necessary and sufficient signs of their existence. The main purpose of this paper is to find conditions for the existence and non-existence results for global solutions of parabolic equations not in divergence form on the basis of the self-similar analysis.

Keywords *nonlinear parabolic systems of equations, not in divergence form, global solutions, self-similar solutions, asymptotic representation of solution*

1 Introduction

Consider in $Q = \{(t, x): \ t > 0, \ x \in R^N\}$ parabolic system of two quasilinear equations not in divergence form

$$|x|^n \frac{\partial u}{\partial t} = u^{\gamma_1} \nabla\left(|\nabla u|^{p-2} \nabla u\right) + |x|^n v^{q_1}, \quad |x|^n \frac{\partial v}{\partial t} = v^{\gamma_2} \nabla\left(|\nabla v|^{p-2} \nabla v\right) + |x|^n u^{q_2}, \tag{1}$$

$$u|_{t=0} = u_0(x) \geq 0, \ \ v|_{t=0} = v_0(x) \geq 0, \ \ \forall x \in R^N \tag{2}$$

where $n, \ p, \ \gamma_i, \ q_i \ (i = 1, 2)-$ the numerical parameters set, $\nabla(\cdot) = grad_x(\cdot)$, t and $x \in R^N-$ respectively, the temporal and spatial coordinates, $u = u(t, x) \geq 0$, $v = v(t, x) \geq 0 -$ are the solutions.

The numerical parameter n characterizes the variable density of the nonlinear medium. The system of equations (1) describes the process of polytrophic filtration in a nonlinear two-componential medium with variable density. The system of equations (1) is called the system of equations of polytrophic filtration, two-componential nonlinear medium.

In this system $u \geq 0, \ v \geq 0-$ means the pressure, $|\nabla u|^{p-2}\nabla u, |\nabla v|^{p-2}\nabla v-$ filtration flow, $u^{q_2}, \ v^{q_1}-$ power volume filtration sources.

The system of equations (1) describes many physical phenomena [2–10]. In particular, at $\gamma_i = 2, \ p = 2, \ q_i = 3, \ n = 0$ for a single equation in (1) it is encountered in plasma physics [2]. This problem, when $N = 1$, arises in a model for the resistive diffusion of a force-free magnetic field in a plasma confined between two walls $\{0 < z < \infty\}$. The magnetic field has the form $B_0(\cos\phi, \sin\phi, 0)$ with B_0 constant and $\phi = \phi(z, t)$.

In [3] Zhou and Yao are studied the Cauchy problem (1)-(2) for $p = 2, \ n = 0$ and the absence of absorption, proved the existence of a single viscous solutions, and in [4] Wang is investigated the existence and uniqueness of a classical solution of the Cauchy problem for $p = 2, \ n = 0$.

In [5] Wang and Wei are considered a degenerate nonlinear parabolic system with localized source $u_t = u^\alpha(\Delta u + u^p(x,t) v^q(x_0,t))$, $v_t = v^\beta(\Delta v + v^m(x,t) u^n(x_0,t))$. In [5] deals with blow-up properties for a degenerate parabolic system with nonlinear localized sources subject to the homogeneous Dirichlet boundary conditions. The main aim of [5] is to study the blow-up rate estimate and the uniform blow-up profile of the blow-up solution. At the end, the blow-up set and blow up rate with respect to the radial variable is considered when the domain Q is a ball.

In [6] Zhi and Li studied the nonlinear degenerate parabolic system $u_t = v^{\gamma_1}(u_{xx} + au)$, $v_t = u^{\gamma_2}(v_{xx} + bv)$ with Dirichlet boundary condition . The regularization method and upper-lower solutions technique are employed to show the local existence of a solution for the nonlinear degenerate parabolic system. The global existence of a solution is discussed. The finite time blow-up result together with an estimate of the blow-up time are found. The blow-up set with positive measure is analyzed in some detail.

In [7] Lu is deals with positive solutions of the following quasilinear parabolic systems not in divergence form:

$$u_{it} = f_i\left(u_{i+1}\right)\left(\Delta u_i + a_i u_i\right),\ i = 1, 2, ..., n-1,$$
$$u_{nt} = f_n\left(u_1\right)\left(\Delta u_n + a_n u_n\right),\ x \in \Omega,\ t > 0$$

with homogenous Dirichlet boundary condition and positive initial condition, where $a_i \geq 0\ (i = 1, 2, ..., n)$ and $f_i\,(i = 1, 2, ..., n)$ satisfy to some conditions are studied. The local existence and uniqueness of classical solution are proved. Moreover, it is proved that: (i) when $\min\{a_1, ..., a_n\} \leq \lambda_1$ then there exists global positive classical solution, and all positive classical solutions can not blow up in finite time in the meaning of maximum norm; (ii) when $\min\{a_1, ..., a_n\} > \lambda_1$ and the initial datum $(u_{10}, u_{20}, ..., u_{n0})$ satisfies some assumptions, then the positive classical solution is unique and blows up in finite time, where λ_1 is the first eigenvalue of $-\Delta$ in Ω with homogeneous Dirichlet boundary condition.

In [8], Chunhua and Jingxue are concerned with the self-similar solutions of the form

$$u\left(t, x\right) = \left(t+1\right)^{-\alpha} f\left(\left(t+1\right)^{\beta} |x|^2\right)$$

for the following degenerate and singular parabolic equation in non-divergence form

$$\frac{\partial u}{\partial t} = u^m div\left(|\nabla u|^{p-2}\nabla u\right),\ \ m \geq 1,\, p > 1.$$

First established the existence and uniqueness of solutions f with compact supports, which implies that the self-similar solution is shrink. On the basis of this, also established the convergent rates of these solutions on the boundary of the supports. On the other hands, also considered the convergent speeds of solutions, and compare which with Dirac function as t tends to infinity.

In [9], Raimbekov studied some properties of the solutions of the Cauchy problem for a nonlinear parabolic equations in non-divergence form with variable density $|x|^n \frac{\partial u}{\partial t} = u^m div\left(|\nabla u|^{p-2}\nabla u\right),\ p > 1,\ 0 \leq m < \frac{(p-2)(N+n)+p+n}{p-N}$ received self-similar solution Barenblatt-Zeldovich-Kompaneets type and methods of the theory of comparisons prove the asymptotic behavior of solutions in the case of fast and slow diffusion. This article also gives some comparative numerical results for the case $m = 0, m = 1$ and $m = 1,5$. Using this result the author talks about the properties of the finite speed of propagation of heat for divergent equations and localization for non-divergent case.

Aripov and Matyakubov [10] studied the asymptotic behavior of self-similar solutions of a parabolic equation (1) for the case $n = 0$. Constructed asymptotic representation of self-similar solutions of nonlinear parabolic systems of equations not in divergence form, depending on the value in the system of the numerical parameters necessary and sufficient signs of their existence.

In this paper using self-similar approach we find a particular solution of the system (1), and it is proved this solution asymptotic of compactly supported solutions. The main purpose of this paper is to find conditions for the existence and non-existence results for global solutions of problem (1)-(2) on the basis of the self-similar analysis [1,12].

2 Self-similar system of equations (1)

Transform the system (1) to the relatively easy to study mind. To receive this auxiliary system of equations is applicable to a system (1) the following transformation

$$u\left(x,t\right) = \left(t+T\right)^{-\alpha_1} f\left(\xi\right),$$
$$v\left(x,t\right) = \left(t+T\right)^{-\alpha_2} \varphi\left(\xi\right), \tag{3}$$
$$\xi = \left(t+T\right)^{-\gamma} |x|,$$

where $\alpha_1 = -\frac{1+q_1}{1-q_1 q_2},\ \alpha_2 = -\frac{1+q_2}{1-q_1 q_2},\ T > 0,$
$n\gamma = 1 + \frac{(1+q_1)(p+\gamma_1-2)}{1-q_1 q_2},\ \ \alpha_1\left(p+\gamma_1-2\right) = \alpha_2\left(p+\gamma_2-2\right)$, we get the self-similar system of equations

$$f^{\gamma_1}\xi^{1-N}\frac{d}{d\xi}\left(\xi^{N-1}\left|\frac{df}{d\xi}\right|^{p-2}\frac{df}{d\xi}\right) + \alpha_1\xi^n f +$$
$$+\gamma\xi^{n+1}\frac{df}{d\xi} + \xi^n\varphi^{q_1} = 0,$$
$$\varphi^{\gamma_2}\xi^{1-N}\frac{d}{d\xi}\left(\xi^{N-1}\left|\frac{d\varphi}{d\xi}\right|^{p-2}\frac{d\varphi}{d\xi}\right) + \alpha_2\xi^n\varphi + \tag{4}$$
$$+\gamma\xi^{n+1}\frac{d\varphi}{d\xi} + \xi^n f^{q_2} = 0.$$

The case $n = 0$ was considered in [10]. In the work [11], the qualitative properties of solutions of system (4) in divergence form are studied based on the self-similar and approximately self-similar approach, one way of construction of the critical exponent and property finite speed of perturbation (FSP) for system (1) are established.

3 Slowly diffusion case: $p+\gamma_i-2 > 0,\ i = 1, 2$. A global solvability of solutions

We prove properties of a global solvability of weak solutions of the system (1) using a comparison principle [13]. For this goal, we construct a new system of equation using the standard equation method as in [1,12]:

$$u_+\left(t,x\right) = \left(t+T\right)^{-\alpha_1}\overline{f}\left(\xi\right),$$
$$v_+\left(t,x\right) = \left(t+T\right)^{-\alpha_2}\overline{\varphi}\left(\xi\right), \tag{5}$$

where $\alpha_1 = \frac{1+q_1}{q_1 q_2 - 1},\ \alpha_2 = \frac{1+q_2}{q_1 q_2 - 1},\ T > 0,$
$\xi = \left(t+T\right)^{-\gamma}|x|,\ n\gamma = 1 + \alpha_1\left(p+\gamma_1-2\right).$
In the case, $\alpha_1\left(p+\gamma_1-2\right) = \alpha_2\left(p+\gamma_2-2\right)$

$$\overline{f}\left(\xi\right) = A_1\left(a - \xi^{\frac{p+n}{p-1}}\right)_+^{\frac{p-1}{p+\gamma_1-2}},$$
$$\overline{\varphi}\left(\xi\right) = A_2\left(a - \xi^{\frac{p+n}{p-1}}\right)_+^{\frac{p-1}{p+\gamma_2-2}}, \tag{6}$$

where $a > 0,\ A_i = \left(\frac{\gamma(p+\gamma_i-2)^{p-1}}{(1-\gamma_i)(p+n)^{p-1}}\right)^{\frac{1}{p+\gamma_i-2}},\ i = 1, 2,$
$b_+ = \max\left(0, b\right)$.
We introduce the notations:

$k_i = \frac{(p-1)q_i}{p+\gamma_{3-i}-2} - \frac{p-1}{p+\gamma_i-2}, \quad h_i = \frac{p+n+(1-n)\gamma_i-2}{n(1-\gamma_i)}$,
$m_i = A_i^{-1} A_{3-i}^{q_i}, \ i = 1, 2.$

Theorem 1. Let $p + \gamma_i - 2 > 0$, $q_i > \frac{p+\gamma_{3-i}-2}{p+\gamma_i-2}$,

$$-\frac{N+n}{n(1-\gamma_i)} + \frac{(1+q_i)\,h_i}{q_1 q_2 - 1} + m_i a^{k_i} \leq 0, \quad i = 1, 2,$$

$$u_+(0,x) \geq u_0(x), \quad v_+(0,x) \geq v_0(x), \quad x \in R^N.$$

Then for sufficiently small $u_0(x)$, $v_0(x)$ the followings holds

$$u(t,x) \leq u_+(t,x), \ v(t,x) \leq v_+(t,x) \quad in \ \ Q, \qquad (7)$$

where the functions $u_+(t,x)$, $v_+(t,x)$ defined as above.

Proof. Theorem 1 is proved by the method of comparison of solutions. As a comparison solution we take the functions $u_+(t,x)$, $v_+(t,x)$. Substituting (5) in (1) we obtain the following inequality

$$\begin{gathered} \overline{f}^{\gamma_1}\xi^{1-N}\frac{d}{d\xi}\left(\xi^{N-1}\left|\frac{d\overline{f}}{d\xi}\right|^{p-2}\frac{d\overline{f}}{d\xi}\right) + \alpha_1 \xi^n \overline{f} + \\ +\gamma\xi^{n+1}\frac{d\overline{f}}{d\xi} + \xi^n \overline{\varphi}^{q_1} \leq 0, \\ \overline{\varphi}^{\gamma_2}\xi^{1-N}\frac{d}{d\xi}\left(\xi^{N-1}\left|\frac{d\overline{\varphi}}{d\xi}\right|^{p-2}\frac{d\overline{\varphi}}{d\xi}\right) + \alpha_2 \xi^n \overline{\varphi} + \\ +\gamma\xi^{n+1}\frac{d\overline{\varphi}}{d\xi} + \xi^n \overline{f}^{q_2} \leq 0. \end{gathered} \qquad (8)$$

Given the specific form (6) of the functions $\overline{f}(\xi)$, $\overline{\varphi}(\xi)$ inequality (8) can be rewritten as follows:

$$-\frac{N+n}{n(1-\gamma_1)} + \frac{(1+q_1)\,h_1}{q_1 q_2 - 1} + m_1\left(a - \xi^{\frac{p+n}{p-1}}\right)^{k_1} \leq 0,$$

$$-\frac{N+n}{n(1-\gamma_2)} + \frac{(1+q_2)\,h_2}{q_1 q_2 - 1} + m_2\left(a - \xi^{\frac{p+n}{p-1}}\right)^{k_2} \leq 0.$$

It is easy to check that $m_1\left(a - \xi^{\frac{p+n}{p-1}}\right)^{k_1} \leq m_1 a^{k_1}$, $m_2\left(a - \xi^{\frac{p+n}{p-1}}\right)^{k_2} \leq m_2 a^{k_2}$.

Then according to the hypotheses of Theorem 1 and comparison principle we have
$u(t,x) \leq u_+(t,x)$, $v(t,x) \leq v_+(t,x)$ in Q,
if $u_+(0,x) \geq u_0(x)$, $v_+(0,x) \geq v_0(x)$, $x \in R^N$.
The proof of the theorem is complete.

4 Asymptotic of the self-similar solutions

Next, we study the asymptotic behavior of the self-similar solutions of the system (4). Self-similar solution of system equations (4) will search for in the form

$$f(\xi) = \overline{f}(\xi) y(\eta), \quad \varphi(\xi) = \overline{\varphi}(\xi) z(\eta), \qquad (9)$$

where $\eta = -\ln\left(a - \xi^{\frac{p+n}{p-1}}\right)$, $\overline{f}(\xi) = \left(a - \xi^{\frac{p+n}{p-1}}\right)^{\frac{p-1}{p+\gamma_1-2}}$, $\overline{\varphi}(\xi) = \left(a - \xi^{\frac{p+n}{p-1}}\right)^{\frac{p-1}{p+\gamma_2-2}}$, $a > 0$.

Then substituting (9) into (4) for the function $y(\eta) > 0$, $z(\eta) > 0$ we have the following nonlinear system of equations

$$\begin{gathered} y^{\gamma_1}\frac{d}{d\eta}(L_1 y) + a_{11}(\eta)\, y^{\gamma_1}(L_1 y) + a_{13}(\eta)\, z^{q_1} + \\ +a_{12}(\eta)\left(\frac{dy}{d\eta} + a_{10}(\eta)\, y\right) + a_{14}(\eta)\, y = 0, \\ z^{\gamma_2}\frac{d}{d\eta}(L_2 z) + a_{21}(\eta)\, z^{\gamma_2}(L_2 z) + a_{23}(\eta)\, y^{q_2} + \\ +a_{22}(\eta)\left(\frac{dz}{d\eta} + a_{20}(\eta)\, z\right) + a_{24}(\eta)\, z = 0. \end{gathered} \qquad (10)$$

Here $a_{i0}(\eta) = -\frac{p-1}{p+\gamma_i-2}$, $a_{i2}(\eta) = \gamma\left(\frac{p-1}{p+n}\right)^{p-1}$,
$a_{i1}(\eta) = \frac{(N+n)(p-1)}{p+n}\frac{e^{-\eta}}{a-e^{-\eta}} - \frac{(p-1)(1-\gamma_i)}{p+\gamma_i-2}$,
$a_{i4}(\eta) = \alpha_i\left(\frac{p-1}{p+n}\right)^p$, $a_{i3}(\eta) = \left(\frac{p-1}{p+n}\right)^p \frac{e^{-s_i\eta}}{a-e^{-\eta}}$,
$s_i = 1 + \frac{(p-1)q_i}{p+\gamma_{3-i}-2} - \frac{p-1}{p+\gamma_i-2} \quad (i = 1, 2).$
$L_1 y = \left|\frac{dy}{d\eta} + a_{10}(\eta)\, y\right|^{p-2}\left(\frac{dy}{d\eta} + a_{10}(\eta)\, y\right)$,
$L_2 z = \left|\frac{dz}{d\eta} + a_{20}(\eta)\, z\right|^{p-2}\left(\frac{dz}{d\eta} + a_{20}(\eta)\, z\right)$.

There was supposed to $\xi \in [\xi_0, \ \xi_1)$, $0 < \xi_0 < \xi_1$, $\xi_1 = a^{\frac{p-1}{p+n}}$.

Therefore, the function $\eta(\xi)$ has properties: $\eta'(\xi) > 0$ at $\xi \in [\xi_0, \xi_1)$, $\eta_0 = \eta(\xi_0) > 0$, $\lim_{\xi_0 \to \xi_1} \eta(\xi) = +\infty$.

Further, in what follows the auxiliary system of equations (7) is investigated in the following limited:

$$\lim_{\eta \to +\infty} a_{ij}(\eta) = a_{ij}^0 \quad (\mathrm{i} = 1, 2; \mathrm{j} = 0, 1, 2, 3, 4)$$

are exists, finite and nonzero, those $0 < \left|a_{ij}^0\right| < +\infty$.

Through the introduction of transformations (3), (9) and properties $\eta \to +\infty$, study of the solutions of (1) reduced to the study of the solutions of (10), each of which is in a neighborhood $+\infty$ satisfies the inequalities

$$y(\eta) > 0, \mathrm{y}' + a_{10}(\eta) y \neq 0 \ ,$$

$$z(\eta) > 0, \mathrm{z}' + a_{20}(\eta) z \neq 0 \ .$$

We now study the asymptotic behavior of positive, have nonzero a finite limit as $\eta \to +\infty$ solutions of (10).

5 The main results

We introduce the notations:
$c_{i1} = \frac{1-\gamma_i}{(p+\gamma_i-2)^p}$, $c_{i2} = \frac{1}{(p+n)^{p-1}}\left(\frac{\alpha_i}{p+n} - \frac{\gamma}{p+\gamma_i-2}\right)$,
$c_{i3} = \frac{1}{(p+n)^p a}$, $(i = 1, 2)$.

Let $y(\eta) = y^0 + o(1)$, $z(\eta) = z^0 + o(1)$ at $\eta \to +\infty$ and is performed the equality $(1 + q_1)(\gamma_1 + p - 2) = (1 + q_2)(\gamma_2 + p - 2)$.

Then are valid the following theorem:

Theorem 2. Let $s_1 = 0$, $s_2 = 0$. Then the self-similar solution of equation (1) has the asymptotic

$$u_A(t,x) = y^0(T+t)^{\frac{1+q_1}{1-q_1q_2}} \times \times\left(a - \left(\frac{|x|}{(t+T)^\gamma}\right)^{\frac{p+n}{p-1}}\right)_+^{\frac{p-1}{p+\gamma_1-2}} (1+o(1)),$$
$$v_A(t,x) = z^0(T+t)^{\frac{1+q_2}{1-q_1q_2}} \times \times\left(a - \left(\frac{|x|}{(t+T)^\gamma}\right)^{\frac{p+n}{p-1}}\right)_+^{\frac{p-1}{p+\gamma_2-2}} (1+o(1)), \quad (11)$$

at $|x| \to a^{\frac{p-1}{p+n}}(t+T)^\gamma$, where $0 < y^0 < +\infty$, $0 < z^0 < +\infty$ and y^0, z^0 are the respectively roots w_1, w_2 the system of nonlinear algebraic equations

$$c_{i1}w_i^{p+\gamma_i-1} + c_{i2}w_i + c_{i3}w_{3-i}^{q_i} = 0 \quad (i=1,2). \quad (12)$$

Theorem 3. Let $s_1 = 0$, $s_2 > 0$. Then the self-similar solution of equation (1) has the asymptotic at $|x| \to a^{\frac{p-1}{p+n}}(t+T)^\gamma$ form (11), where $0 < y^0 < +\infty$, $0 < z^0 < +\infty$ and y^0, z^0 are the respectively roots w_1, w_2 the system of nonlinear algebraic equations

$$\begin{aligned} c_{11}w_1^{p+\gamma_1-1} + c_{12}w_1 + c_{13}w_2^{q_1} &= 0, \\ c_{21}w_2^{p+\gamma_2-1} + c_{22}w_2 &= 0. \end{aligned}$$

Theorem 4. Let $s_1 > 0$, $s_2 = 0$. Then the self-similar solution of equation (1) has the asymptotic at $|x| \to a^{\frac{p-1}{p+n}}(t+T)^\gamma$ form (11), where $0 < y^0 < +\infty$, $0 < z^0 < +\infty$ and y^0, z^0 are the respectively roots w_1, w_2 the system of nonlinear algebraic equations

$$\begin{aligned} c_{11}w_1^{p+\gamma_1-2} + c_{12}w_1 &= 0, \\ c_{21}w_2^{p+\gamma_2-2} + c_{22}w_2 + c_{23}w_1^{q_2} &= 0. \end{aligned}$$

Theorem 5. Let $s_1 > 0$, $s_2 > 0$. Then the self-similar solution of equation (1) has the asymptotic at $|x| \to a^{\frac{p-1}{p+n}}(t+T)^\gamma$ form (11), where $0 < y^0 < +\infty$, $0 < z^0 < +\infty$ and y^0, z^0 are the respectively roots w_1, w_2 the system of nonlinear algebraic equations

$$\begin{aligned} c_{11}w_1^{p+\gamma_1-2} + c_{12}w_1 &= 0, \\ c_{21}w_2^{p+\gamma_2-2} + c_{22}w_2 &= 0. \end{aligned}$$

The proof. Assuming that the system (10)

$$\vartheta_1(\eta) = L_1 y, \ \vartheta_2(\eta) = L_2 z \quad (13)$$

obtain the identity

$$\begin{aligned} \vartheta_1'(\eta) \equiv & -a_{11}(\eta)\vartheta_1(\eta) - a_{12}(\eta)y^{-\gamma_1}\vartheta_1^{\frac{1}{p-1}}(\eta) - \\ & -a_{13}(\eta)y^{-\gamma_1}z^{q_1} - a_{14}(\eta)y^{1-\gamma_1}, \\ \vartheta_2'(\eta) \equiv & -a_{21}(\eta)\vartheta_2(\eta) - a_{22}(\eta)z^{-\gamma_2}\vartheta_2^{\frac{1}{p-1}}(\eta) - \\ & -a_{23}(\eta)z^{-\gamma_2}y^{q_2} - a_{24}(\eta)z^{1-\gamma_2}. \end{aligned} \quad (14)$$

Now consider the function

$$\begin{aligned} g_1(\lambda_1,\eta) \equiv & -a_{11}(\eta)\lambda_1 - a_{12}(\eta)y^{-\gamma_1}\lambda_1^{\frac{1}{p-1}} - \\ & -a_{13}(\eta)y^{-\gamma_1}z^{q_1} - a_{14}(\eta)y^{1-\gamma_1}, \\ g_2(\lambda_2,\eta) \equiv & -a_{21}(\eta)\lambda_2 - a_{22}(\eta)z^{-\gamma_2}\lambda_2^{\frac{1}{p-1}} - \\ & -a_{23}(\eta)z^{-\gamma_2}y^{q_2} - a_{24}(\eta)z^{1-\gamma_2}. \end{aligned} \quad (15)$$

where $\lambda_i \in R$, $(i=1,2)$.

Suppose first $s_i = 0$ $(i=1,2)$. Then the functions $g_i(\lambda_i,\eta)$ $(i=1,2)$ preserves a sign on an interval $[\eta_1, +\infty) \subset [\eta_0, +\infty)$ for each fixed value λ_i $(i=1,2)$, different from the values satisfying the system

$$\begin{aligned} & -a_{11}^0\lambda_1 - a_{12}^0(y^0)^{-\gamma_1}\lambda_1^{\frac{1}{p-1}} - a_{13}^0(y^0)^{-\gamma_1}(z^0)^{q_1} - \\ & - a_{14}^0(y^0)^{1-\gamma_1} = 0, \\ & -a_{21}^0\lambda_2 - a_{22}^0(z^0)^{-\gamma_2}\lambda_2^{\frac{1}{p-1}} - a_{23}^0(z^0)^{-\gamma_2}(y^0)^{q_2} - \\ & - a_{24}^0(z^0)^{1-\gamma_2} = 0. \end{aligned}$$

Now let $s_i > 0$ $(i=1,2)$. It is easy to see that the functions $g_i(\lambda_i,\eta)$ $(i=1,2)$ for each fixed value λ_i $(i=1,2)$, different from the values satisfying the system

$$\begin{aligned} -a_{11}^0\lambda_1 - a_{12}^0(y^0)^{-\gamma_1}\lambda_1^{\frac{1}{p-1}} - a_{14}^0(y^0)^{1-\gamma_1} &= 0, \\ -a_{21}^0\lambda_2 - a_{22}^0(z^0)^{-\gamma_2}\lambda_2^{\frac{1}{p-1}} - a_{24}^0(z^0)^{1-\gamma_2} &= 0. \end{aligned}$$

preserves a sign on an interval $[\eta_2, +\infty) \subset [\eta_0, +\infty)$.

And in the case $s_i < 0$ $(i=1,2)$ the functions $g_i(\lambda_i,\eta)$ $(i=1,2)$ rewritten in the following form

$$\begin{aligned} g_1(\lambda_1,\eta) \equiv & -a_{11}(\eta)\lambda_1 - a_{12}(\eta)y^{-\gamma_1}\lambda_1^{\frac{1}{p-1}} - \\ & -a_{13}(\eta)y^{1-\gamma_1}\left(y^{-1}z^{q_1} - a_{14}(\eta)a_{13}^{-1}(\eta)\right), \\ g_2(\lambda_2,\eta) \equiv & -a_{21}(\eta)\lambda_2 - a_{22}(\eta)z^{-\gamma_2}\lambda_2^{\frac{1}{p-1}} - \\ & -a_{23}(\eta)z^{1-\gamma_2}\left(z^{-1}y^{q_2} - a_{24}(\eta)a_{23}^{-1}(\eta)\right). \end{aligned}$$

From here mean $\lim\limits_{\eta\to+\infty} a_{i1}(\eta) = -\frac{(p-1)(1-\gamma_i)}{p+\gamma_i-2}$, $\lim\limits_{\eta\to+\infty} a_{i2}(\eta) = \gamma\left(\frac{p-1}{p+n}\right)^{p-1}$, $\lim\limits_{\eta\to+\infty} a_{i3}(\eta) = \infty$, $\lim\limits_{\eta\to+\infty} a_{i4}(\eta) = \alpha_i\left(\frac{p-1}{p+n}\right)^p$ $(i=1,2)$ implies that the functions $g_i(\lambda_i,\eta)$ $(i=1,2)$ preserve sign on the interval $[\eta_2, +\infty) \subset [\eta_0, +\infty)$, where $\lambda_i \neq 0$ $(i=1,2)$. That means the functions $g_i(\lambda_i,\eta)$ $(i=1,2)$ for all $\eta \in [\eta_i, +\infty)$ $(i=1,2)$ satisfy one of the inequalities

$$g_i(\lambda_i,\eta) > 0 \quad or \quad g_i(\lambda_i,\eta) < 0 \quad (i=1,2). \quad (16)$$

Suppose now that for the functions $\vartheta_i(\eta)$ $(i=1,2)$ limit as $\eta \to +\infty$ does not exists. Consider the case where executed any of the inequalities (16). In view the oscillation of the functions $\vartheta_i(\eta)$ $(i=1,2)$ straight line $\overline{\vartheta}_i = \lambda_i$ $(i=1,2)$ its graph intersects the infinite number of times in the interval $[\eta_i, +\infty)$ $(i=1,2)$. But this is impossible, since on the interval $[\eta_i, +\infty)$ $(i=1,2)$ rightly one of the inequalities (16) and, therefore, the identity (15) follows, that the graph of the functions $\vartheta_i(\eta)$ $(i=1,2)$ intersects the straight lines $\overline{\vartheta}_i = \lambda_i$ $(i=1,2)$ only once on the interval $[\eta_i, +\infty)$ $(i=1,2)$. Therefore, for the functions $\vartheta_i(\eta)$ $(i=1,2)$ are exists limit at $\eta \to +\infty$.

By assumption, $y(\eta) = y^0 + o(1)$, $z(\eta) = z^0 + o(1)$ at $\eta \to +\infty$, and the functions $\vartheta_i(\eta)$ $(i=1,2)$ identified in accordance with (13) and has a limit at $\eta \to +\infty$. Then $y'(\eta)$ and $z'(\eta)$ has a limit at $\eta \to +\infty$, and is equal to zero.

Then

$$\begin{aligned} \vartheta_1(\eta) &= \left|\frac{dy}{d\eta} + a_{10}(\eta)\,y\right|^{p-2}\left(\frac{dy}{d\eta} + a_{10}(\eta)\,y\right) = \\ &= \left|a_{10}^0 y^0\right|^{p-2} a_{10}^0 y^0 + o(1), \\ \vartheta_2(\eta) &= \left|\frac{dz}{d\eta} + a_{20}(\eta)\,z\right|^{p-2}\left(\frac{dz}{d\eta} + a_{20}(\eta)\,z\right) = \\ &= \left|a_{20}^0 z^0\right|^{p-2} a_{20}^0 z^0 + o(1) \end{aligned}$$

at $\eta \to +\infty$ and by (14) derivative of functions $\vartheta_i(\eta)$ $(i = 1, 2)$ has a limit at $\eta \to +\infty$, which is obviously equal to zero.

Therefore, it is necessary in order to

$$\lim_{\eta\to+\infty}\left(a_{11}(\eta)\vartheta_1(\eta)+a_{12}(\eta)y^{-\gamma_1}\vartheta_1^{\frac{1}{p-1}}(\eta)\right)+$$

$$+\lim_{\eta\to+\infty}\left(a_{13}(\eta)y^{-\gamma_1}z^{q_1}+a_{14}(\eta)y^{1-\gamma_1}\right)=0\,,$$

$$\lim_{\eta\to+\infty}\left(a_{21}(\eta)\vartheta_2(\eta)+a_{22}(\eta)z^{-\gamma_2}\vartheta_2^{\frac{1}{p-1}}(\eta)\right)+$$

$$+\lim_{\eta\to+\infty}\left(a_{23}(\eta)z^{-\gamma_2}y^{q_2}+a_{24}(\eta)z^{1-\gamma_2}\right)=0\,.$$

From here easy to be convinced in the fact that at $s_i < 0$ $(i = 1, 2)$ system (13) can not have solutions $(y(\eta), z(\eta))$ with a finite non-zero limit, at $\eta \to +\infty$, and at $s_i \geq 0$ $(i = 1, 2)$ for the existence of such solutions is necessary, order to comply with the conditions of the theorem 2,3,4,5.

Consequently, by the transformations introduced by (3) and (9), self-similar solution of the system equation (1) has an asymptotic at $|x| \to a^{\frac{p-1}{p+n}}(t+T)^{\gamma}$ and has the following form

$$u_A(t,x)\simeq y^0\,(T+t)^{\frac{1+q_1}{1-q_1q_2}}\left(a-\left(\frac{|x|}{(t+T)^{\gamma}}\right)^{\frac{p+n}{p-1}}\right)_+^{\frac{p-1}{p+\gamma_1-2}}$$

$$v_A(t,x)\simeq z^0(T+t)^{\frac{1+q_2}{1-q_1q_2}}\left(a-\left(\frac{|x|}{(t+T)^{\gamma}}\right)^{\frac{p+n}{p-1}}\right)_+^{\frac{p-1}{p+\gamma_2-2}}.$$

The theorems are proved.

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